

Dr. Heckl István:

Theory of Digital Computation practice course



**A felsőfokú informatikai oktatás
minőségének fejlesztése,
modernizációja**

TÁMOP-4.1.2.A/1-11/1-2011-0104



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Szegedi Tudományegyetem
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2014



MAGYARORSZÁG MEGÚJUL



A projekt az Európai Unió támogatásával, az Európai Szociális Alap társfinanszírozásával valósul meg.

Element of the Theory of the Computation

Lecture 1

Sets: Example 1

- Decide whether the following statements are true or not!
 - $5 \in \{1, 2, 3, 4, 5\}$
 - $0 \in \{1, 2, 3, 4, 5\}$
 - $\{2, 3\} \in \{\{1, 3\}, 2, \{2, 3, 4\}, \{2, 3, \emptyset\}\}$
 - $\emptyset \in \{1, 2, 3, 4, 5\}$
 - $7 \in \{1, 3, 5, 7\}$
 - $1 \in \{\{1\}, 2, 3, 5, 7\}$
 - $\{2, 3\} \in \{1, 2, \{3, 2\}, 4\}$
 - $\{1, 2\} \in \{1, 2, \{2, 3\}\}$
 - $\emptyset \in \{\emptyset, 1, \{2, 4\}\}$

Sets: Example 2

- Decide whether the following statements are true or not!
 - $\{1, 2\} \subseteq \{2, 1\}$
 - $\{\{1\}\} \subseteq \{1, 2\}$
 - $\{1\} \subseteq \{1, 2\}$
 - $\{1\} \subseteq \{3, 2\}$
 - $\emptyset \subseteq \{1, 2\}$
 - $\{2, 3\} \subseteq \{1, 3, \{2, 4\}\}$
 - $\emptyset \subseteq \{\emptyset, 3, \{3, 4\}\}$
 - $\{\{2, 3\}\} \subseteq \{1, 3, \{2, 3\}\}$

Sets: Example 3

- Decide whether the following statements are true or not!
 - $\emptyset \subseteq \emptyset$
 - $\emptyset \in \emptyset$
 - $\emptyset \subseteq \{\emptyset\}$
 - $\emptyset \in \{\emptyset\}$
 - $\{a, b\} \subseteq \{a, b, c\}$
 - $\{a, b\} \in \{a, b\}$
 - $\{a, b\} \subseteq \{a, b, \{a, b\}\}$
 - $\{\{a, b\}\} \subseteq \{a, b, \{a, b\}\}$
 - $\{a, b\} \in \{a, b, \{b, a\}\}$

Power sets: Example 1

- Which elements are inside the power set?
 - $P(\{a, b\}) =$
0: excluded, 1: included

a	b	$P(\{a, b\})$
0	0	\emptyset
0	1	{b}
1	0	{a}
1	1	{a, b}

- $P(\{a, b\}) = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$

Power sets: Example 2

- Which elements are inside the power sets?
 - $2^{\{5, 7\}} =$

0: excluded, 1: included

5	7	$P(\{5, 7\})$
0	0	\emptyset
0	1	{7}
1	0	{5}
1	1	{5, 7}

- $2^{\{5, 7\}} = \{\emptyset, \{5\}, \{7\}, \{5, 7\}\}$

Power sets: Example 3

- Which elements are inside the power sets?

- $P(\{2, 3, 5\}) =$

2	3	5	$P(\{2, 3, 5\})$
0	0	0	\emptyset
0	0	1	{5}
0	1	0	{3}
0	1	1	{3, 5}
1	0	0	{2}
1	0	1	{2, 5}
1	1	0	{2, 3}
1	1	1	{2, 3, 5}

- $P(\{2, 3, 5\}) = \{\emptyset, \{2\}, \{3\}, \{5\}, \{2, 3\}, \{2, 5\}, \{3, 5\}, \{2, 3, 5\}\}$

Power sets: Example 4

- Which elements are inside the power sets?
 - $P(\{x\}) =$

0: excluded, 1: included

x	P({x})
0	{}
1	{x}

- $P(\{x\}) = \{\emptyset, \{x\}\}$

Power sets: Example 4

- Which elements are inside the power sets?
 - $P(\{\emptyset\}) =$

0: excluded, 1: included

\emptyset	$P(\{\emptyset\})$
0	{}
1	{ \emptyset }

- $P(\{\emptyset\}) = \{\{\}, \{\emptyset\}\} = \{\emptyset, \{\emptyset\}\}$

Power sets: Example 5

- Which elements are inside the power sets?
 - $P(\{\{\emptyset\}, \emptyset\}) =$
0: excluded, 1: included

$\{\emptyset\}$	\emptyset	$P(\{\{\emptyset\}, \emptyset\})$
0	0	\emptyset
0	1	$\{\emptyset\}$
1	0	$\{\{\emptyset\}\}$
1	1	$\{\{\emptyset\}, \emptyset\}$

- $P(\{\{\emptyset\}, \emptyset\}) = \{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}, \{\{\emptyset\}, \emptyset\}\}$

Power sets: Example 6

- Decide whether the following statements true or not!
 - $\{a, b\} \subseteq P(\{a, b\})$
 - $\emptyset \subseteq P(\emptyset)$
 - $\{a, b\} \in P(\{a, b\})$
 - $\emptyset \in P(\emptyset)$
 - $\{3, 5\} \subseteq 2^{\{3, 5\}}$
 - $\{3\} \in 2^{\{3, 5\}}$
 - $\{\emptyset, \{a, b\}\} \subseteq P(\{a, b\})$
 - $\{\{a\}, \{b\}\} \in P(\{a, b\})$
 - $\{\emptyset\} \in 2^{\{3, 5\}}$

Operations with sets: Example 1

- Perform the operations!
 - $(\{1, 2, 5\} \cup \{2, 1\}) \cap \{2, 5, 7\} =$
 $= \{1, 2, 5\} \cap \{2, 5, 7\} =$
 $= \{2, 5\}$
 - $\{0, 3, 5\} \cup (\{3, 7\} \cap \{3, 5, 7\}) =$
 $= \{0, 3, 5\} \cup \{3, 7\} =$
 $= \{0, 3, 5, 7\}$

Operations with sets: Example 2

- Perform the operations!
 - $(\cup\{\{3, 5\}, \{1, 2, 3\}, \{7\}\}) \cap (\cap\{\{3, 1, 5\}, \{3, 2\}, \{7\}\}) =$
 $= \{1, 2, 3, 5, 7\} \cap \{\} =$
 $= \{\}$
 - $(\cup\{\{3\}, \{3, 5\}, \{7\}\}) \cup (\cap\{\{1, 2, 3\}, \{2, 3, 4\}\}) =$
 $= \{3, 5, 7\} \cup \{2, 3\} =$
 $= \{2, 3, 5, 7\}$

Operations with sets: Example 3

- Perform the operations!

$$- (\{1, 2, 5\} - \{5, 7, 9\}) \cup (\{5, 7, 9\} - \{1, 2, 5\}) =$$

$$= \{1, 2\} \cup \{7, 9\} =$$

$$= \{1, 2, 7, 9\}$$

$$- (\{3, 5, 7\} - \{1, 2, 5\}) \cap (\{1, 3, 7, 9\} - \{1, 2, 7\}) =$$

$$= \{3, 7\} \cap \{3, 9\} =$$

$$= \{3\}$$

Operations with sets: Example 4

- Perform the assigned operations!
 - $P(\{2, 3, 5\}) - P(\{3, 5\}) =$
 $= \{\emptyset, \{2\}, \{3\}, \{5\}, \{2, 3\}, \{2, 5\}, \{3, 5\}, \{2, 3, 5\}\} -$
 $\{\emptyset, \{3\}, \{5\}, \{3, 5\}\} =$
 $= \{\{2\}, \{2, 3\}, \{2, 5\}, \{2, 3, 5\}\}$
 - $(2^{\{5, 7\}} - 2^{\{5, 9\}}) \cup 2^{\{3, 5\}} =$
 $= (\{\emptyset, \{5\}, \{7\}, \{5, 7\}\} - \{\emptyset, \{5\}, \{9\}, \{5, 9\}\}) \cup$
 $\{\emptyset, \{3\}, \{5\}, \{3, 5\}\} =$
 $= \{\{7\}, \{5, 7\}\} \cup \{\emptyset, \{3\}, \{5\}, \{3, 5\}\} =$
 $= \{\emptyset, \{3\}, \{5\}, \{7\}, \{5, 7\}, \{3, 5\}\}$

Operations with sets: Example 5

- Perform the operations!
 - $(\{1, 2\} \cup \{2, 3\}) \cap (\{4, 5, 6\} \cap \{3, 5, 7\}) =$
 $= \{1, 2, 3\} \cap \{5\} =$
 $= \emptyset$
 - $((\{1, 4, 5, 7\} \cup \{2, 4, 8, 3\}) \cap \{1, 2, 3, 4, 5, 6, 7\}) \cap ((\{4, 5, 6\} \cup \{3, 5, 7\}) - \{4, 6, 7\}) =$
 $= (\{1, 2, 3, 4, 5, 7, 8\} \cap \{1, 2, 3, 4, 5, 6, 7\}) \cap$
 $= (\{3, 4, 5, 6, 7\} - \{4, 6, 7\}) =$
 $= \{1, 2, 3, 4, 5, 7\} \cap \{3, 5\} =$
 $= \{3, 5\}$

Operations with sets: Example 6

- Perform the operations!
 - A, B are sets, \triangle is defined as:
$$A \triangle B = (A \cup B) - (A \cap B)$$
 - $(\{1, 2\} \cap \{2, 3\}) \cup (\{4, 5, 6\} \triangle \{3, 5, 7\}) =$
$$= \{2\} \cup \{3, 4, 6, 7\} =$$
$$= \{2, 3, 4, 6, 7\}$$
 - $(\{1, 2, 3, 4, 5, 6, 7, 8, 9\} \triangle \{1, 2, 3, 4, 5, 6, 9\}) \triangle \{1, 3, 6, 8\} =$
$$= \{7, 8\} \triangle \{1, 3, 6, 8\} =$$
$$= \{1, 3, 6, 7\}$$

Operations with sets: Example 7

- Perform the operations!

- A, B are sets, Δ is defined as:

$$A \Delta B = (A \cup B) - (A \cap B)$$

- $(\{1, 2, 3, 9\} - \{4, 5, 6, 7\}) \Delta \{1, 5\} =$

$$= \{1, 2, 3, 9\} \Delta \{1, 5\} =$$

$$= \{2, 3, 5, 9\}$$

- $((\{1, 9\} - \{3, 8, 9\}) \cup (\{4, 6, 8\} \cup \{1, 3, 6, 8\})) \Delta (\{1, 2, 5\} \Delta (\{3, 6, 9\} \cap \{1, 2, 3, 4, 5\})) =$

$$= (\{1\} \cup \{1, 3, 4, 6, 8\}) \Delta (\{1, 2, 5\} \Delta \{3\}) =$$

$$= \{1, 3, 4, 6, 8\} \Delta \{1, 2, 3, 5\}$$

$$= \{2, 4, 5, 6, 8\}$$

Operations with sets: Example 8

- Perform the operations!
 - A: {1, 2, 3, 7, 8}, B: {3, 4, 5}, C: {5, 6, 7}
 - $A \cup B =$
 $\{1, 2, 3, 7, 8\} \cup \{3, 4, 5\} = \{1, 2, 3, 4, 5, 7, 8\}$
 - $B \cap C =$
 $\{3, 4, 5\} \cap \{5, 6, 7\} = \{5\}$
 - $A \triangle (B \cup (C \cup A)) =$
 $\{1, 2, 3, 7, 8\} \triangle (\{3, 4, 5\} \cup (\{5, 6, 7\} \cup \{1, 2, 3, 7, 8\})) =$
 $\{1, 2, 3, 7, 8\} \triangle (\{3, 4, 5\} \cup \{1, 2, 3, 5, 6, 7, 8\}) =$
 $\{1, 2, 3, 7, 8\} \triangle \{1, 2, 3, 4, 5, 6, 7, 8\} = \{4, 5, 6\}$

Cartesian products

- Perform the operations!
 - $\{1, 5, 9\} \times \{b, c, d\} =$
 $\{(1, b), (1, c), (1, d), (5, b), (5, c), (5, d), (9, b),$
 $(9, c), (9, d)\}$
 - $\{0\} \times \{1, 2\} \times \{1, 2, 3\} =$
 $\{(0, 1, 1), (0, 1, 2), (0, 1, 3), (0, 2, 1), (0, 2, 2),$
 $(0, 2, 3)\}$

Cartesian products

- Perform the operations!
 - $P(\{4, 5\}) \times \{1, 2\} =$
 $\{\emptyset, \{4\}, \{5\}, \{4, 5\}\} \times \{1, 2\} =$
 $\{(\emptyset, 1), (\emptyset, 2), (\{4\}, 1), (\{4\}, 2), (\{5\}, 1), (\{5\}, 2),$
 $(\{4, 5\}, 1), (\{4, 5\}, 2)\}$
 - $P(\{2, 4\}) \times P(\{3, 5\}) =$
 $\{\emptyset, \{2\}, \{4\}, \{2, 4\}\} \times \{\emptyset, \{3\}, \{5\}, \{3, 5\}\} =$
 $\{(\emptyset, \emptyset), (\emptyset, \{3\}), (\emptyset, \{5\}), (\emptyset, \{3, 5\}), (\{2\}, \emptyset),$
 $(\{2\}, \{3\}), (\{2\}, \{5\}), (\{2\}, \{3, 5\}), (\{4\}, \emptyset), (\{4\}, \{3\}),$
 $(\{4\}, \{5\}), (\{4\}, \{3, 5\}), (\{2, 4\}, \emptyset), (\{2, 4\}, \{3\}),$
 $(\{2, 4\}, \{5\}), (\{2, 4\}, \{3, 5\})\}$

Cartesian products

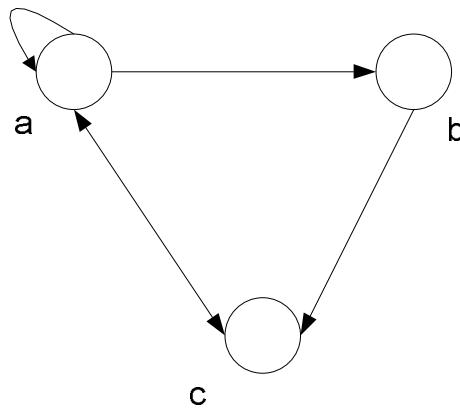
- Decide whether the following statements are true or not!
 - $\{a\} \times \{b\} = \{(a, b)\}$
 - $\{a, b\} \subseteq \{a\} \times \{b\}$
 - $\{a, b\} \in \{a\} \times \{b\}$
 - $(a, b) \in \{a\} \times \{b\}$
 - $\{(a, b)\} \subseteq \{a\} \times \{b\}$

Cartesian products

- Decide whether the following statements are true or not!
 - $(a, b) \in (a, b) \times \{a, b\}$
= $\{(a, b), a\}, \{(a, b), b\}$
 - $\{a, b\} \in \{b, a\} \times \{b\}$
= $\{(b, b), (a, b)\}$
 - $(a, b) \in \{a, (a, b)\} \times \{(b, a), b\} =$
 $\{(a, (b, a)), (a, b), ((a, b), (b, a)), ((a, b), b)\} =$
 $\{(a, b, a), (a, b), (a, b, b, a), (a, b, b)\}$

Relations: Example 1

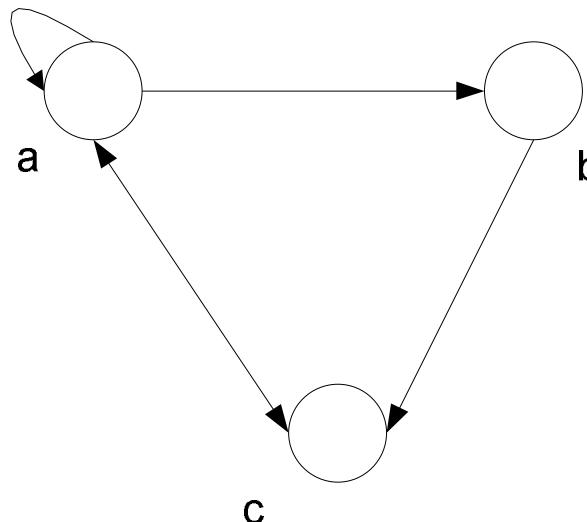
- Is the graph given below is reflexive?
 - $R_1 = \{(a, a), (a, b), (a, c), (b, c), (c, a)\}$
 $R_1 \subseteq \{a, b, c\} \times \{a, b, c\}$



- answer: no, because there is no loop at each node

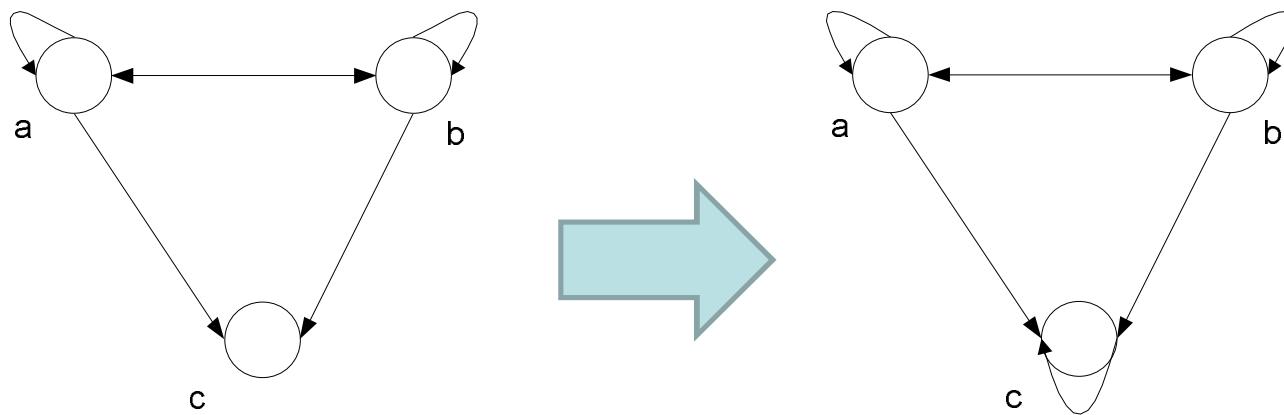
Relations: Example 2

- Decide whether the following statements are true or not!
 - the graph given below is:
 - symmetrical
 - transitive
 - anti-symmetrical
- there are no (b, a), (c, b) edges
(c, b) is missing
for (a, c) there is (c, a) edge too



Relations: Example 3

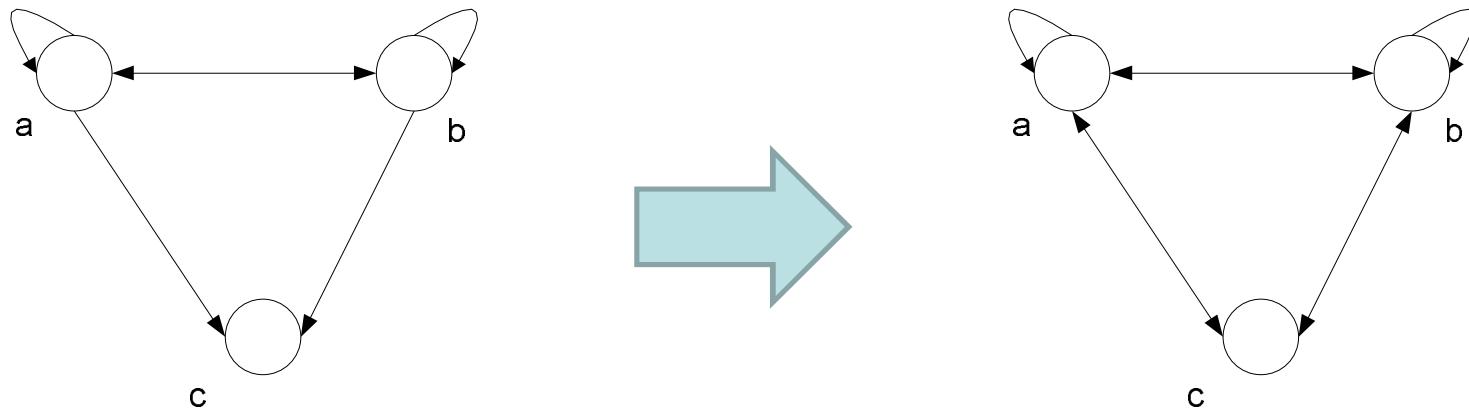
- Calculate the reflexive closure of the graph given below!
 - $R_1 = \{(a, a), (a, b), (b, a), (b, b), (a, c), (b, c)\}$



- $R_2 = \{(a, a), (a, b), (b, a), (b, b), (a, c), (b, c), (c, c)\}$

Relations: Example 4

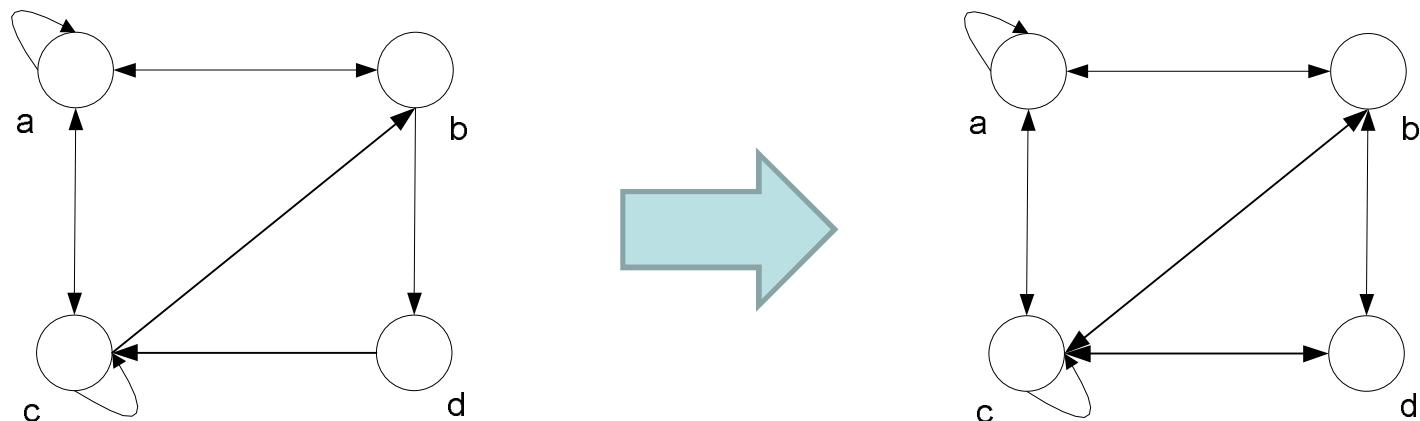
- Calculate the symmetrical closure of the graph given below!
 - $R_1 = \{(a, a), (a, b), (b, a), (b, b), (a, c), (b, c)\}$



- $R_2 = \{(a, a), (a, b), (b, a), (b, b), (a, c), (b, c), (c, a), (c, b)\}$

Relations: Example 5

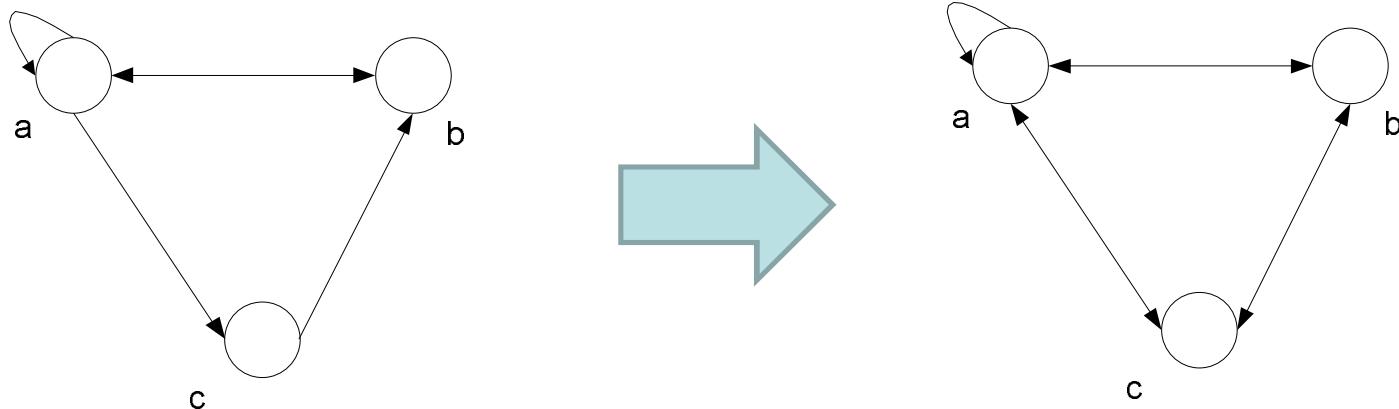
- Calculate the symmetrical closure of the graph given below!
 - $R_1 = \{(a, a), (c, c), (a, b), (b, a), (a, c), (c, a), (d, c), (b, d), (c, b)\}$



- $R_2 = \{(a, a), (c, c), (a, b), (b, a), (a, c), (c, a), (d, c), (b, d), (c, b), (b, c), (c, d), (d, b)\}$

Relations: Example 6

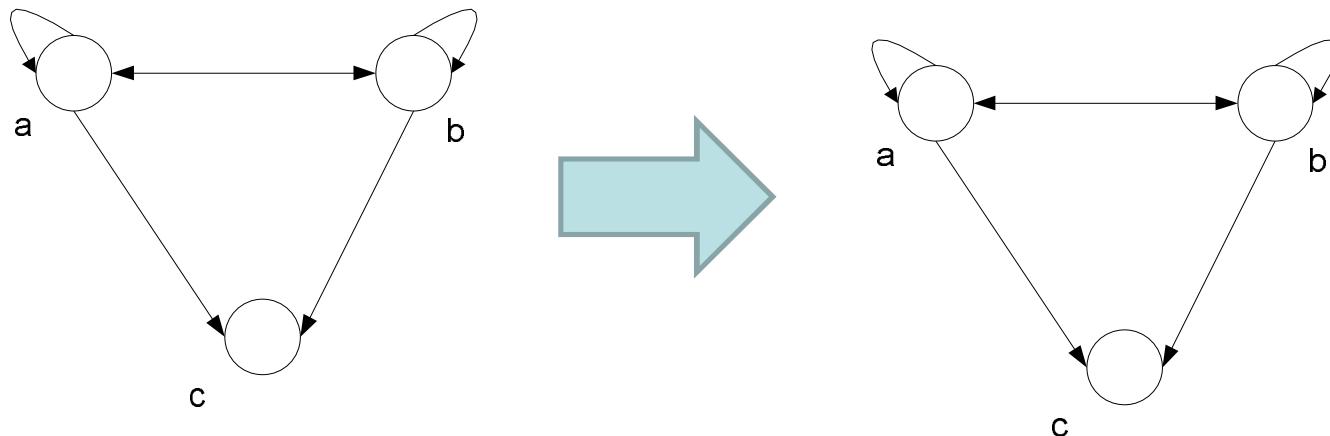
- Calculate the transitive closure of the graph given below!
 - $R_1 = \{(a, a), (a, b), (a, c), (b, a), (c, b)\}$



- $R_2 = \{(a, a), (a, b), (a, c), (b, a), (c, b), (b, c), (c, a)\}$

Relations: Example 7

- Calculate the transitive closure of the graph given below!
 - $R_1 = \{(a, a), (a, b), (b, a), (b, b), (a, c), (b, c)\}$

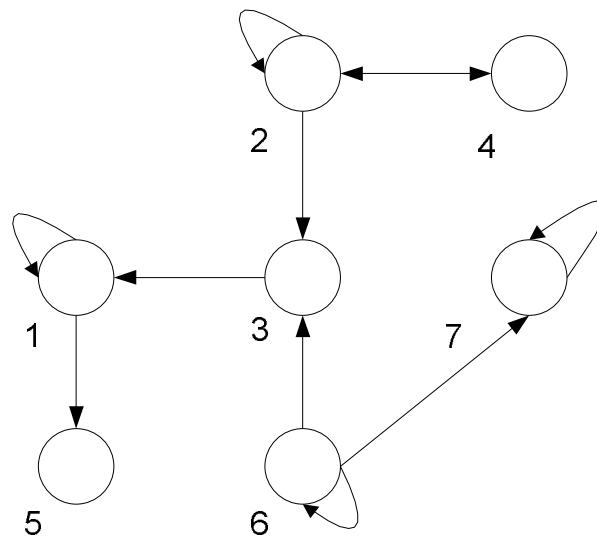


- $R_2 = \{(a, a), (a, b), (b, a), (b, b), (a, c), (b, c)\}$

Relations: Example 8

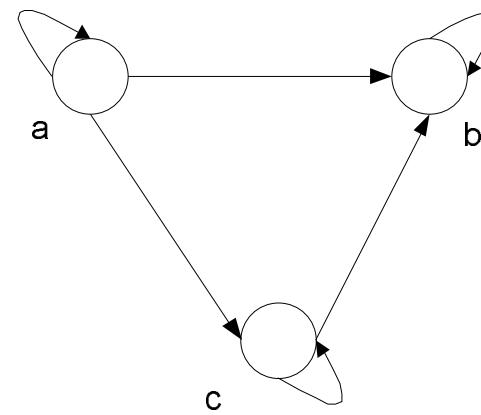
- Describe the properties of this relation!
 - $R_1 = \{(1, 1), (1, 5), (2, 2), (2, 4), (2, 3), (3, 1), (4, 2), (6, 3), (6, 6), (7, 7), (6, 7)\}$

- not reflexive
- not symmetric
- not transitive
- not anti-symmetric



Relations: Example 9

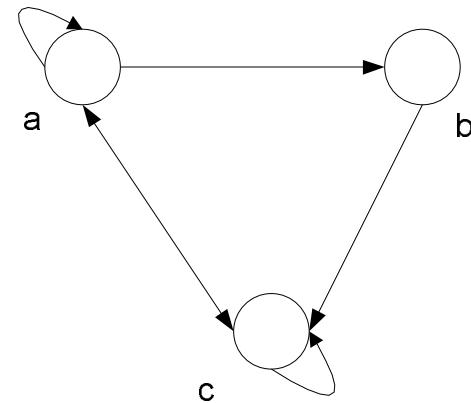
- Describe the properties of this relation!
 - $R_1 = \{(a, b), (b, a), (a, c), (b, b), (c, b), (c, c)\}$



- reflexive
- transitive
- anti-symmetric

Relations: Example 10

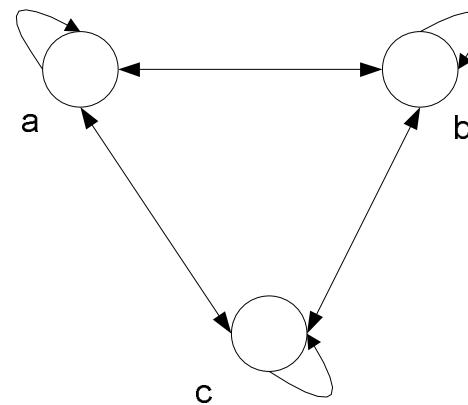
- Describe the properties of this relation!
 - $R_2 = \{(a, a), (a, b), (b, c), (a, c), (c, a), (c, c)\}$



- not reflexive
- not symmetric
- not transitive
- not anti-symmetric

Relations: Example 11

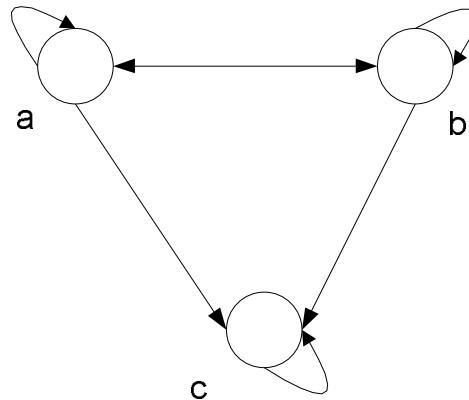
- Describe the properties of the union of previous two relations!
 - $R_1 \cup R_2 = \{(a, a), (a, b), (a, c), (b, a), (b, b), (b, c), (c, a), (c, b), (c, c)\}$
 - reflexive
 - symmetric
 - transitive



Relations: Example 12

- Decide whether the following statements true or not!
 - $R = \{(a, a), (b, b), (c, c), (a, b), (b, a), (a, c), (b, c)\}$

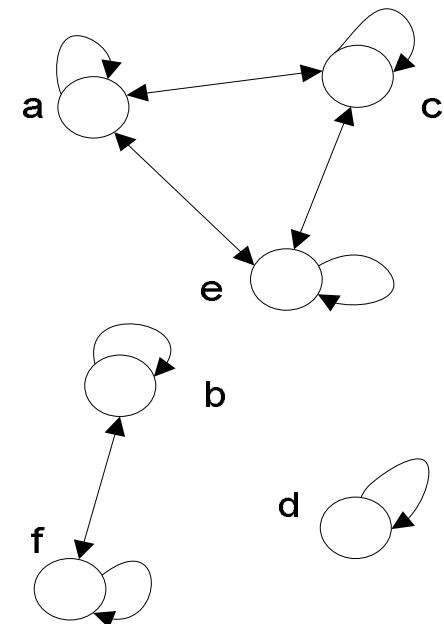
- the graph is given below:
 - reflexive
 - symmetric
 - anti-symmetric
 - transitive
 - equivalence



Relations: Example 13

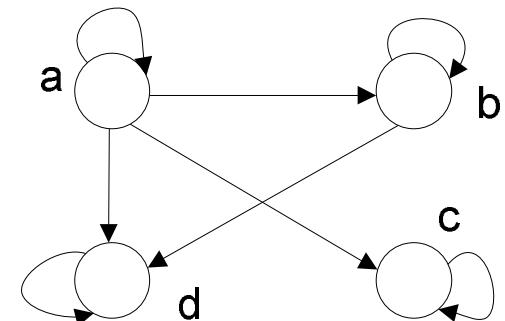
- Decide whether the following statements true or not!
 - $R = \{(a, a), (a, c), (c, a), (c, c), (c, e), (e, c), (e, e), (e, a), (a, e), (b, b), (b, f), (f, b), (f, f), (d, d)\}$

- reflexive
- symmetric
- anti-symmetric
- transitive
- equivalence



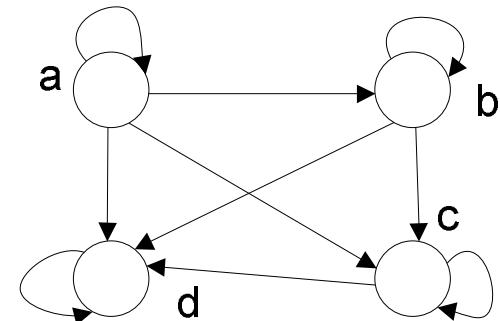
Relations: Example 14

- Decide whether the following statements true or not!
 - $R = \{(a, a), (b, b), (c, c), (d, d), (a, b), (a, c), (a, d), (b, d)\}$
 - the graph is given below:
 - reflexive
 - symmetric
 - anti-symmetric
 - transitive
 - equivalence



Relations: Example 15

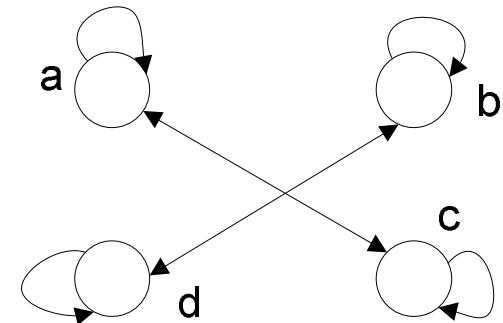
- Decide whether the following statements true or not!
 - $R = \{(a, a), (b, b), (c, c), (d, d), (a, b), (b, d), (a, d), (a, c), (c, d), (b, c)\}$
 - the graph is given below:
 - reflexive
 - symmetric
 - anti-symmetric
 - transitive
 - equivalence



Relations: Example 16

- Decide whether the following statements true or not!
 - $R = \{(a, a), (b, b), (c, c), (d, d), (a, c), (c, a), (b, d), (d, b)\}$

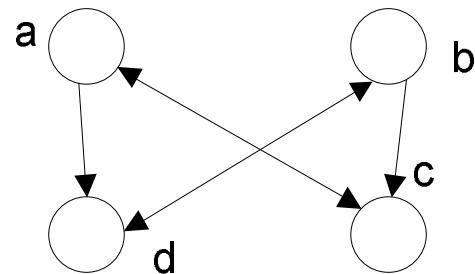
- the graph is given below:
 - reflexive
 - symmetric
 - anti-symmetric
 - transitive
 - equivalence



Relations: Example 17

- Decide whether the following statements true or not!
 - $R = \{(a, c), (c, a), (b, d), (d, b), (a, d), (b, c)\}$

- the graph is given below:
 - reflexive
 - symmetric
 - anti-symmetric
 - transitive
 - equivalence



Element of the Theory of the Computation

Lecture 2

Algorithm complexity: Example 1

- Let $f(n) = 9n^2 + 3n + 2$
 - is it true: $f(n) \in O(n^2)$?
 - $f(n) = 9n^2 + 3n + 2 < 9n^2 + 3n^2 + 2 = 12n^2 + 2$
 - $f(n) \leq c * n^2 + d$
 - $c = 12, d = 2$
- Let $f(n) = 9n^2 + 3n + 2$
 - is it true: $n^2 \in O(f)$?
 - $n^2 \leq c (9n^2 + 3n + 2) + d$
 - $c = 1, d = 0$

Algorithm complexity: Example 1

- Let $f(n) = 9n^2 + 3n + 2$
 - is it true: $n^2 \approx f$?
 - $f(n) \in O(n^2)$, $n^2 \in O(f)$

Algorithm complexity: Example 2

- Let $f(n) = 2n^2 + 3n$
 - is it true: $f(n) \in O(n^3)$?
 - $f(n) = 2n^2 + 3n < 2n^3 + 3n^3 = 5n^3$
 - $f(n) \leq c * n^3 + d$
 - $c = 5, d = 0$
- Let $f(n) = 2n^2 + 3n$
 - is it true: $n^3 \in O(f)$?
 - $n^3 \leq c * (2n^2 + 3n) + d$
 - there's no c and d to satisfy the equation for all n

Algorithm complexity: Example 2

- Let $f(n) = 2n^2 + 3n$
 - is it true: $n^3 \approx f$?
 - no, because
 - $n^3 \notin O(f)$
 - $f \in O(n^3)$

Algorithm complexity: Example 3

- Is it true or false?

- $f(n) = 2^n + n^7$, $g(n) = 5n^7$

$$f(n) \in O(g(n))$$

$$g(n) \in O(f(n))$$

$$f(n) \approx g(n)$$

- $f(n) = 100$, $g(n) = n$

$$f(n) \in O(g(n))$$

$$g(n) \in O(f(n))$$

$$f(n) \approx g(n)$$

Algorithm complexity: Example 4

- Is it true or false?

- $f(n) = 5n^4 + n^2 - 700$, $g(n) = 2n^4 + n^3 + 100$

$$f(n) \in O(g(n))$$

$$g(n) \in O(f(n))$$

$$f(n) \approx g(n)$$

- $f(n) = 20n^{3.1} + 6n^2 + 10n + 6$, $g(n) = 10n^3 + n^{0.1} + 6$

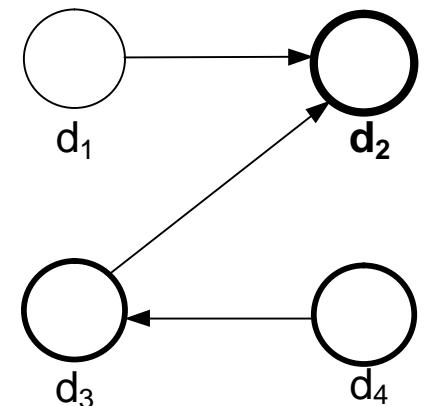
$$f(n) \in O(g(n))$$

$$g(n) \in O(f(n))$$

$$f(n) \approx g(n)$$

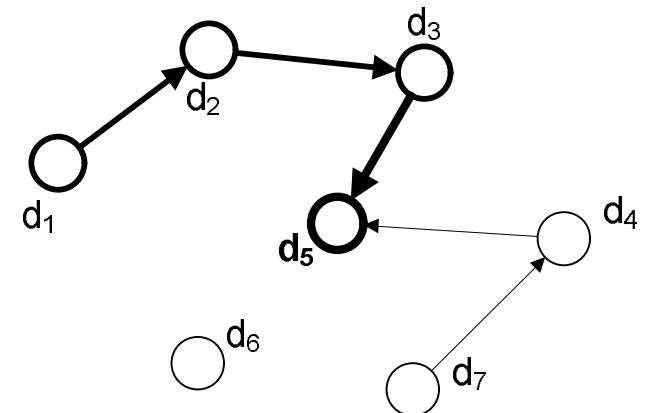
Closure: Example 1

- A binary relation is given on $D \times D$. Give the closure of set A on this relation!
 - $D = \{d_1, d_2, d_3, d_4\}$
 - $A = \{d_4\}$
 - $(d_4, d_3) \in R, d_4 \in A^* \rightarrow d_3 \in A^*$
 - $A^* = \{d_3, d_4\}$
 - $(d_3, d_2) \in R, d_3 \in A^* \rightarrow d_2 \in A^*$
 - $A^* = \{d_2, d_3, d_4\}$



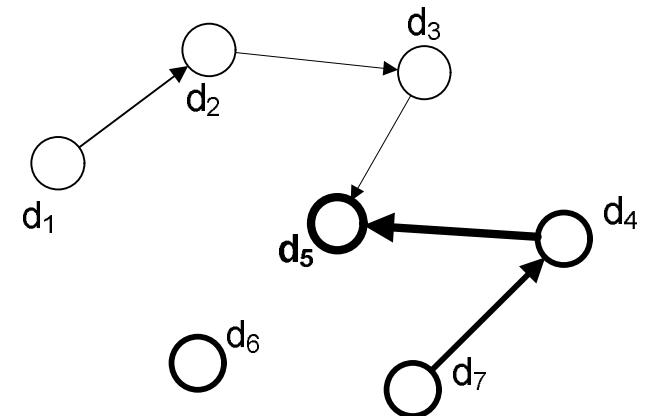
Closure: Example 2

- A binary relation is given on $D \times D$. Give the closure of set A on this relation!
 - $D = \{d_1, d_2, d_3, d_4, d_5, d_6, d_7\}$
 - $A = \{d_1, d_2\}$
 - $A^* = \{d_1, d_2\}$
 - $A^* = \{d_1, d_2, d_3\}$
 - $A^* = \{d_1, d_2, d_3, d_5\}$



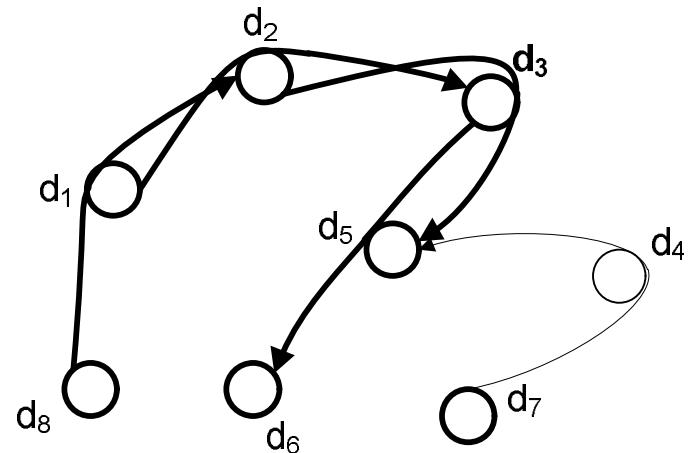
Closure: Example 3

- A binary relation is given on $D \times D$. Give the closure of set A on this relation!
 - $D = \{d_1, d_2, d_3, d_4, d_5, d_6, d_7\}$
 - $A = \{d_6, d_7\}$
 - $A^* = \{d_6, d_7\}$
 - $A^* = \{d_7, d_6, d_4\}$
 - $A^* = \{d_6, d_7, d_4, d_5\}$



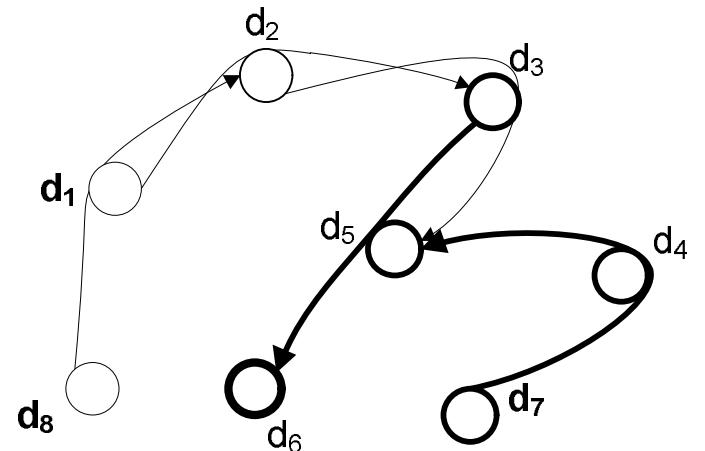
Closure: Example 4

- A ternary relation is given on $D \times D \times D$. Give the closure of set A on this relation!
 - $D = \{d_1, d_2, d_3, d_4, d_5, d_6, d_7, d_8, d_9, d_{10}\}$
 - $A = \{d_8, d_7, d_1\}$
 - $A^* = \{d_8, d_7, d_1\}$
 - $A^* = \{d_8, d_7, d_1, d_2\}$
 - $A^* = \{d_8, d_7, d_1, d_2, d_3\}$
 - $A^* = \{d_8, d_7, d_1, d_2, d_3, d_5\}$
 - $A^* = \{d_8, d_7, d_1, d_2, d_3, d_5, d_6\}$



Closure: Example 5

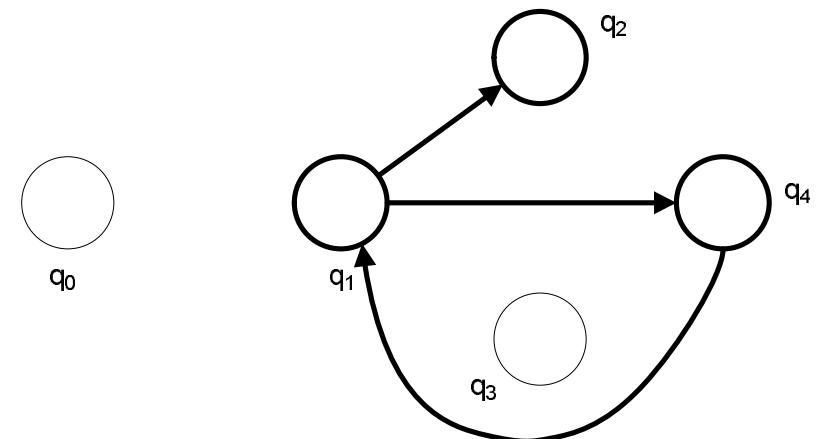
- A ternary relation is given on $D \times D \times D$. Give the closure of set A on this relation!
 - $D = \{d_1, d_2, d_3, d_4, d_5, d_6, d_7, d_8, d_9, d_{10}\}$
 - $A = \{d_4, d_7, d_3\}$
 - $A^* = \{d_4, d_7, d_3\}$
 - $A^* = \{d_4, d_7, d_3, d_5\}$
 - $A^* = \{d_4, d_7, d_3, d_5, d_6\}$



Closure: Example 6

- Give the $E(q_1)$ set of the following NFA!

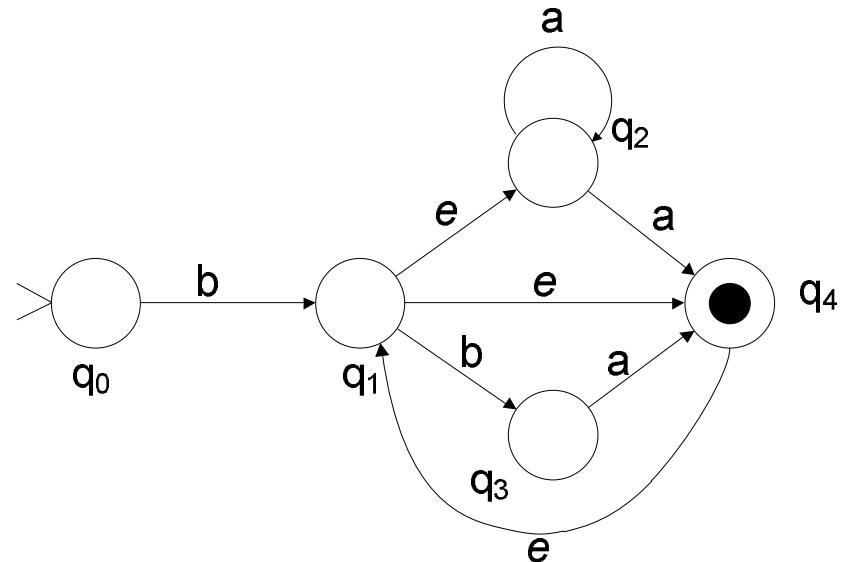
- $E(q_1) = \{q_1\}$
- $E(q_1) = \{q_1, q_2\}$
- $E(q_1) = \{q_1, q_2, q_4\}$



Closure: Example 6

- Give the "E" sets of the states of the following NFA!

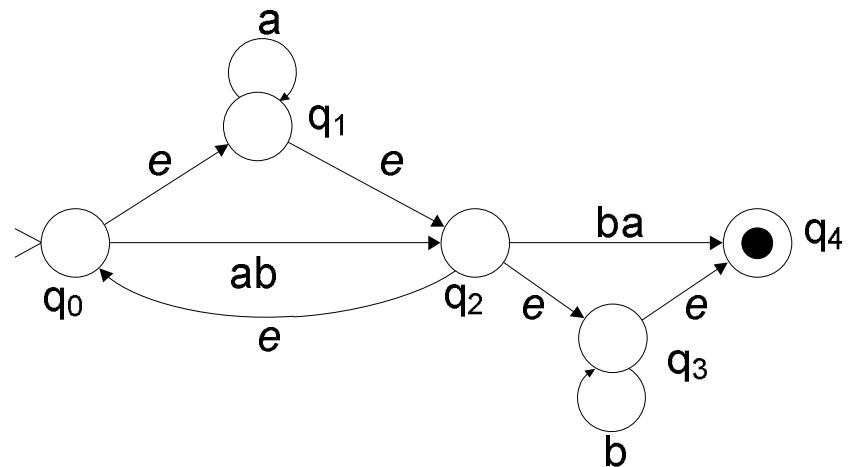
- $E(q_0) = \{q_0\}$
- $E(q_1) = \{q_1, q_2, q_4\}$
- $E(q_2) = \{q_2\}$
- $E(q_3) = \{q_3\}$
- $E(q_4) = \{q_4, q_1, q_2\}$



Closure: Example 7

- Give the "E" sets of the states of the following NFA!

- $E(q_0) = \{q_0, q_1, q_2, q_3, q_4\}$
- $E(q_1) = \{q_0, q_1, q_2, q_3, q_4\}$
- $E(q_2) = \{q_0, q_1, q_2, q_3, q_4\}$
- $E(q_3) = \{q_3, q_4\}$
- $E(q_4) = \{q_4\}$



Element of the Theory of the Computation

Lecture 3

Language: Example 1

- Define the next language: $L(a \cup b)L(c \cup d) =$
- Solution:
 - = $(L(a) \cup L(b))(L(c) \cup L(d)) =$
 - = $(\{a\} \cup \{b\})(\{c\} \cup \{d\}) =$
 - = $\{a, b\}\{c, d\} =$
 - = $\{ac, ad, bc, bd\}$

Language: Example 2

- Define the next language: $L(abc \cup ab)L(cd \cup d) =$
- Solution:
 - = $(L(abc) \cup L(ab))(L(cd) \cup L(d)) =$
 - = $(\{abc\} \cup \{ab\})(\{cd\} \cup \{d\}) =$
 - = $\{abc, ab\}\{cd, d\} =$
 - = $\{abccd, abcd, abd\}$

Language: Example 3

- Define the next language: $L(a \cup \emptyset)L(ab \cup ba) =$
- Solution:
 - = $L(a \cup \emptyset)L(ab \cup ba) =$
 - = $(L(a) \cup L(\emptyset))(L(ab) \cup L(ba)) =$
 - = $(\{a\} \cup \emptyset)(\{ab\} \cup \{ba\}) =$
 - = $\{a\}\{ab, ba\} =$
 - = $\{aab, aba\}$

Language: Example 4

- Define the next language: $L(a)(L(a \cup b)L(c \cup \emptyset^*))$
- Solution:
 - = $L(a)((L(a) \cup L(b))(L(c) \cup L(\emptyset^*))) =$
 - = $\{a\}((\{a\} \cup \{b\})(\{c\} \cup \{\epsilon\})) =$
 - = $\{a\}(\{a, b\}\{c, \epsilon\}) =$
 - = $\{a\}\{ac, a, bc, b\} =$
 - = $\{aac, aa, abc, ab\}$

Language: Example 5

- Is the statement, $a \in L(a^*b^*a^*b^*)$, true?
- Is the statement, $L(b^*a^*) \cap L(a^*b^*) = L(a^* \cup b^*)$, true?
- Is the statement, $L(a^*b^*) \cap L(c^*d^*) = \emptyset$, true?
= {e}
- Is the statement, $abcd \in L((a(cd)^*b^*)^*)$, true?
 - false, because the first iteration of the outermost * can generate "ab" but after that there is a compulsory "a"
 - $abacd \in L((a(cd)^*b^*)^*)$

Language: Example 6

- Which strings are the elements of the following language?
 $\{w \in \Sigma^* \mid w = uu^R u, u \in \Sigma^*\}, \Sigma = \{a, b\}$
- Solution:
 - {abbaab, baabba, aaaaaa, bbbbb, ...}
- Which strings are the elements of the following language?
 $\{w \in \Sigma^* \mid ww = www\}, \Sigma = \{a, b\}$
- Solution:
 - {e}

Regular Expression: Example 1

- Give regular expression RE such that
 $L(RE) = \{w \in \{a, b\}^*\}$
- Solution:
 - $RE = (a^*b^*)^*$ or $RE = (a \cup b)^*$
- Give regular expression RE such that
 $L(RE) = \{w \in \{a, b\}^* \mid \#a \text{ is odd}\}$
- Solution:
 - $RE = b^*ab^*(ab^*ab^*)^*$

Regular Expression: Example 2

- Give regular expression RE such that
 $L(RE) = \{w \in \{a, b\}^* \mid \#a \text{ is even}\}$
- Solution:
 - $RE = b^* \cup (b^*ab^*ab^*)^*$
- Give regular expression RE such that
 $L(RE) = \{w \in \{a, b\}^* \mid w \text{ contains substring "abba"}\}$
- Solution:
 - $RE = (a \cup b)^*abba(a \cup b)^*$

Regular Expression: Example 3

- Give regular expression RE such that
 $L(RE) = \{w \in \{a, b\}^* \mid w \text{ not contains substring "ab"}\}$
- Solution:
 - $RE = b^*a^*$
- Give regular expression RE such that
 $L(RE) = \{w \in \{a, b\}^* \mid \#a = 0\}$
- Solution:
 - $RE = b^*$

Regular Expression: Example 4

- Give regular expression RE such that
 $L(RE) = \{w \in \{a, b\}^* \mid \#a = 2\}$
- Solution:
 - $RE = b^*ab^*ab^*$
- Give regular expression RE such that
 $L(RE) = \{w \in \{a, b, c\}^* \mid \#a = \{0, 2, 4\}\}$
- Solution:
 - $RE = (b \cup c)^* \cup ((b \cup c)^*a(b \cup c)^*a(b \cup c)^*) \cup ((b \cup c)^*a(b \cup c)^*a(b \cup c)^*a(b \cup c)^*a(b \cup c)^*)$

Regular Expression: Example 5

- Give regular expression RE such that
 $L(RE) = \{w \in \{a, b\}^* \mid \#a \geq 0\}$
- Solution:
 - $RE = (a \cup b)^*$
- Give regular expression RE such that
 $L(RE) = \{w \in \{a, b\}^* \mid \#a \geq 2\}$
- Solution:
 - $RE = b^*ab^*a(b \cup a)^*$

Regular Expression: Example 6

- Give regular expression RE such that
 $L(RE) = \{w \in \{a, b, c\}^* \mid \#a = 2\}$
- Solution:
 - $RE = (b \cup c)^*a(b \cup c)^*a(b \cup c)^*$
- Give regular expression RE such that
 $L(RE) = \{w \in \{a, b\}^* \mid \#a \leq 2\}$
- Solution:
 - $RE = b^*(a \cup \emptyset^*)b^*(a \cup \emptyset^*)b^*$

Regular Expression: Example 7

- Give regular expression RE such that
 $L(RE) = \{w \in \{a, b\}^* \mid 3 \leq \#a \leq 6\}$
- Solution:
 - $RE = b^*ab^*ab^*ab^*(a \cup \emptyset^*)b^*(a \cup \emptyset^*)b^*(a \cup \emptyset^*)b^*$
- Give regular expression RE such that
 $L(RE) = \{w \in \{a, b\}^* \mid \#a \bmod 4 = 0\}$
- Solution:
 - $RE = b^* \cup (b^*ab^*ab^*ab^*)^*$

Regular Expression: Example 8

- Give regular expression RE such that
 $L(RE) = \{w \in \{a, b\}^* \mid \#a \bmod 6 = 2\}$
- Solution:
 - $RE = (b^*ab^*ab^*ab^*ab^*ab^*)^*b^*ab^*ab^*$
- Give regular expression RE such that
 $L(RE) = \{w \in \{a, b\}^* \mid \#a \bmod 4 = 2 \text{ or } 3\}$
- Solution:
 - $RE = (b^*ab^*ab^*ab^*ab^*)^*b^*ab^*ab^*(a \cup \emptyset^*)b^*$

Regular Expression: Example 9

- Give regular expression RE such that
 $L(RE) = \{w \in \{a, b, c\}^* \mid \#a + \#c = 4\}$
- Solution:
 - $RE = b^*(a \cup c)b^*(a \cup c)b^*(a \cup c)b^*(a \cup c)b^*$
- Give regular expression RE such that
 $L(RE) = \{w \in \{a, b, c\}^* \mid (\#a + \#c) \text{ mod } 4 = 2\}$
- Solution:
 - $RE = (b^*(a \cup c)b^*(a \cup c)b^*(a \cup c)b^*)^* b^*(a \cup c)b^*(a \cup c)b^*$

Regular Expression: Example 10

- Give regular expression RE such that
 $L(RE) = \{w \in \{a, b\}^* \mid \#a = 2+3k, k \geq 0\}$
- Solution:
 - $L(RE) = \{w \in \{a, b\}^* \mid \#a \bmod 3 = 2\}$
 - $RE = (b^*ab^*ab^*ab^*)^*(b^*ab^*ab^*)$
- Give regular expression RE such that
 $L(RE) = \{w \in \{a, b\}^* \mid \#a \text{ is even or } \#a \bmod 3 = 0\}$
- Solution:
 - $RE = (b^*ab^*ab^*ab^*)^* \cup (b^*ab^*ab^*)^* \cup b^*$

Regular Expression: Example 11

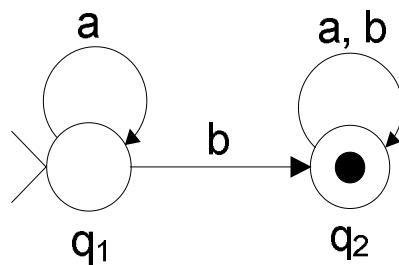
- Give regular expression RE such that
 $L(RE) = \{w \in \{a, b\}^* \mid \#a \text{ is even and } \#a \bmod 3 = 0\}$
- Solution:
 - $L(RE) = \{w \in \{a, b\}^* \mid \#a \bmod 6 = 0\}$
 - $RE = (b^*ab^*ab^*ab^*ab^*ab^*)^* \cup b^*$
- Give regular expression RE such that
 $L(RE) = \{w \in \{a, b\}^* \mid \#a \text{ is odd and } \#a \bmod 3 = 0\}$
- Solution:
 - $L(RE) = \{w \in \{a, b\}^* \mid \#a \bmod 6 = 3\}$
 - $RE = b^*ab^*ab^*(b^*ab^*ab^*ab^*ab^*)^*$

Element of the Theory of the Computation

Lecture 4

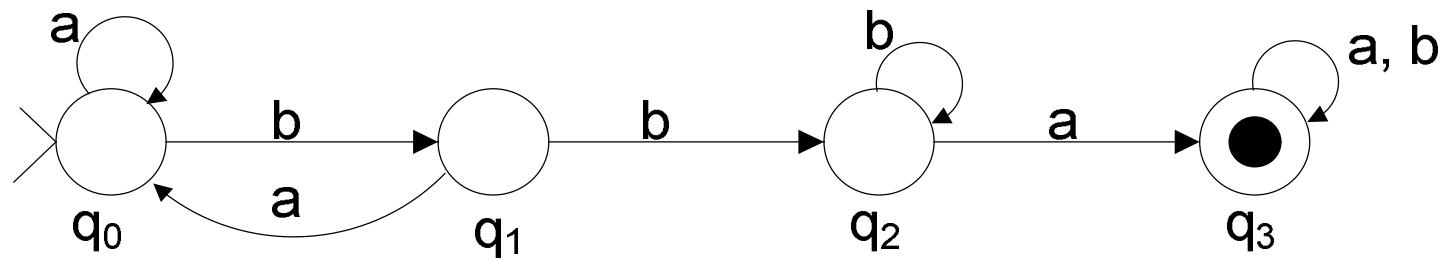
DFA: Example 1

- Define DFA M such that
 - $L(M) = \{w \in \{a, b\}^* \mid w \text{ contains } b\}$
- States:
 - q_1 : DFA haven't read 'b' yet
 - q_2 : final state – DFA have already read 'b'



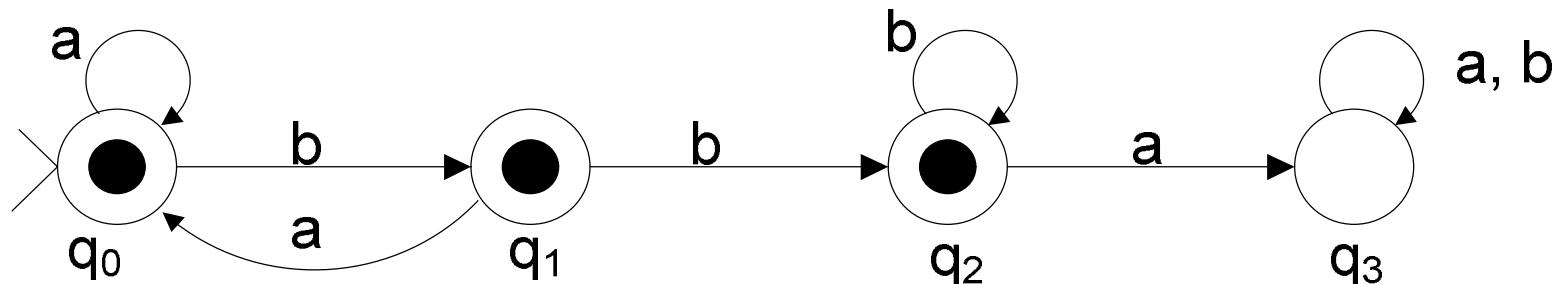
DFA: Example 2

- Define DFA M such that
 - $L(M) = \{w \in \{a, b\}^* \mid w \text{ contains 'bba'}\}$
- States:
 - q_0 : 0 symbol is read from 'bba'
 - q_1 : 1 symbol is read from 'bba'
 - q_2 : 2 symbols are read from 'bba'
 - q_3 : 3 symbols are read from 'bba', final state



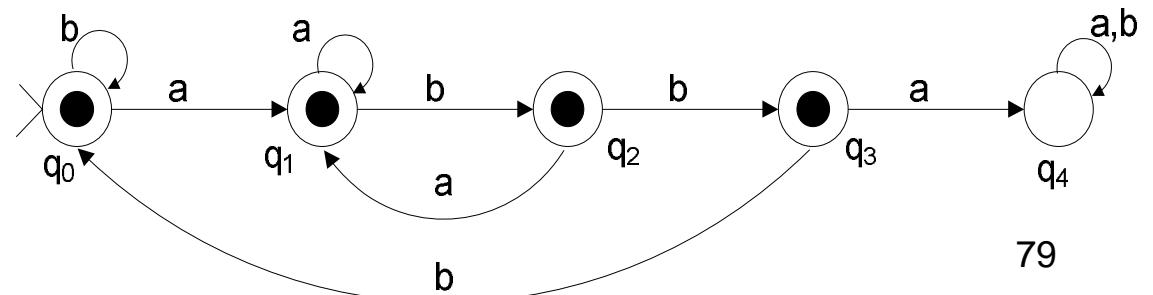
DFA: Example 3

- Define DFA M such that
 - $L(M) = \{w \in \{a, b\}^* \mid w \text{ does not contain "bba"}\}$
- States:
 - q_0 : 0 symbol is read from "bba"
 - q_1 : 1 symbol is read from "bba"
 - q_2 : 2 symbols are read from "bba"
 - q_3 : "bba" is read



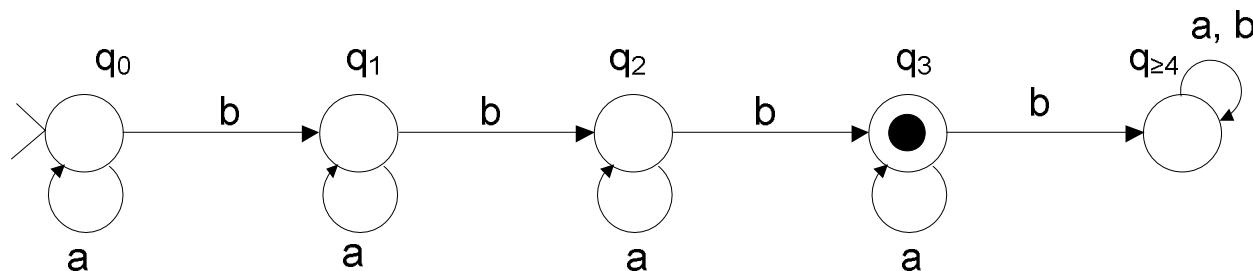
DFA: Example 4

- Define DFA M such that
 - $L(M) = \{w \in \{a, b\}^* \mid w \text{ does not contain 'abba'}\}$
- States:
 - q_0 : 0 symbol is read from 'abba'
 - q_1 : 1 symbol is read from 'abba'
 - q_2 : 2 symbols are read from 'abba'
 - q_3 : 3 symbols are read from 'abba'
 - q_4 : 4 symbols are read from 'abba'



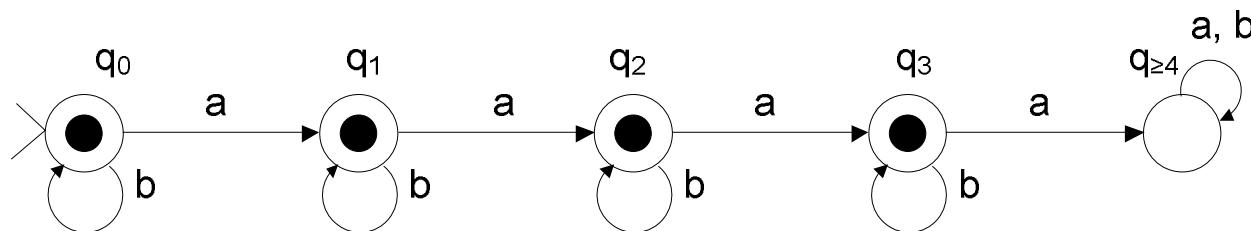
DFA: Example 5

- Define DFA M such that
 - $L(M) = \{w \in \{a, b\}^* \mid \#b = 3\}$
- States:
 - q_0 : $\#b = 0$
 - q_1 : $\#b = 1$
 - q_2 : $\#b = 2$
 - q_3 : $\#b = 3$
 - $q_{\geq 4}$: $\#b \geq 4$



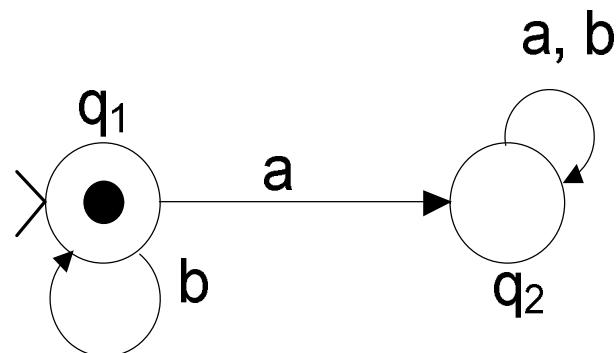
DFA: Example 6

- Define DFA M such that
 - $L(M) = \{w \in \{a, b\}^* \mid \#a < 4\}$
- States:
 - q_0 : $\#a = 0$
 - q_1 : $\#a = 1$
 - q_2 : $\#a = 2$
 - q_3 : $\#a = 3$
 - $q_{\geq 4}$: $\#a \geq 4$



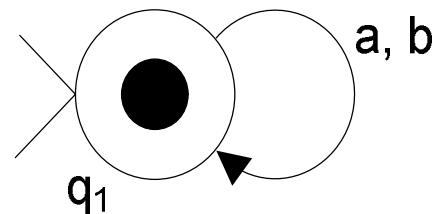
DFA: Example 7

- Define DFA M such that
 - $L(M) = \{w \in \{a, b\}^* \mid \#a = 0\}$
- States:
 - q_1 : 0 'a' is read
 - q_2 : 1 or more 'a' is read



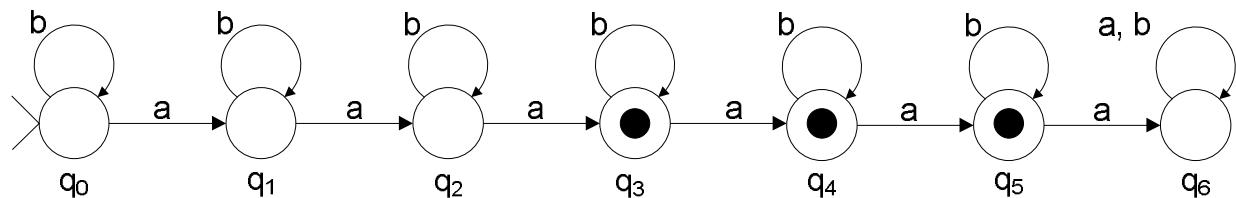
DFA: Example 8

- Define DFA M such that
 - $L(M) = \{w \in \{a, b\}^* \mid \#a \geq 0\}$
- State:
 - q_1 : initial and final state



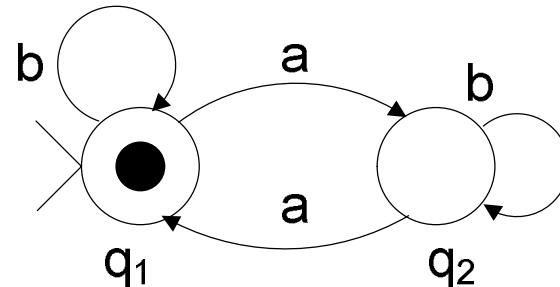
DFA: Example 9

- Define DFA M such that
 - $L(M) = \{w \in \{a, b\}^* \mid 2 < \#a \leq 5\}$
- States:
 - q_0 : $\#a = 0$
 - q_1 : $\#a = 1$
 - q_2 : $\#a = 2$
 - q_3 : $\#a = 3$
 - q_4 : $\#a = 4$
 - q_5 : $\#a = 5$
 - q_6 : $\#a \geq 6$



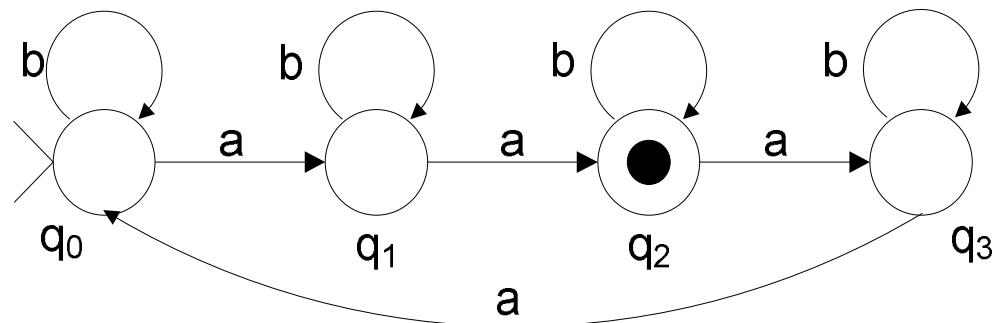
DFA: Example 10

- Define DFA M such that
 - $L(M) = \{w \in \{a, b\}^* \mid \#a \bmod 2 = 0\}$
- States:
 - q_1 : #a is even
 - q_2 : #a is odd



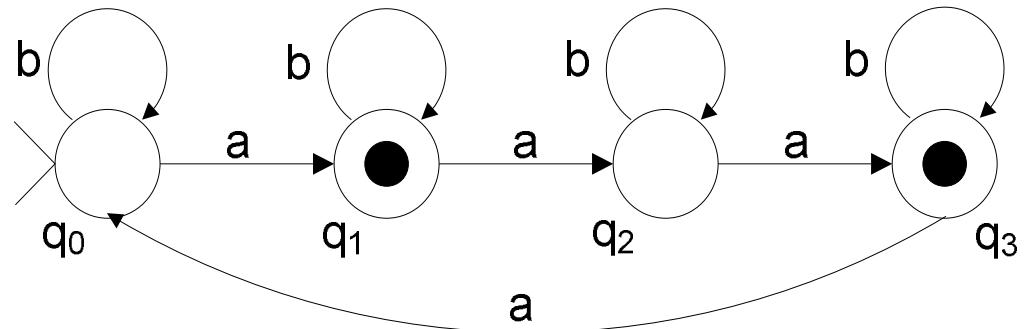
DFA: Example 11

- Define DFA M such that
 - $L(M) = \{w \in \{a, b\}^* \mid \#a \bmod 4 = 2\}$
- States:
 - q_0 : $\#a \bmod 4 = 0$
 - q_1 : $\#a \bmod 4 = 1$
 - q_2 : $\#a \bmod 4 = 2$
 - q_3 : $\#a \bmod 4 = 3$



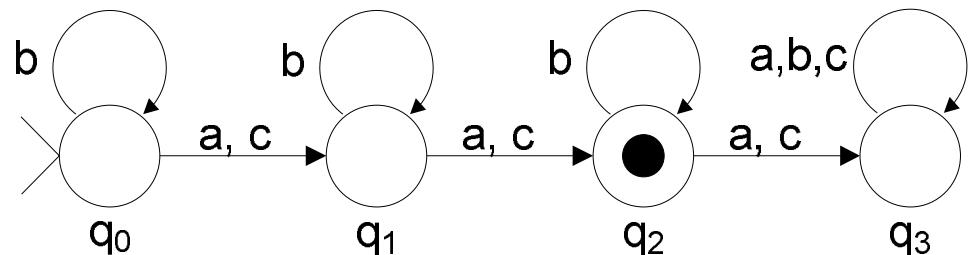
DFA: Example 12

- Define DFA M such that
 - $L(M) = \{w \in \{a, b\}^* \mid \#a \bmod 4 = 1 \text{ or } 3\}$
- States:
 - q_0 : $\#a \bmod 4 = 0$
 - q_1 : $\#a \bmod 4 = 1$
 - q_2 : $\#a \bmod 4 = 2$
 - q_3 : $\#a \bmod 4 = 3$



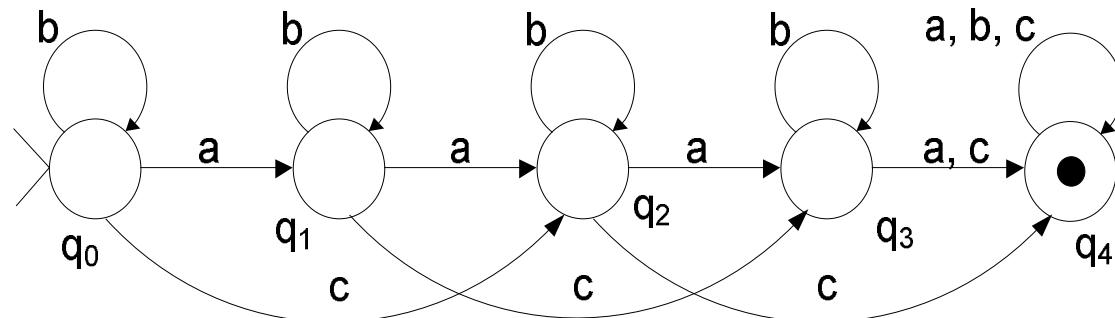
DFA: Example 13

- Define DFA M such that
 - $L(M) = \{w \in \{a, b, c\}^* \mid \#a + \#c = 2\}$
- States:
 - q_0 : $\#a + \#c = 0$
 - q_1 : $\#a + \#c = 1$
 - q_2 : $\#a + \#c = 2$
 - q_3 : $\#a + \#c \geq 3$



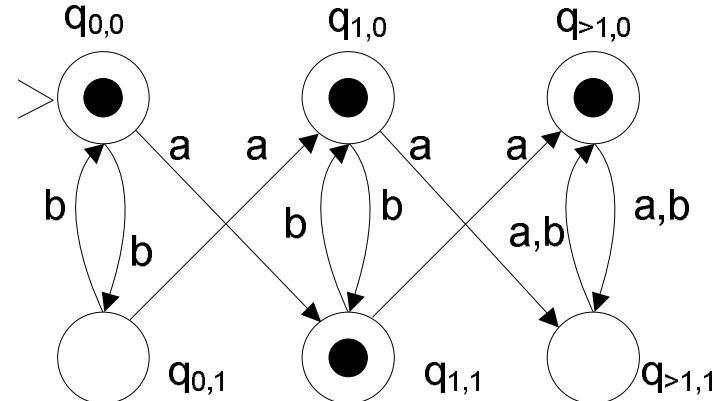
DFA: Example 14

- Define DFA M such that
 - $L(M) = \{w \in \{a, b, c\}^* \mid \#a + 2\#c \geq 4\}$
- States:
 - q_0 : $\#a + 2\#c = 0$
 - q_1 : $\#a + 2\#c = 1$
 - q_2 : $\#a + 2\#c = 2$
 - q_3 : $\#a + 2\#c = 3$
 - q_4 : $\#a + 2\#c \geq 4$



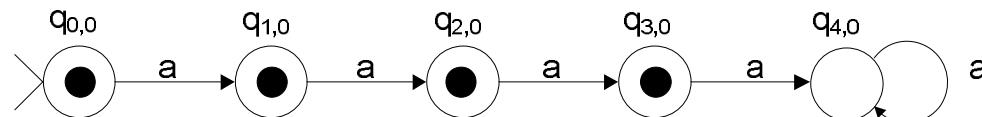
DFA: Example 15

- Define DFA M such that
 - $L(M) = \{w \in \{a, b\}^* \mid (\#a = 1) \text{ or } ((\#a + \#b) \bmod 2 = 0)\}$
- States: two independent conditions
 - $q_{0,j}$: $\#a = 0$
 - $q_{1,j}$: $\#a = 1$
 - $q_{>1,j}$: $\#a > 1$
 - $q_{i,0}$: $(\#a + \#b) \bmod 2 = 0$
 - $q_{i,1}$: $(\#a + \#b) \bmod 2 = 1$



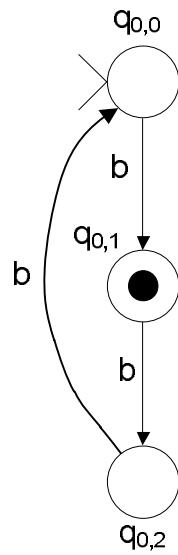
DFA: Example 16

- Define DFA M such that
 - $L(M) = \{w \in \{a, b\}^* \mid (\#a \leq 3) \& (\#b \bmod 3 = 1)\}$
- States: $q_{i,j}$: $\#a = i$ and $\#b \bmod 3 = j$



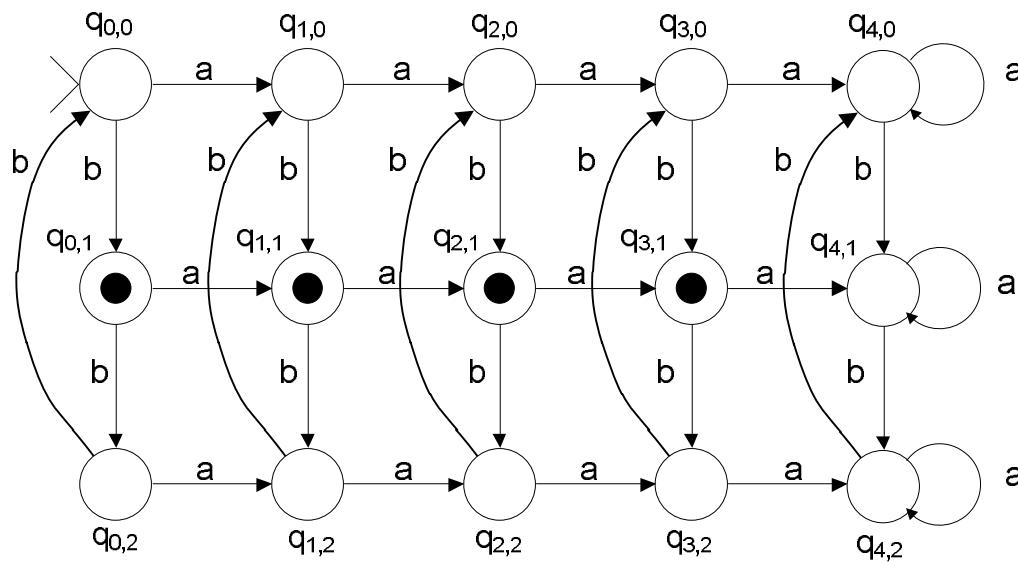
DFA: Example 16

- Define DFA M such that
 - $L(M) = \{w \in \{a, b\}^* \mid (\#a \leq 3) \& (\#b \bmod 3 = 1)\}$
- States: $q_{i,j}$: $\#a = i$ and $\#b \bmod 3 = j$



DFA: Example 16

- Define DFA M such that
 - $L(M) = \{w \in \{a, b\}^* \mid (\#a \leq 3) \& (\#b \bmod 3 = 1)\}$
- States: $q_{i,j}$: $\#a = i$ and $\#b \bmod 3 = j$



DFA: Examples

- Define DFA M such that
 - $L = \{w \in \{a, b\}^* \mid w \text{ contains "aabbaabb"}\}$
 - $L = \{w \in \{a, b\}^* \mid \#b = 3 \text{ or } 6\}$
 - $L = \{w \in \{a, b\}^* \mid 3 < a\# \leq 6\}$
 - $L = \{w \in \{a, b\}^* \mid \#a \bmod 5 = 3\}$
 - $L = \{w \in \{a, b\}^* \mid \#b \bmod 3 = 1 \text{ or } 2\}$
 - $L = \{w \in \{a, b, c\}^* \mid \#a + 3\#c = 1\}$
 - $L = \{w \in \{a, b, c\}^* \mid 2\#a + \#b + 3\#c < 7\}$
 - $L(M) = \{w \in \{a, b\}^* \mid (\#b \leq 4) \text{ \& } (\#b \bmod 5 = 2 \text{ or } 3)\}$

DFA

- Give the transition diagram of the DFA!
 - $M = (K, \Sigma, \delta, s, F)$
 - $K = \{q_0, q_1\}$
 - $\Sigma = \{a, b\}$
 - $s = q_0$
 - $F = \{q_1\}$
- $L(M) = \{w: \#a \text{ is odd}\}$

q	σ	$\delta(q, \sigma)$
q_0	a	q_1
q_0	b	q_0
q_1	a	q_0
q_1	b	q_1

DFA

- Process bbaba with the DFA!
 $(q_0, \text{bbaba}) \xrightarrow{M} (q_0, \text{baba})$
 $\xrightarrow{M} (q_0, \text{aba})$
 $\xrightarrow{M} (q_1, \text{ba})$
 $\xrightarrow{M} (q_1, \text{a})$
 $\xrightarrow{M} (q_0, \text{e})$
- bbaba is rejected, because q_0 is not a final state

DFA

- Give the transition diagram of the DFA!
 - $M = (K, \Sigma, \delta, s, F)$
 - $K = \{q_0, q_1, q_2\}$
 - $\Sigma = \{a, b\}$
 - $s = q_0$
 - $F = \{q_1\}$
- $L(M) = \{w : \#a \geq 2\}$

q	σ	$\delta(q, \sigma)$
q_0	a	q_1
q_0	b	q_0
q_1	a	q_2
q_1	b	q_1
q_2	a	q_2
q_2	b	q_2

DFA

- Process abbbaba with the DFA!
 $(q_0, \text{abbbaba}) \xrightarrow{-M} (q_1, \text{bbbaba})$
 $\xrightarrow{-M} (q_1, \text{bbaba})$
 $\xrightarrow{-M} (q_1, \text{baba})$
 $\xrightarrow{-M} (q_1, \text{aba})$
 $\xrightarrow{-M} (q_2, \text{ba})$
 $\xrightarrow{-M} (q_2, \text{a})$
 $\xrightarrow{-M} (q_2, \text{e})$
- abbbaba is accepted

Element of the Theory of the Computation

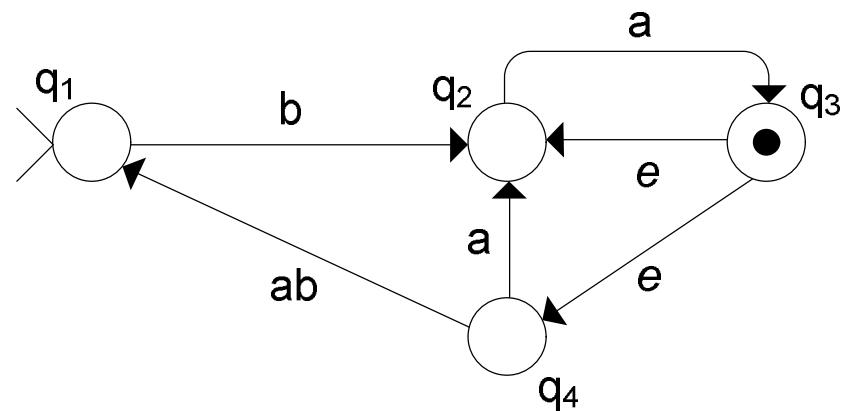
Lecture 5

NFA → DFA

- Steps of NFA to DFA conversion
 - eliminating string transitions by introducing new states
 - definition of e-sets
 - definition of new initial state
 - definition of transition function
 - definition of finite states

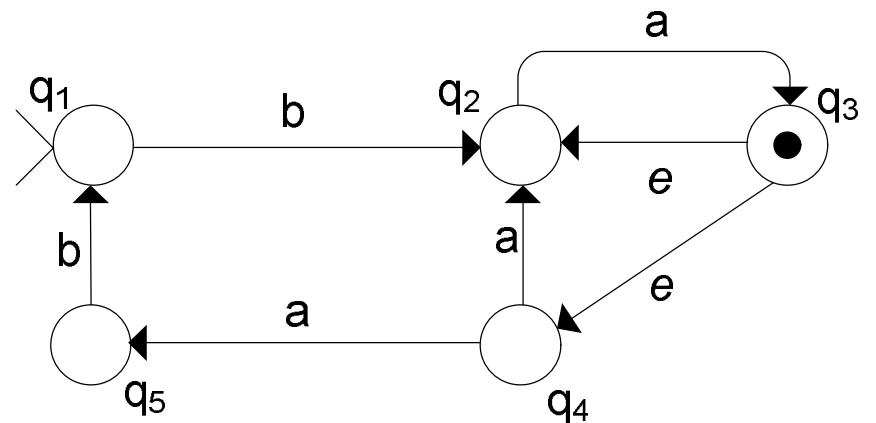
NFA \rightarrow DFA: Example 1

- Give DFA M' such that $L(M') = L(M)$!
 - M is given below



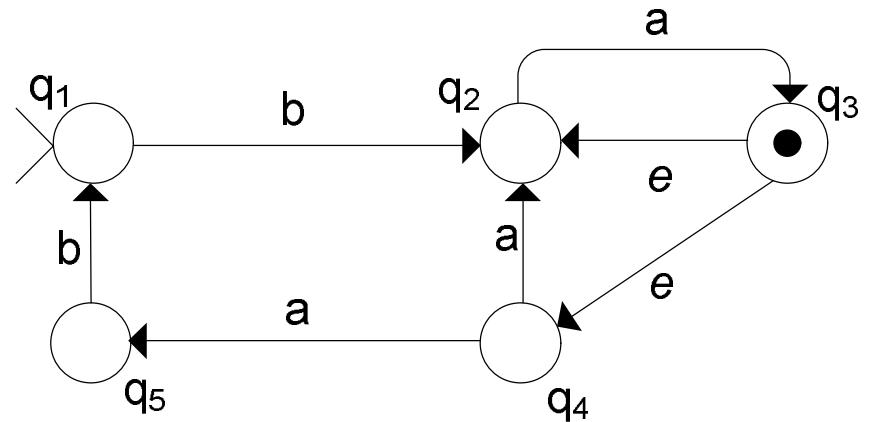
NFA → DFA: Example 1

- Eliminating string transitions by introducing new states



NFA \rightarrow DFA: Example 1

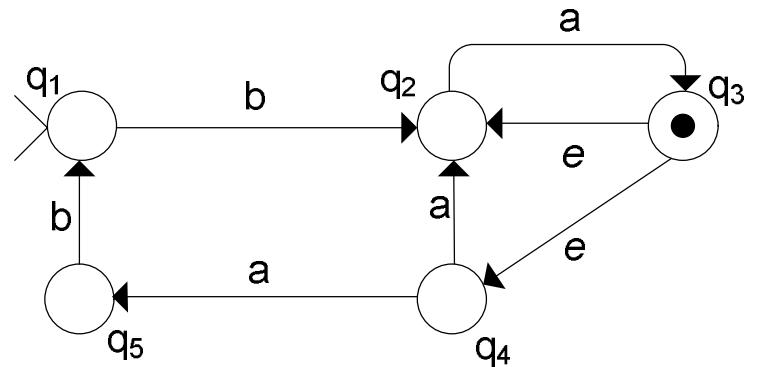
- Definition of E-sets:
 - $E(q_1) = \{q_1\}$
 - $E(q_2) = \{q_2\}$
 - $E(q_3) = \{q_2, q_3, q_4\}$
 - $E(q_4) = \{q_4\}$
 - $E(q_5) = \{q_5\}$
- Definition of new initial state
 - $s_3 = E(q_1) = \{q_1\} = Q_1$



NFA → DFA: Example 1

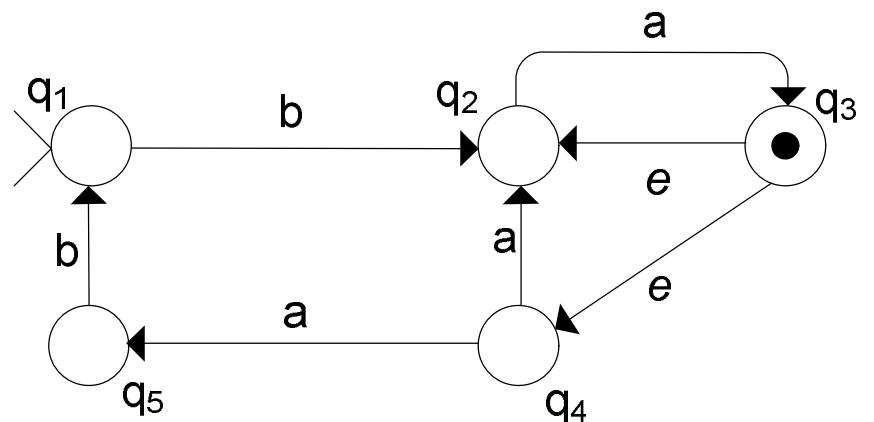
- Definition of transition function:

- $\delta(\{q_1\}, a) = \emptyset = Q_2$
- $\delta(\{q_1\}, b) = E(q_2) = \{q_2\} = Q_3$
- $\delta(\emptyset, a) = \emptyset = Q_2$
- $\delta(\emptyset, b) = \emptyset = Q_2$
- $\delta(\{q_2\}, a) = E(q_3) = \{q_2, q_3, q_4\} = Q_4$
- $\delta(\{q_2\}, b) = \emptyset = Q_2$
- $\delta(\{q_2, q_3, q_4\}, a) = E(q_2) \cup E(q_3) \cup E(q_5) = \{q_2, q_3, q_4, q_5\} = Q_5$
- $\delta(\{q_2, q_3, q_4\}, b) = \emptyset = Q_2$

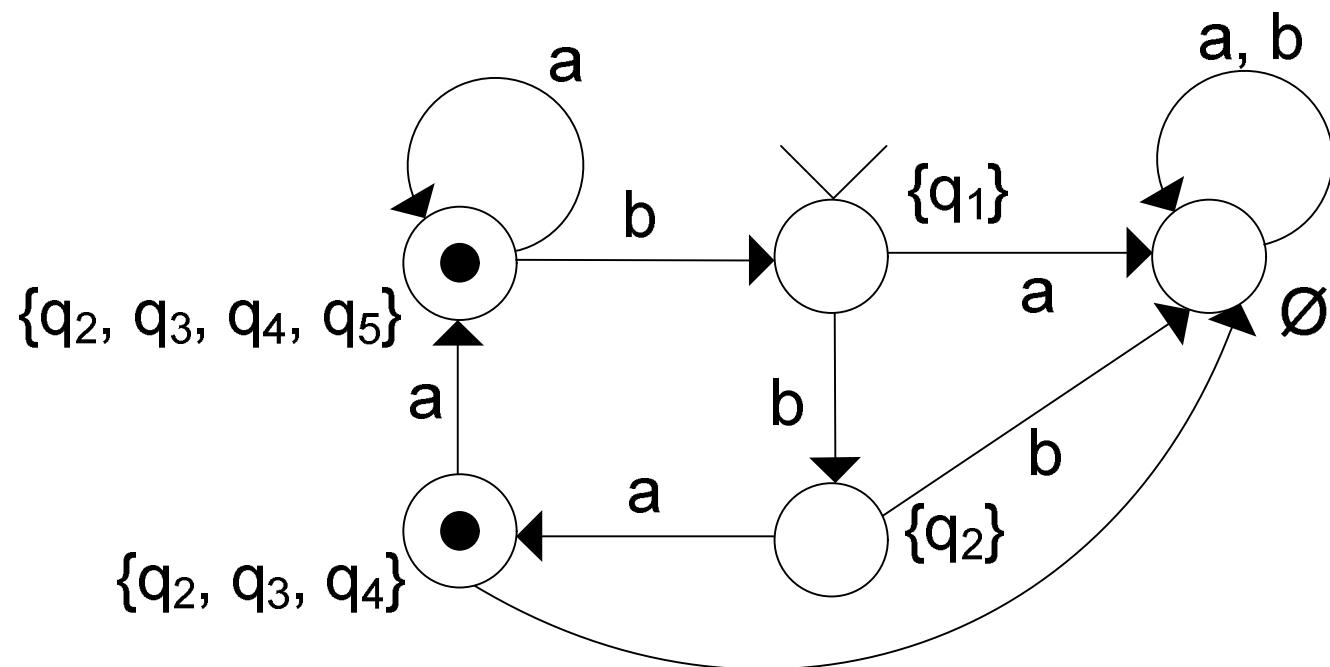


NFA \rightarrow DFA: Example 1

- $\delta(\{q_2, q_3, q_4, q_5\}, a) = E(q_2) \cup E(q_3) \cup E(q_5) = \{q_2, q_3, q_4, q_5\} = Q_5$
- $\delta(\{q_2, q_3, q_4, q_5\}, b) = E(q_1) = \{q_1\} = Q_1$
- Definition of final states
 - $F_3 = \{\{q_2, q_3, q_4\}, \{q_2, q_3, q_4, q_5\}\}$

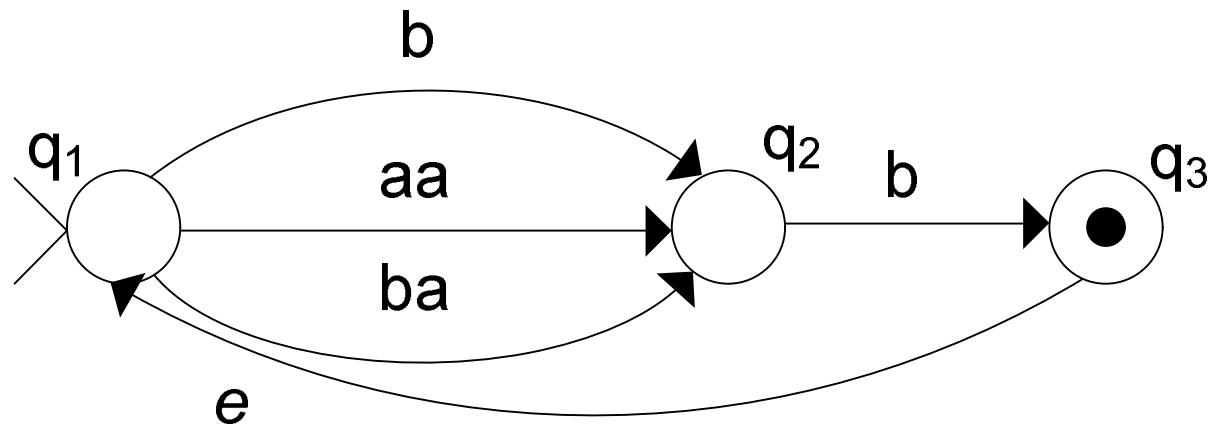


NFA \rightarrow DFA: Example 1



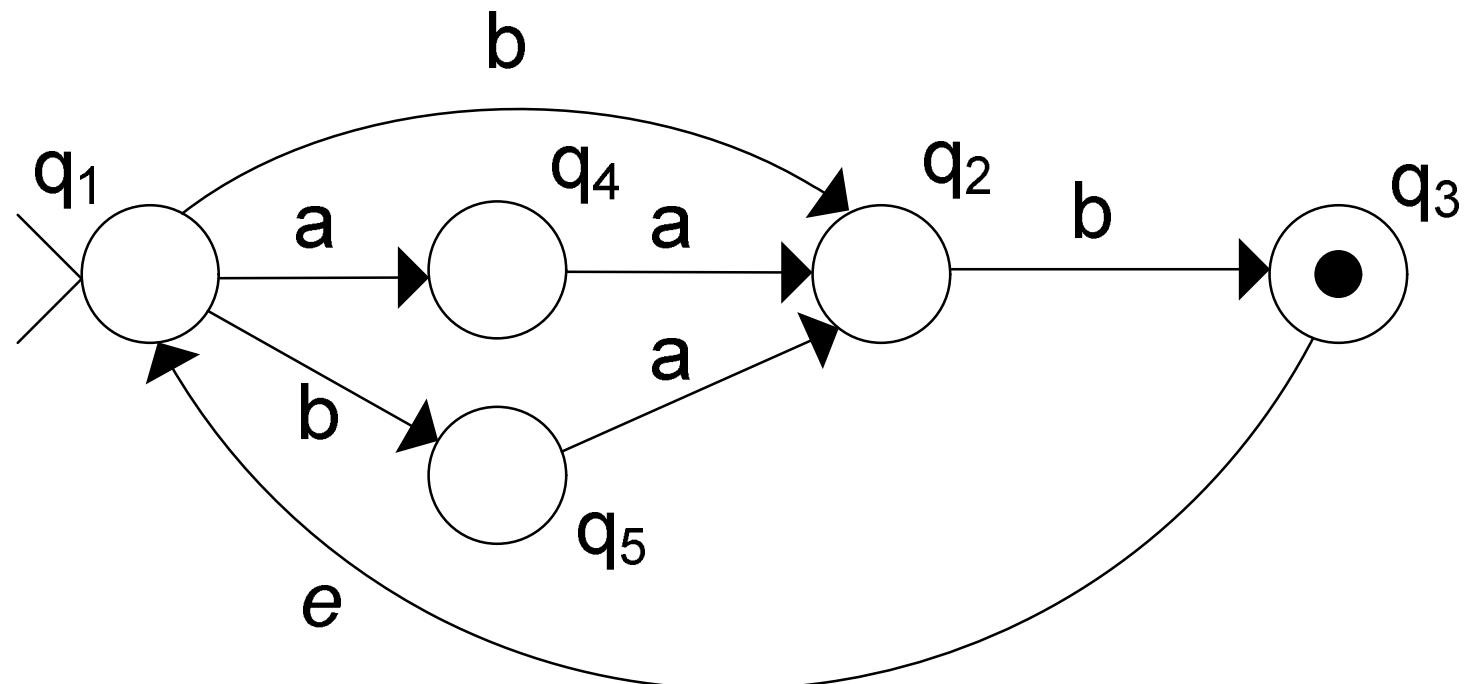
NFA \rightarrow DFA: Example 2

- Give DFA M' such that $L(M') = L(M)$!
 - M is given below



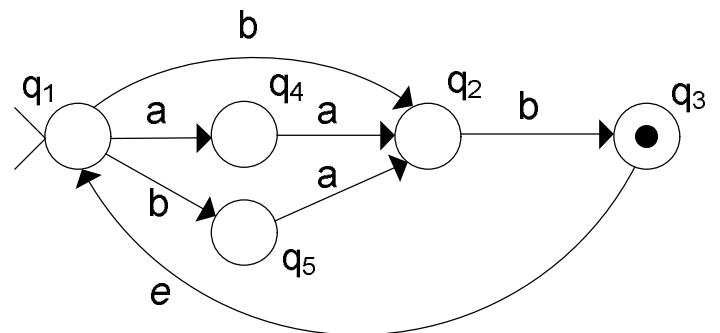
NFA → DFA: Example 2

- Eliminating string transitions by introducing new states



NFA \rightarrow DFA: Example 2

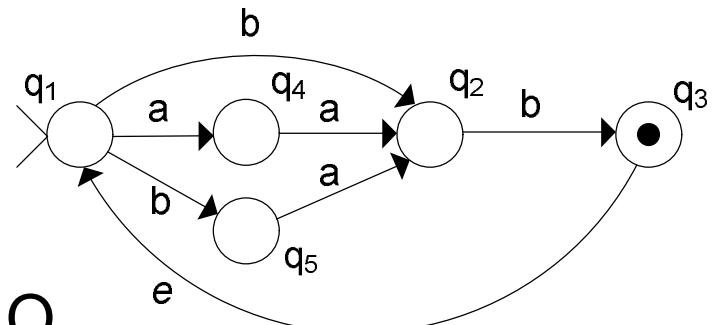
- Definition of E-sets:
 - $E(q_1) = \{q_1\}$
 - $E(q_2) = \{q_2\}$
 - $E(q_3) = \{q_1, q_3\}$
 - $E(q_4) = \{q_4\}$
 - $E(q_5) = \{q_5\}$
- Definition of new initial state
 - $s_3 = E(q_1) = \{q_1\} = Q_1$



NFA → DFA: Example 2

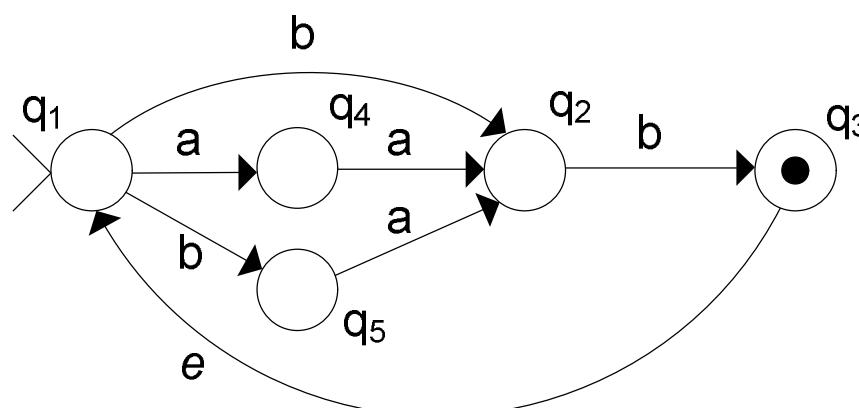
- Definition of transition function:

- $\delta(\{q_1\}, a) = E(q_4) = \{q_4\} = Q_2$
- $\delta(\{q_1\}, b) = E(q_2) \cup E(q_5) = \{q_2, q_5\} = Q_3$
- $\delta(\{q_4\}, a) = E(q_2) = \{q_2\} = Q_4$
- $\delta(\{q_4\}, b) = \emptyset = Q_5$
- $\delta(\{q_2, q_5\}, a) = E(q_2) = \{q_2\} = Q_4$
- $\delta(\{q_2, q_5\}, b) = E(q_3) = \{q_1, q_3\} = Q_6$
- $\delta(\{q_2\}, a) = \emptyset = Q_5$
- $\delta(\{q_2\}, b) = E(q_3) = \{q_1, q_3\} = Q_6$

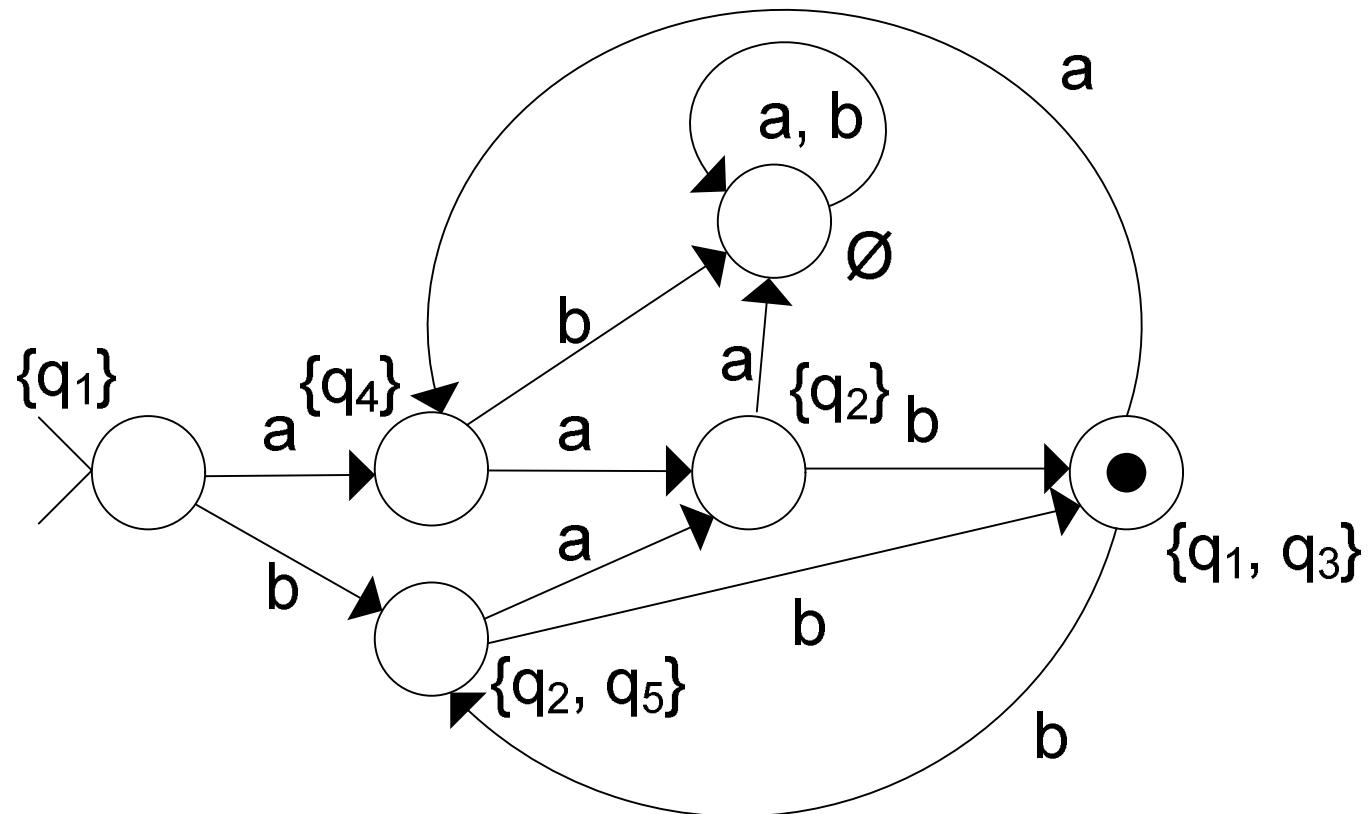


NFA \rightarrow DFA: Example 2

- $\delta(\emptyset, a) = \emptyset = Q_5$
- $\delta(\emptyset, b) = \emptyset = Q_5$
- $\delta(\{q_1, q_3\}, a) = E(q_4) = \{q_4\} = Q_2$
- $\delta(\{q_1, q_3\}, b) = E(q_2) \cup E(q_5) = \{q_2, q_5\} = Q_3$
- Definition of final states
 - $F_3 = \{\{q_1, q_3\}\}$

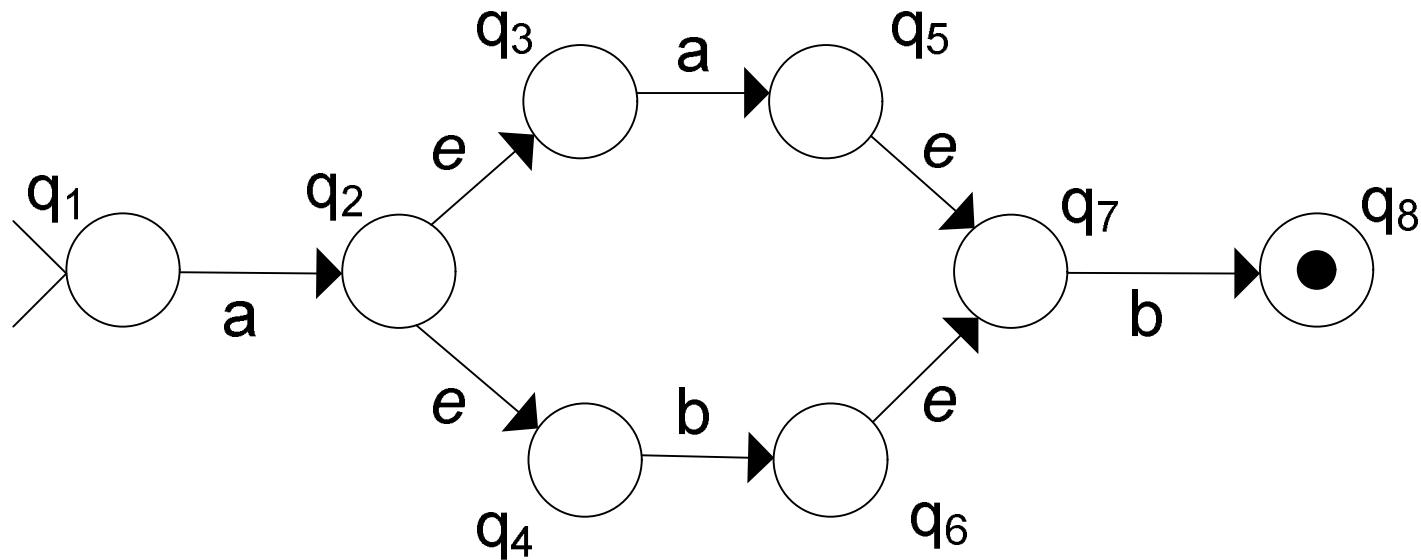


NFA \rightarrow DFA: Example 2



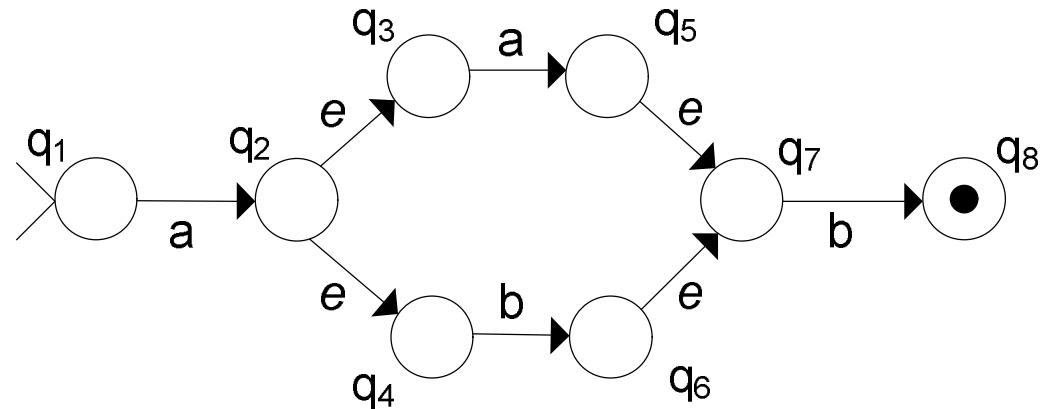
NFA \rightarrow DFA: Example 3

- Give DFA M' such that $L(M') = L(M)$!
 - M is given below



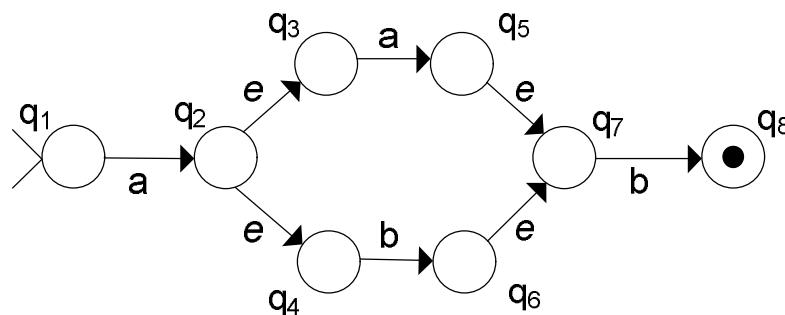
NFA \rightarrow DFA: Example 3

- Definition of E-sets:
 - $E(q_1) = \{q_1\}$
 - $E(q_2) = \{q_2, q_3, q_4\}$
 - $E(q_3) = \{q_3\}$
 - $E(q_4) = \{q_4\}$
 - $E(q_5) = \{q_5, q_7\}$
 - $E(q_6) = \{q_6, q_7\}$
 - $E(q_7) = \{q_7\}$
 - $E(q_8) = \{q_8\}$



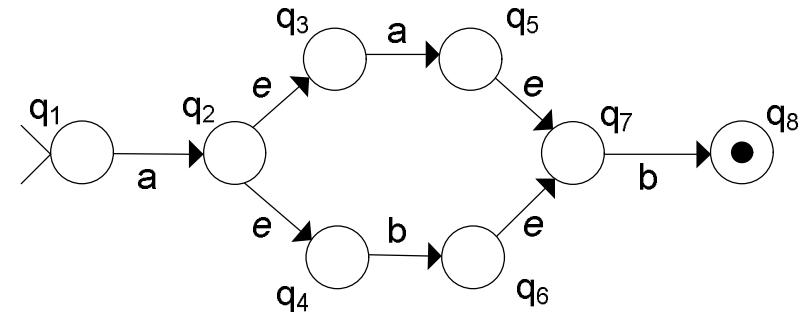
NFA \rightarrow DFA: Example 3

- Definition of new initial state
 - $s_3 = E(q_1) = \{q_1\} = Q_1$
- Definition of transition function:
 - $\delta(\{q_1\}, a) = E(q_2) = \{q_2, q_3, q_4\} = Q_2$
 - $\delta(\{q_1\}, b) = \emptyset = Q_3$
 - $\delta(\{q_2, q_3, q_4\}, a) = E(q_5) = \{q_5, q_7\} = Q_4$
 - $\delta(\{q_2, q_3, q_4\}, b) = E(q_6) = \{q_6, q_7\} = Q_5$
 - $\delta(\emptyset, a) = \emptyset = Q_3$
 - $\delta(\emptyset, b) = \emptyset = Q_3$

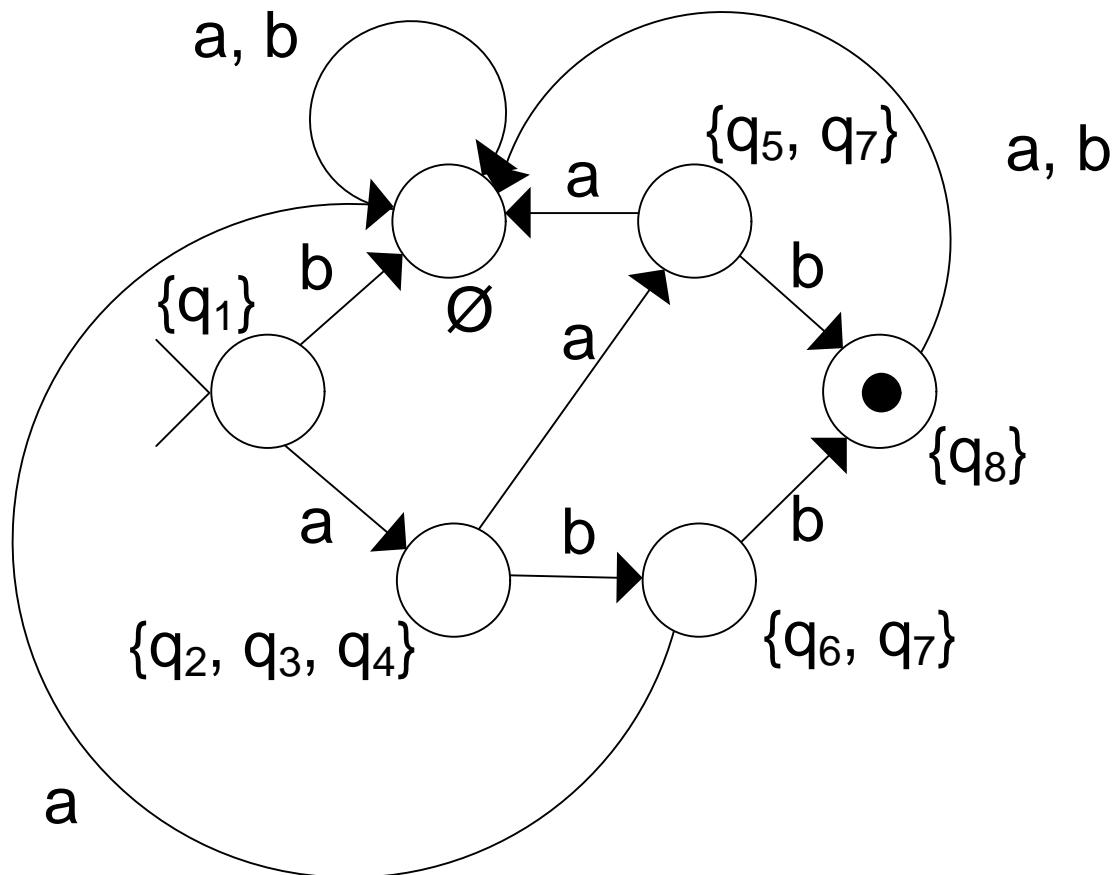


NFA → DFA: Example 3

- $\delta(\{q_5, q_7\}, a) = \emptyset = Q_3$
- $\delta(\{q_5, q_7\}, b) = E(q_8) = \{q_8\} = Q_6$
- $\delta(\{q_6, q_7\}, a) = \emptyset = Q_3$
- $\delta(\{q_6, q_7\}, b) = E(q_8) = \{q_8\} = Q_6$
- $\delta(\{q_8\}, a) = \emptyset = Q_3$
- $\delta(\{q_8\}, b) = \emptyset = Q_3$
- Definition of final states
 - $F_3 = \{\{q_8\}\}$

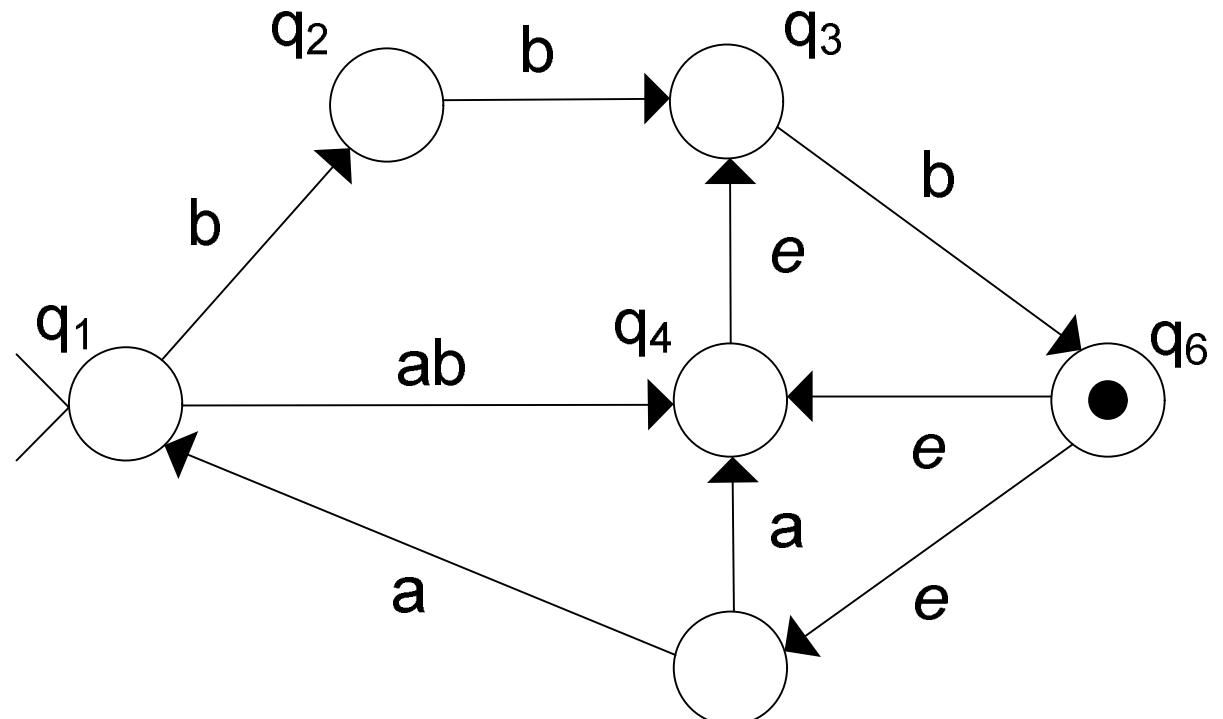


NFA \rightarrow DFA: Example 3



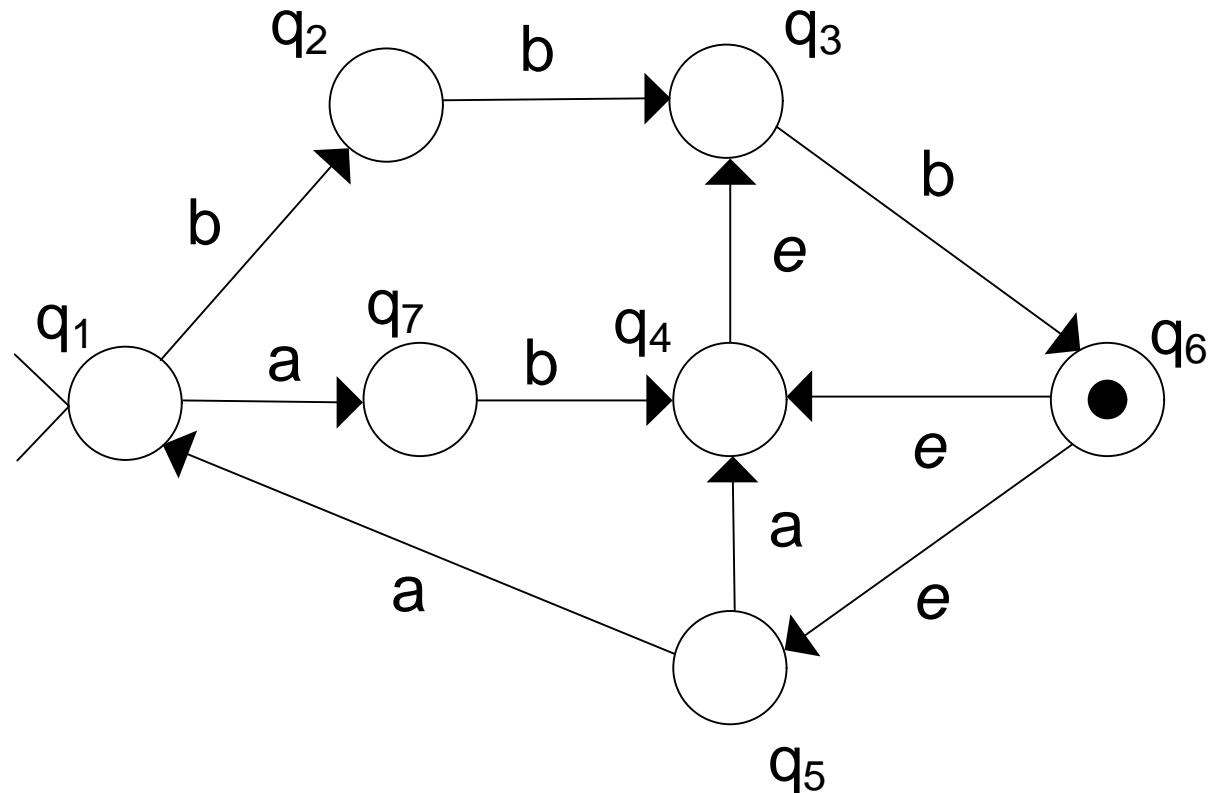
NFA \rightarrow DFA: Example 4

- Give DFA M' such that $L(M') = L(M)$!
 - M is given below



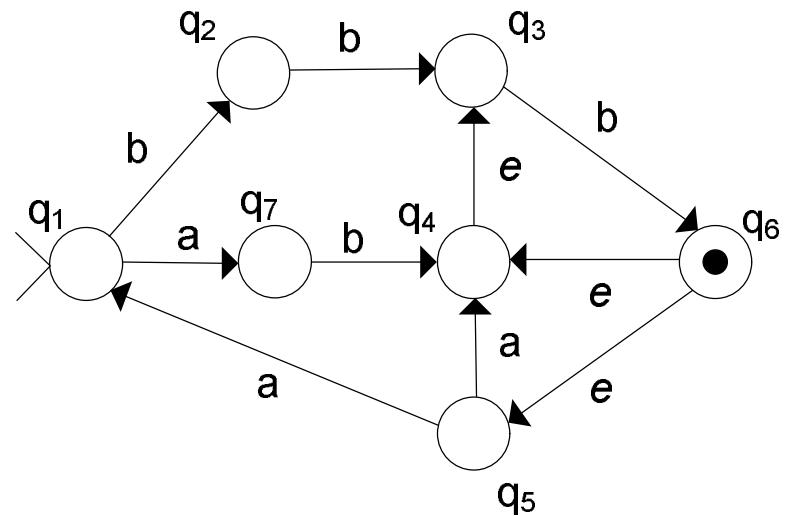
NFA \rightarrow DFA: Example 4

- Eliminating string transitions by introducing new states



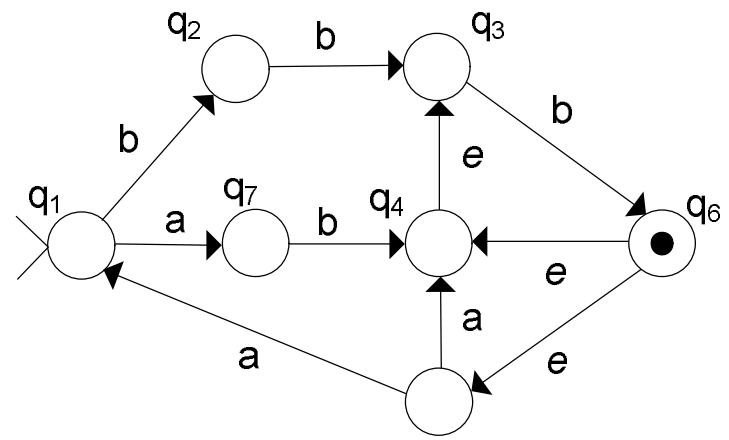
NFA \rightarrow DFA: Example 4

- Definition of E-sets:
 - $E(q_1) = \{q_1\}$
 - $E(q_2) = \{q_2\}$
 - $E(q_3) = \{q_3\}$
 - $E(q_4) = \{q_3, q_4\}$
 - $E(q_5) = \{q_5\}$
 - $E(q_6) = \{q_3, q_4, q_5, q_6\}$
 - $E(q_7) = \{q_7\}$



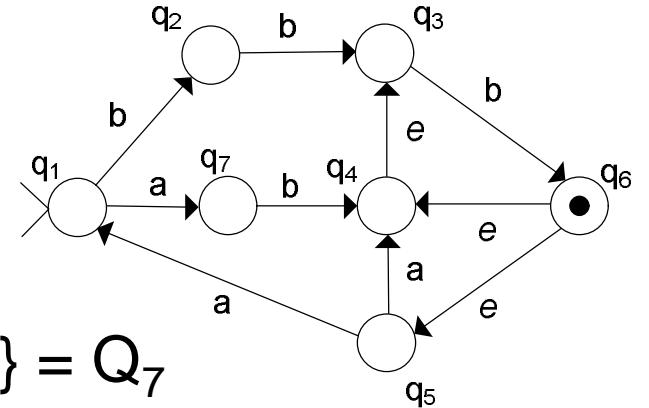
NFA \rightarrow DFA: Example 4

- Definition of new initial state
 - $s_3 = E(q_1) = \{q_1\} = Q_1$
- Definition of transition function:
 - $\delta(\{q_1\}, a) = E(q_7) = \{q_7\} = Q_2$
 - $\delta(\{q_1\}, b) = E(q_2) = \{q_2\} = Q_3$
 - $\delta(\{q_7\}, a) = \emptyset = Q_4$
 - $\delta(\{q_7\}, b) = E(q_3) \cup E(q_4) = \{q_3, q_4\} = Q_5$
 - $\delta(\{q_2\}, a) = \emptyset = Q_4$
 - $\delta(\{q_2\}, b) = E(q_3) = \{q_3\} = Q_6$
 - $\delta(\emptyset, a) = \emptyset = Q_4$
 - $\delta(\emptyset, b) = \emptyset = Q_4$



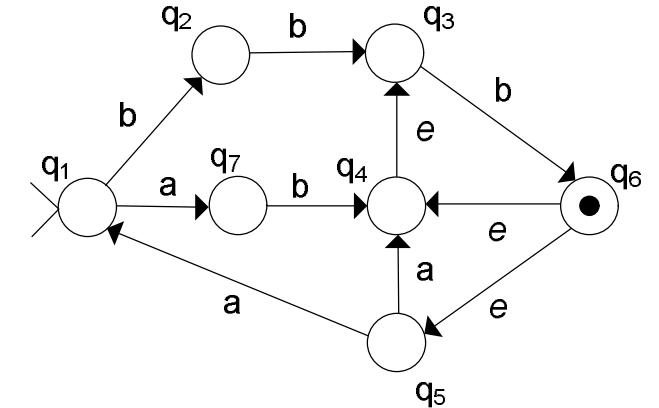
NFA → DFA: Example 4

- $\delta(\{q_3, q_4\}, a) = \emptyset = Q_4$
- $\delta(\{q_3, q_4\}, b) = E(q_6) = \{q_3, q_4, q_5, q_6\} = Q_7$
- $\delta(\{q_3\}, a) = \emptyset = Q_4$
- $\delta(\{q_3\}, b) = E(q_6) = \{q_3, q_4, q_5, q_6\} = Q_7$
- $\delta(\{q_3, q_4, q_5, q_6\}, a) = E(q_1) \cup E(q_4) = \{q_1, q_3, q_4\} = Q_8$
- $\delta(\{q_3, q_4, q_5, q_6\}, b) = E(q_6) = \{q_3, q_4, q_5, q_6\} = Q_7$

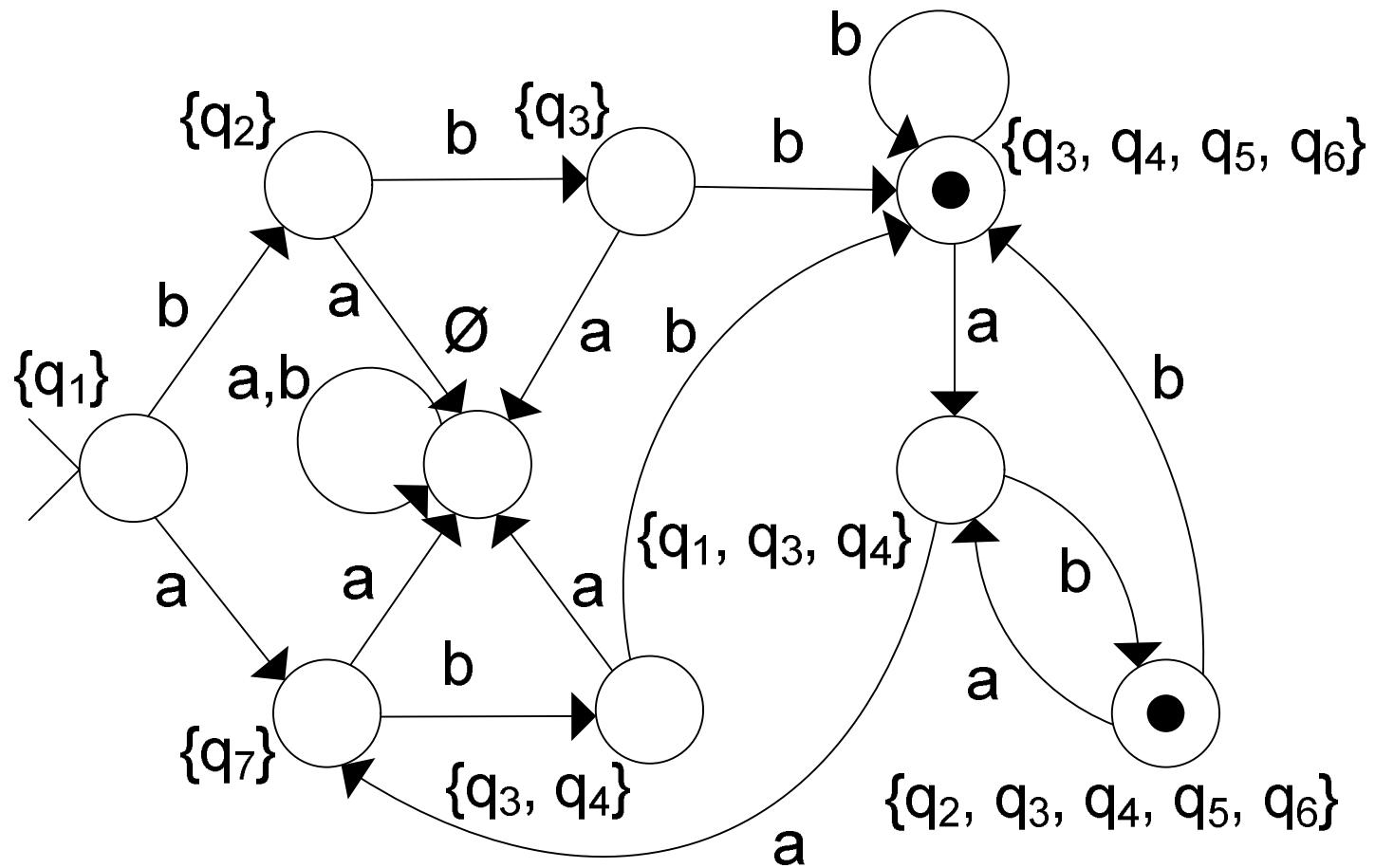


NFA → DFA: Example 4

- $\delta(\{q_1, q_3, q_4\}, a) = E(q_7) = \{q_7\} = Q_2$
- $\delta(\{q_1, q_3, q_4\}, b) = E(q_2) \cup E(q_6) = \{q_2, q_3, q_4, q_5, q_6\} = Q_9$
- $\delta(\{q_2, q_3, q_4, q_5, q_6\}, a) = E(q_1) \cup E(q_4) = \{q_1, q_3, q_4\} = Q_8$
- $\delta(\{q_2, q_3, q_4, q_5, q_6\}, b) = E(q_3) \cup E(q_6) = \{q_3, q_4, q_5, q_6\} = Q_7$
- Definition of final states
 - $F_3 = \{\{q_3, q_4, q_5, q_6\}, \{q_2, q_3, q_4, q_5, q_6\}\}$

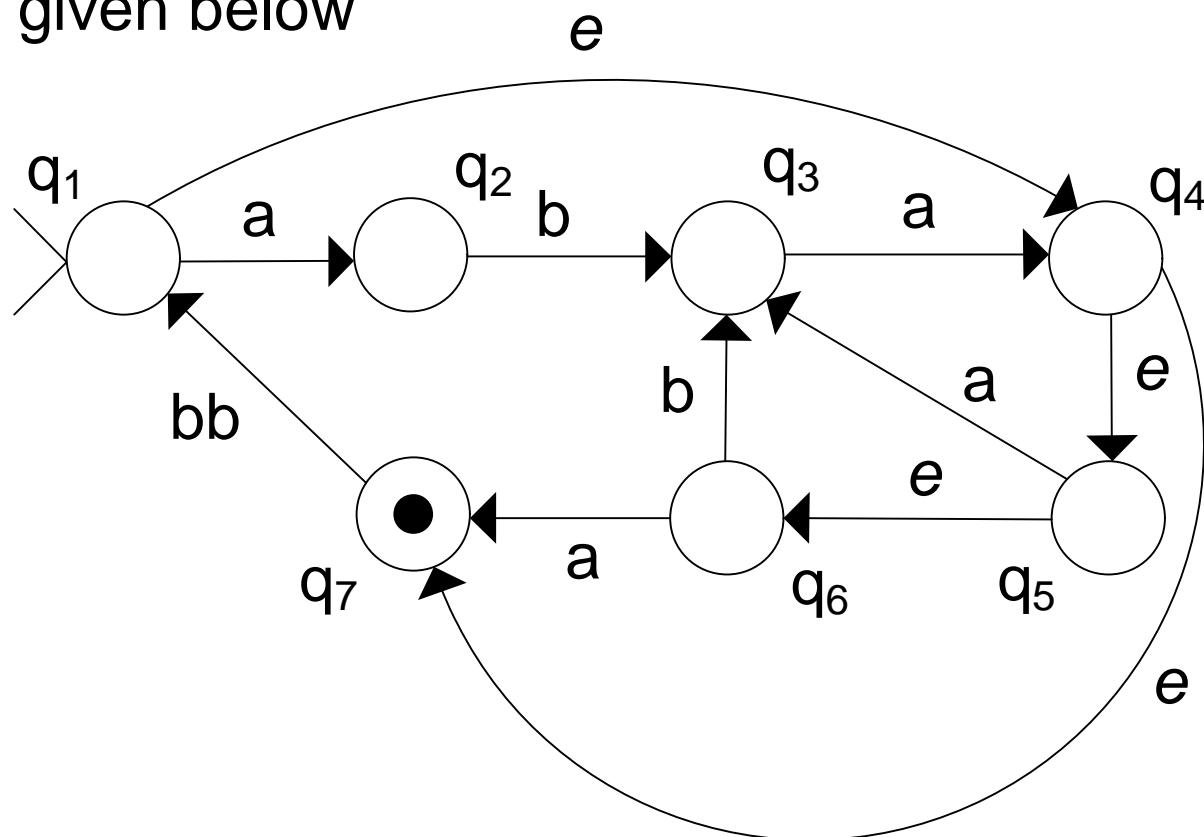


NFA \rightarrow DFA: Example 4



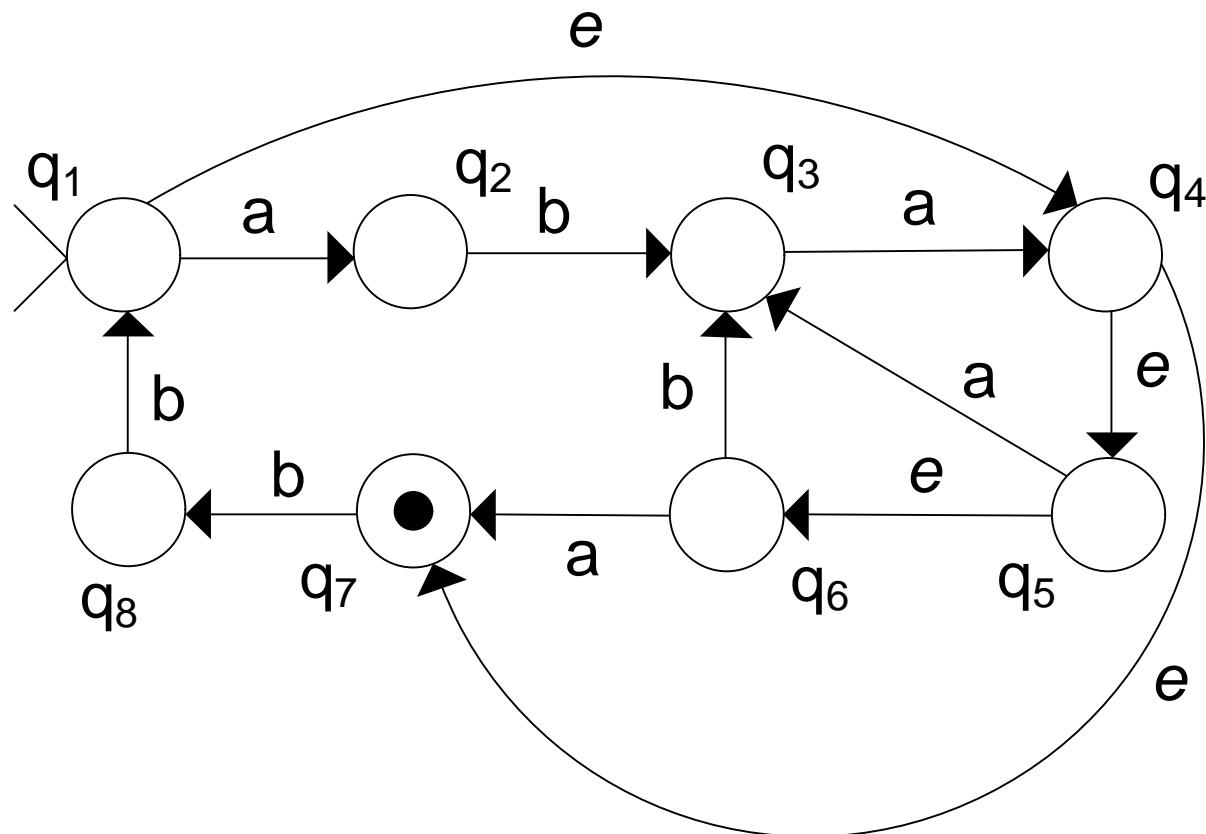
NFA \rightarrow DFA: Example 5

- Give DFA M' such that $L(M') = L(M)$!
 - M is given below



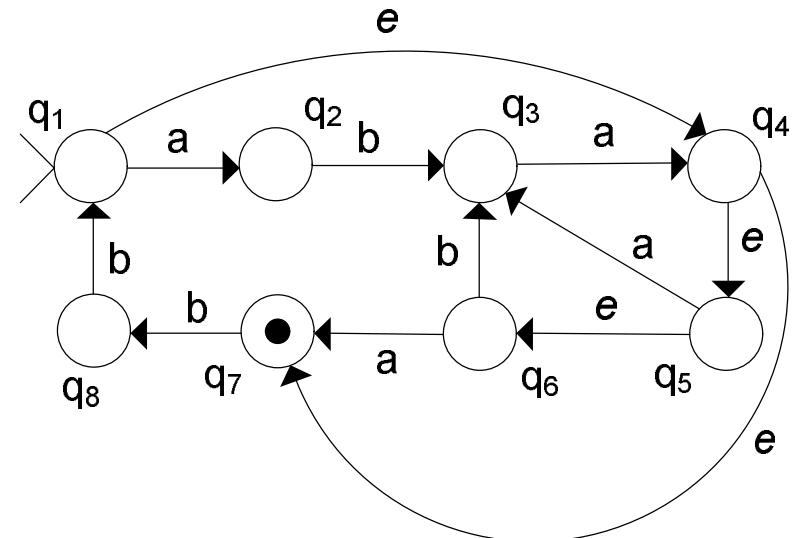
NFA \rightarrow DFA: Example 5

- Eliminating string transitions by introducing new states

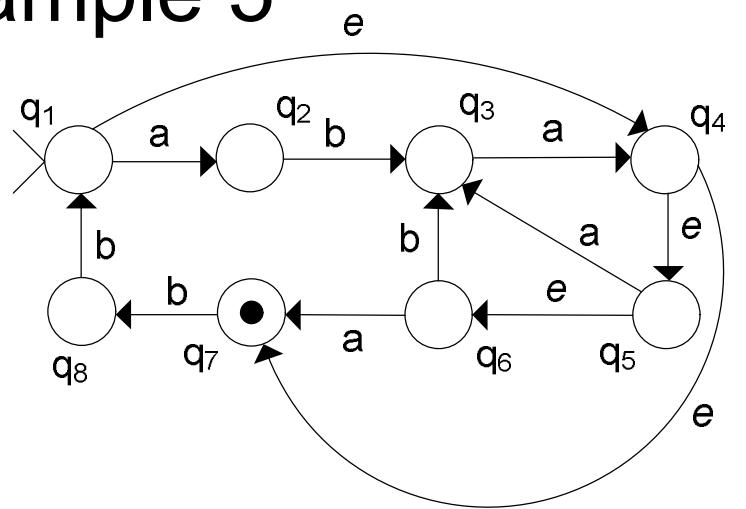


NFA \rightarrow DFA: Example 5

- Definition of E-sets:
 - $E(q_1) = \{q_1, q_4, q_5, q_6, q_7\}$
 - $E(q_2) = \{q_2\}$
 - $E(q_3) = \{q_3\}$
 - $E(q_4) = \{q_4, q_5, q_6, q_7\}$
 - $E(q_5) = \{q_5, q_6\}$
 - $E(q_6) = \{q_6\}$
 - $E(q_7) = \{q_7\}$
 - $E(q_8) = \{q_8\}$



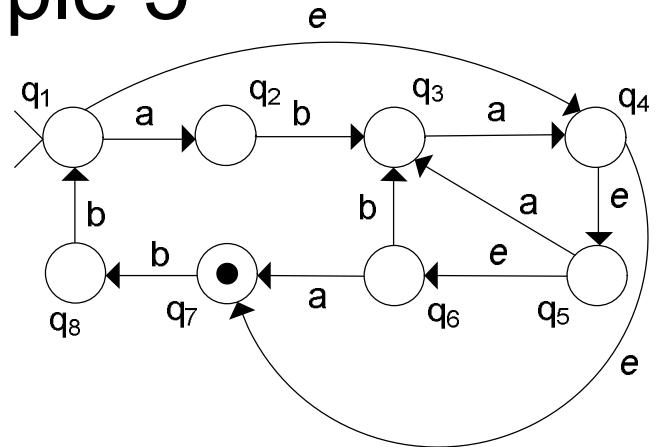
NFA → DFA: Example 5



- Definition of new initial state
 - $s_3 = E(q_1) = \{q_1, q_4, q_5, q_6, q_7\} = Q_1$
- Definition of transition function:
 - $\delta(\{q_1, q_4, q_5, q_6, q_7\}, a) = E(q_2) \cup E(q_3) \cup E(q_7) = \{q_2, q_3, q_7\} = Q_2$
 - $\delta(\{q_1, q_4, q_5, q_6, q_7\}, b) = E(q_3) \cup E(q_8) = \{q_3, q_8\} = Q_3$

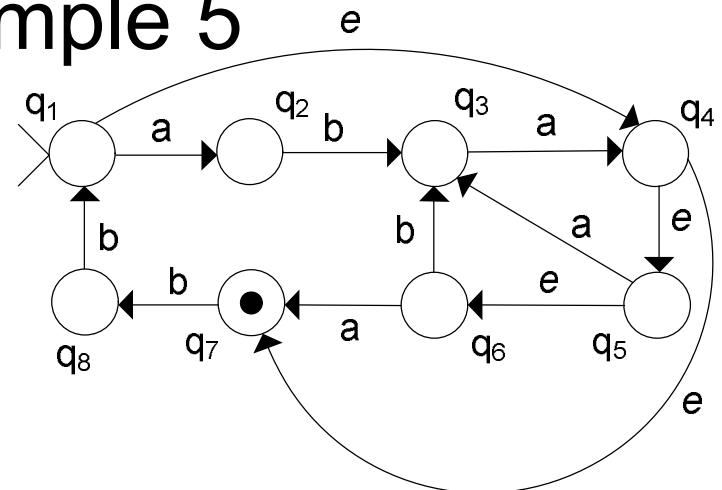
NFA → DFA: Example 5

- $\delta(\{q_2, q_3, q_7\}, a) = E(q_4) = \{q_4, q_5, q_6, q_7\} = Q_4$
- $\delta(\{q_2, q_3, q_7\}, b) = E(q_3) \cup E(q_8) = \{q_3, q_8\} = Q_3$
- $\delta(\{q_3, q_8\}, a) = E(q_4) = \{q_4, q_5, q_6, q_7\} = Q_4$
- $\delta(\{q_3, q_8\}, b) = E(q_1) = \{q_1, q_4, q_5, q_6, q_7\} = Q_1$
- $\delta(\{q_4, q_5, q_6, q_7\}, a) = E(q_3) \cup E(q_7) = \{q_3, q_7\} = Q_5$
- $\delta(\{q_4, q_5, q_6, q_7\}, b) = E(q_3) \cup E(q_8) = \{q_3, q_8\} = Q_3$



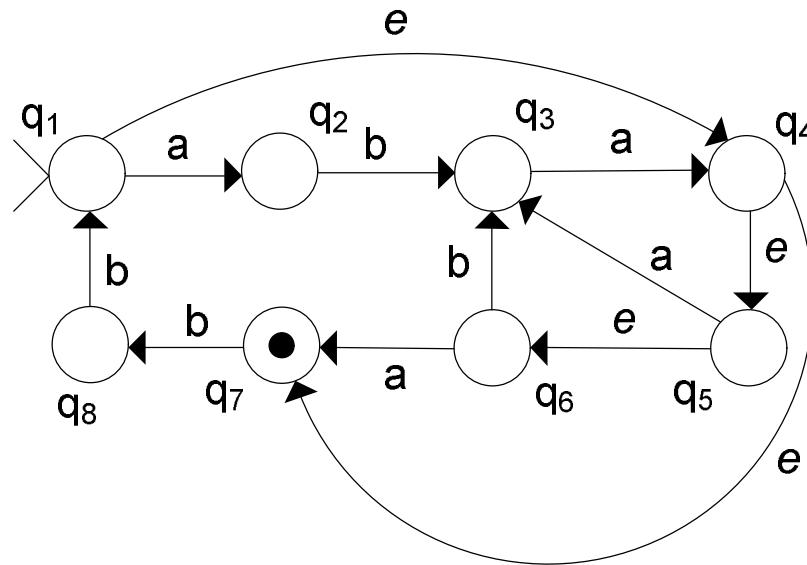
NFA → DFA: Example 5

- $\delta(\{q_3, q_7\}, a) = E(q_4) = \{q_4, q_5, q_6, q_7\} = Q_4$
- $\delta(\{q_3, q_7\}, b) = E(q_8) = \{q_8\} = Q_6$
- $\delta(\{q_8\}, a) = \emptyset = Q_7$
- $\delta(\{q_8\}, b) = E(q_1) = \{q_1, q_4, q_5, q_6, q_7\} = Q_1$
- $\delta(\{\}, a) = \emptyset = Q_7$
- $\delta(\{\}, b) = \emptyset = Q_7$

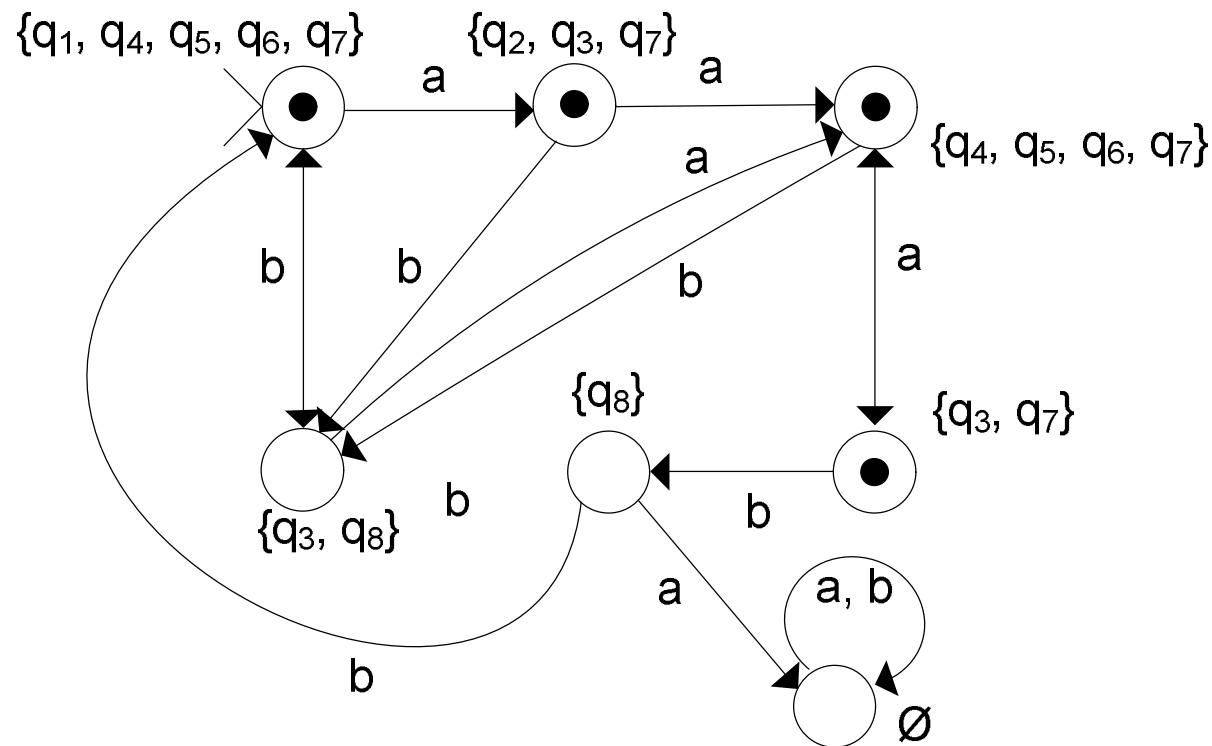


NFA \rightarrow DFA: Example 5

- Definition of final states
 - $F_3 = \{\{q_1, q_4, q_5, q_6, q_7\}, \{q_2, q_3, q_7\}, \{q_4, q_5, q_6, q_7\}, \{q_3, q_7\}\}$



NFA \rightarrow DFA: Example 5

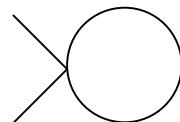


Element of the Theory of the Computation

Lecture 6

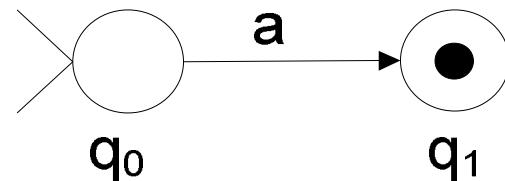
$\text{RE} \rightarrow \text{NFA}$: Example 1

- Define NFA M , where $L(M) = L(R)$ and $R = \emptyset$!



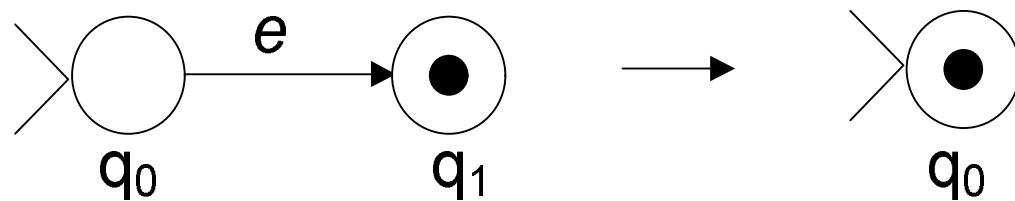
RE → NFA: Example 2

- Define NFA M, where $L(M) = L(R)$ and $R = a^*$



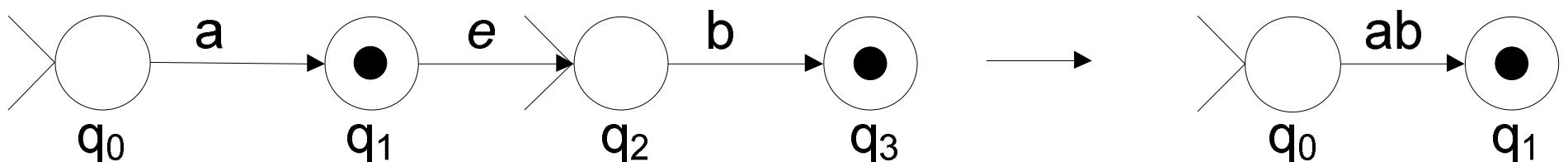
RE → NFA: Example 3

- Define NFA M, where $L(M) = L(R)$ and $R = \emptyset^*$!



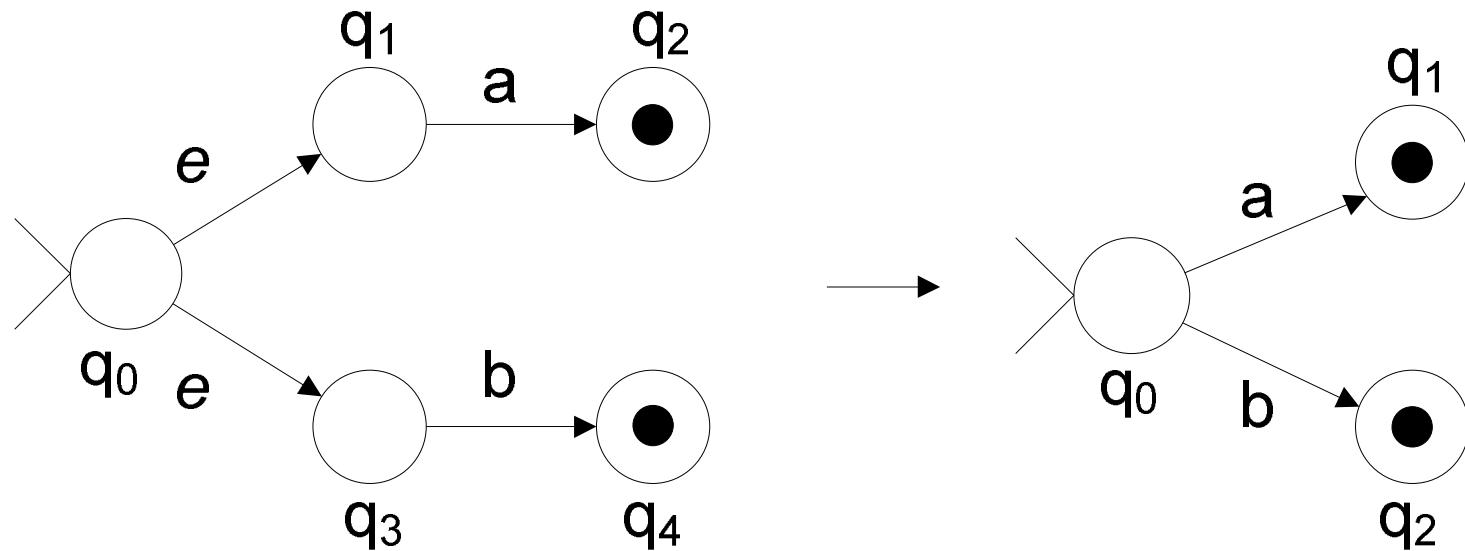
RE → NFA: Example 4

- Define NFA M, where $L(M) = L(R)$ and $R = ab^*$



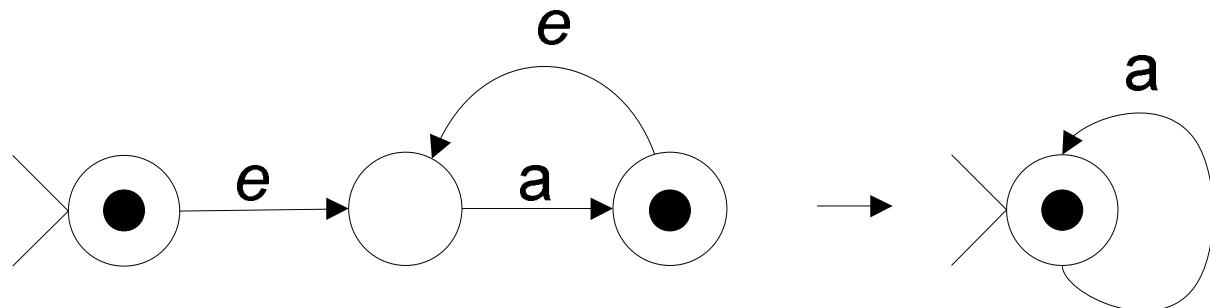
RE → NFA: Example 5

- Define NFA M, where $L(M) = L(R)$ and $R = a \cup b^*$!



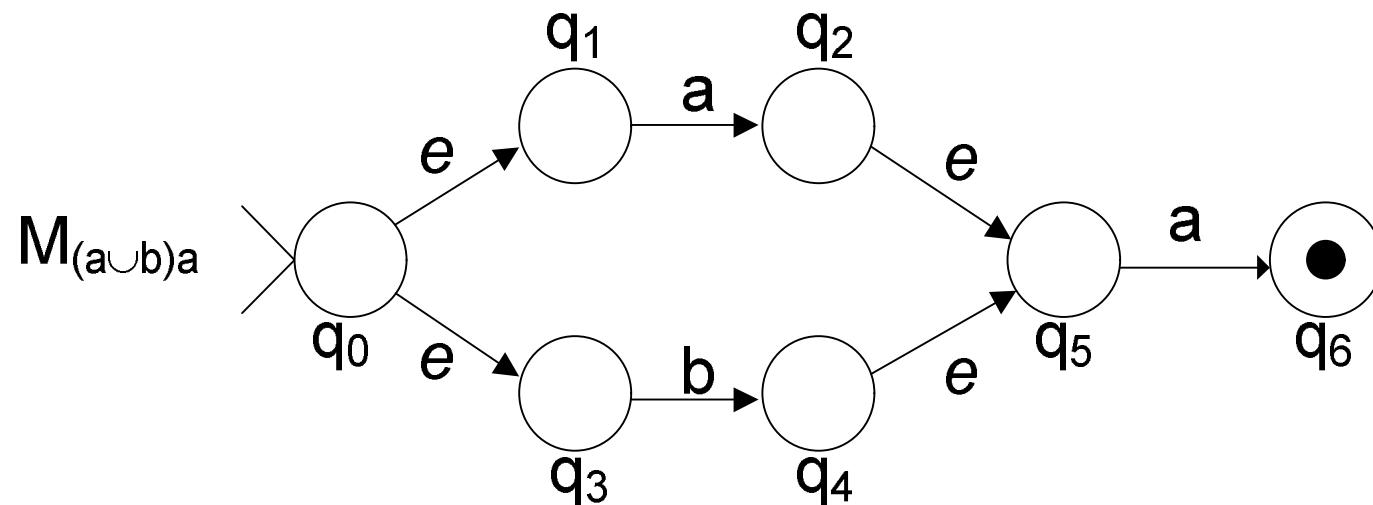
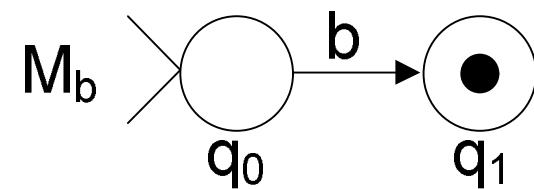
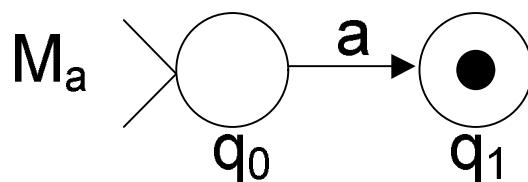
RE \rightarrow NFA: Example 6

- Define NFA M , where $L(M) = L(R)$ and $R = a^* !$
 - simplify carefully if possible



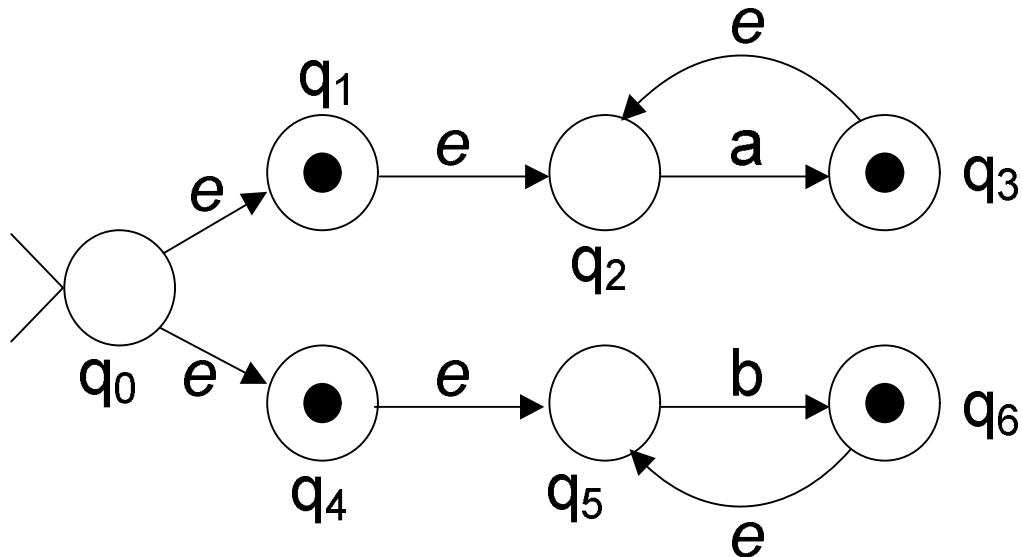
RE → NFA: Example 7

- Define NFA M , where $L(M) = L(R)$ and R is given below!
Use the construction theorems!
 - $R = (a \cup b)a$



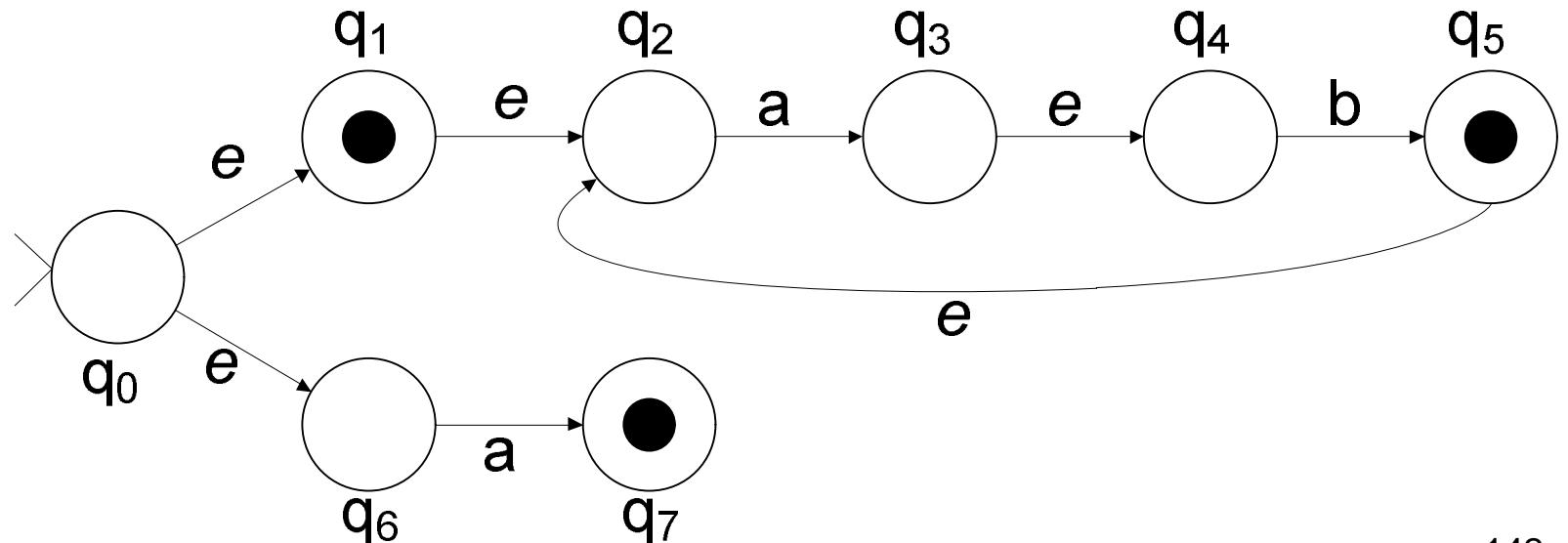
RE → NFA: Example 8

- Define NFA M, where $L(M) = L(R)$ and R is given below!
Use the construction theorems!
 - $R = a^* \cup b^*$



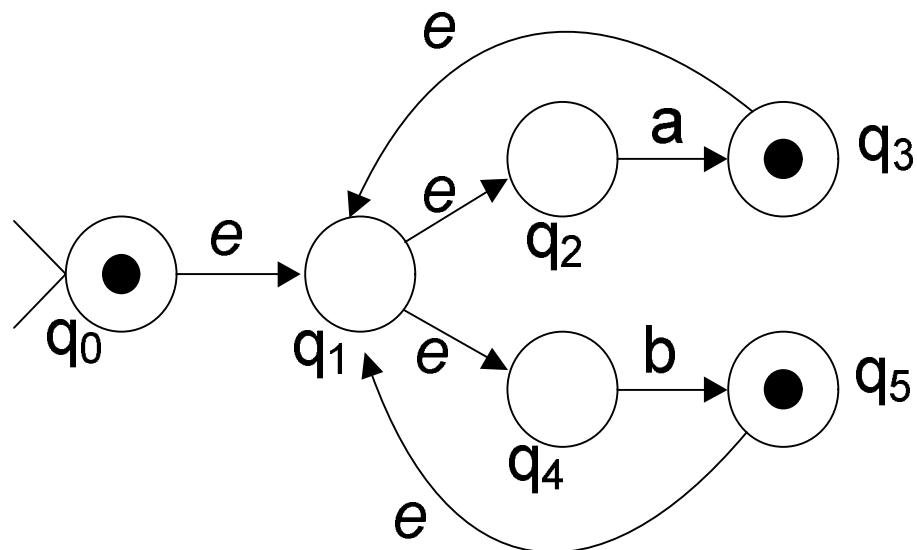
RE → NFA: Example 9

- Define NFA M, where $L(M) = L(R)$ and R is given below!
Use the construction theorems!
 - $R = a \cup (ab)^*$



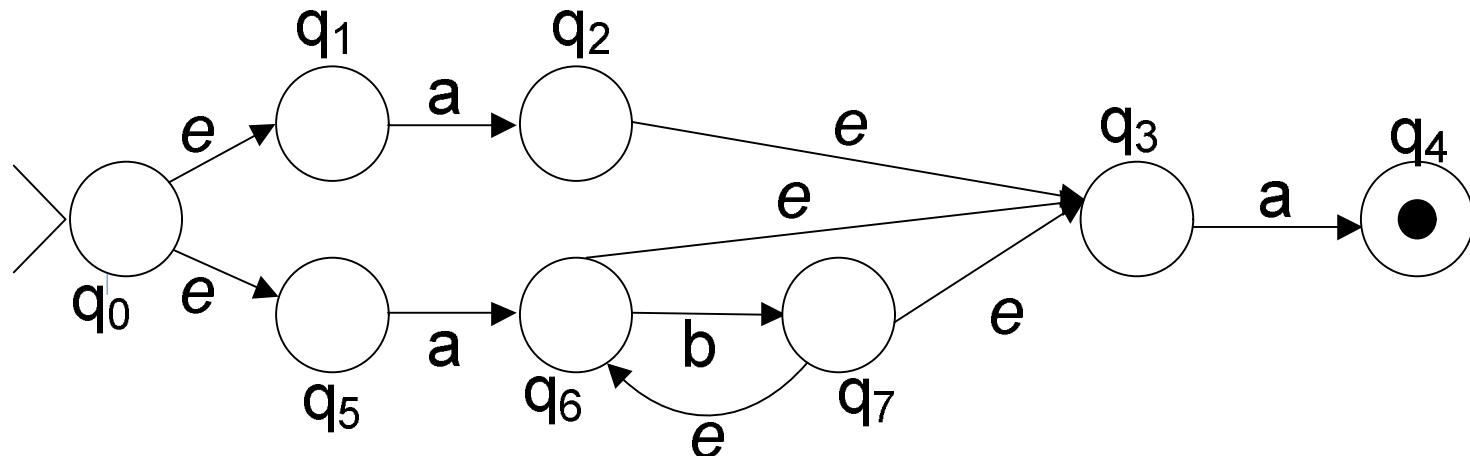
RE → NFA: Example 10

- Define NFA M, where $L(M) = L(R)$ and R is given below!
Use the construction theorems!
 - $R = (a \cup b)^*$



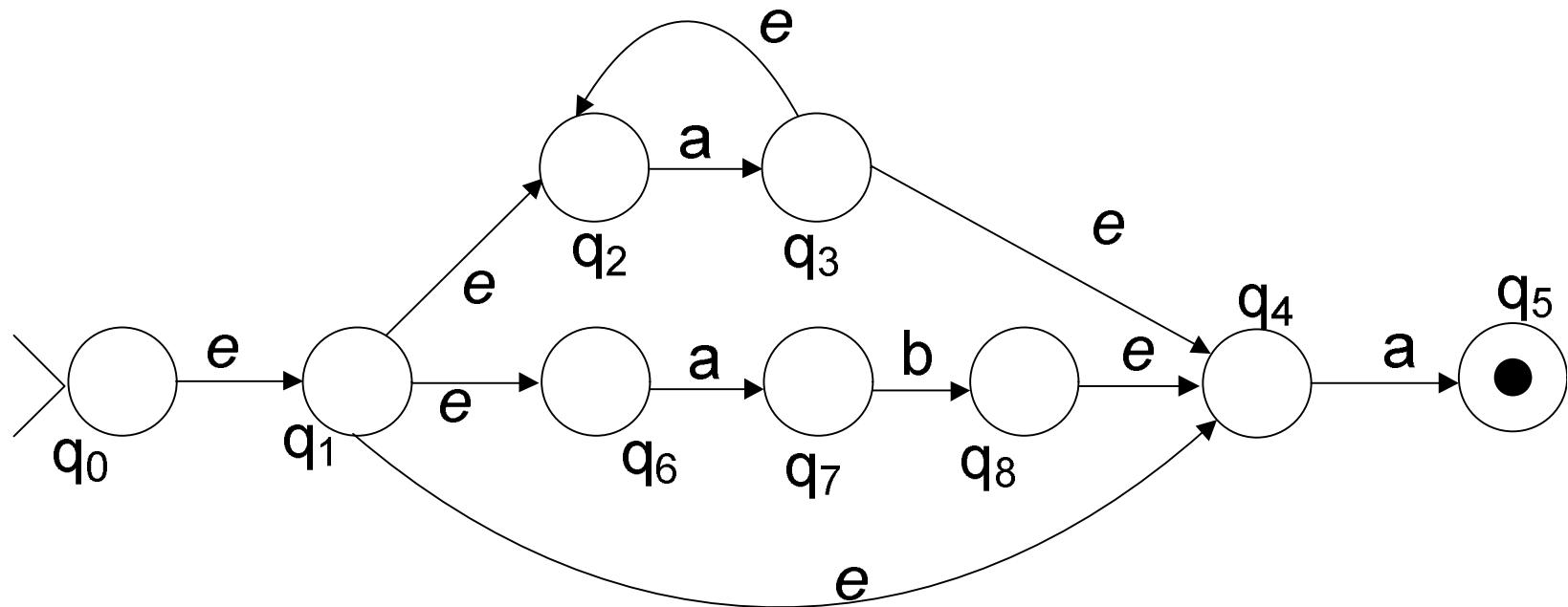
RE → NFA: Example 11

- Define NFA M, where $L(M) = L(R)$ and R is given below!
 - $R = (a \cup ab^*)a$



RE \rightarrow NFA: Example 12

- Define NFA M, where $L(M) = L(R)$ and R is given below!
 - $R = (a^* \cup ab \cup \emptyset^*)a$

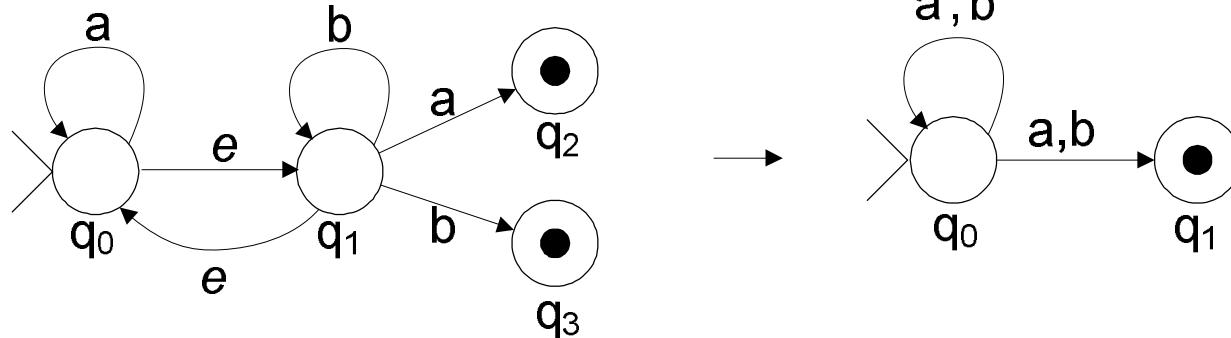


RE → NFA: Example 13

- Define NFA M, where $L(M) = L(R)$ and R is given below!
 - $R = (a^*b^*)^*(a \cup b)$

- $K = \{q_0, q_1, q_2, q_3\}$
- $\Sigma = \{a, b\}$
- $S = q_0$
- $F = \{q_2, q_3\}$

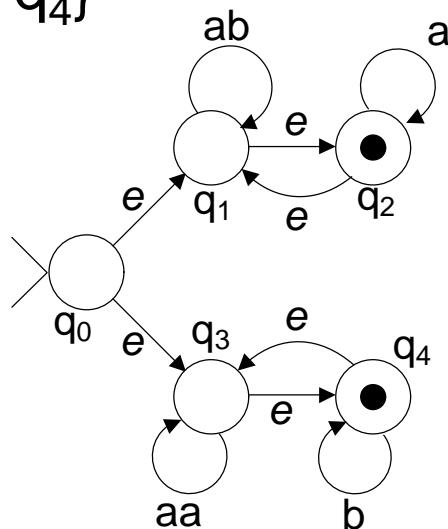
- $K = \{q_0, q_1\}$
- $\Sigma = \{a, b\}$
- $S = q_0$
- $F = \{q_1\}$



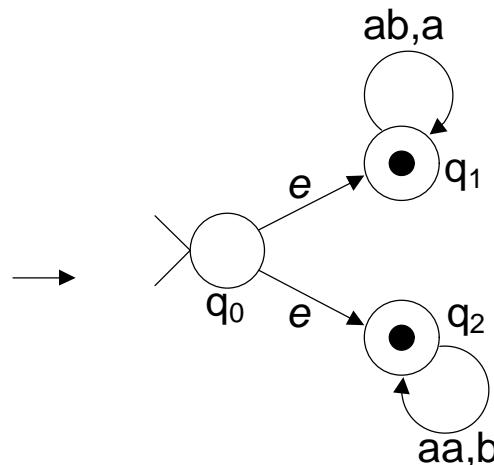
RE → NFA: Example 14

- Define NFA M, where $L(M) = L(R)$ and R is given below!
 - $R = (((ab)^*a^*)^* \cup ((aa)^*b^*)^*)$

- $K = \{q_0, q_1, q_2, q_3, q_4\}$
- $\Sigma = \{a, b\}$
- $S = q_0$
- $F = \{q_2, q_4\}$

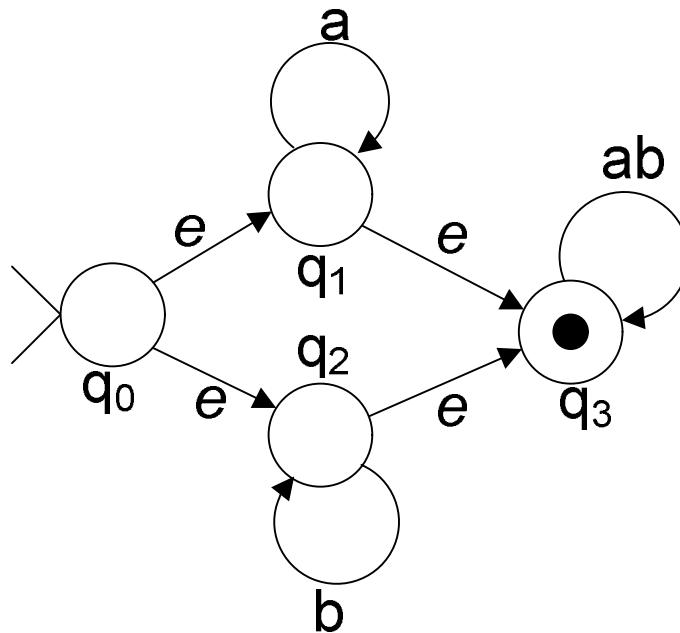


- $K = \{q_0, q_1, q_2\}$
- $\Sigma = \{a, b\}$
- $S = q_0$
- $F = \{q_1, q_2\}$



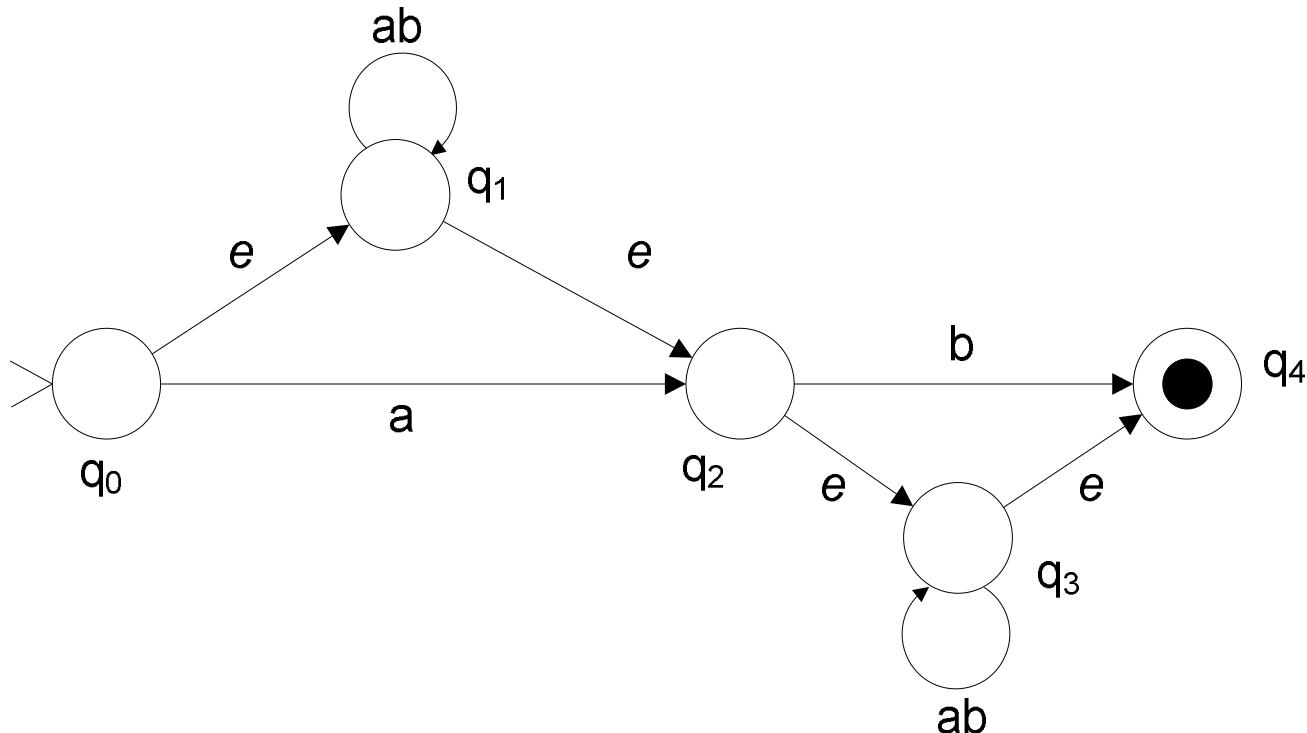
RE \rightarrow NFA: Example 15

- Define NFA M, where $L(M) = L(R)$ and R is given below!
 - $R = (a^* \cup b^*)(ab)^*$



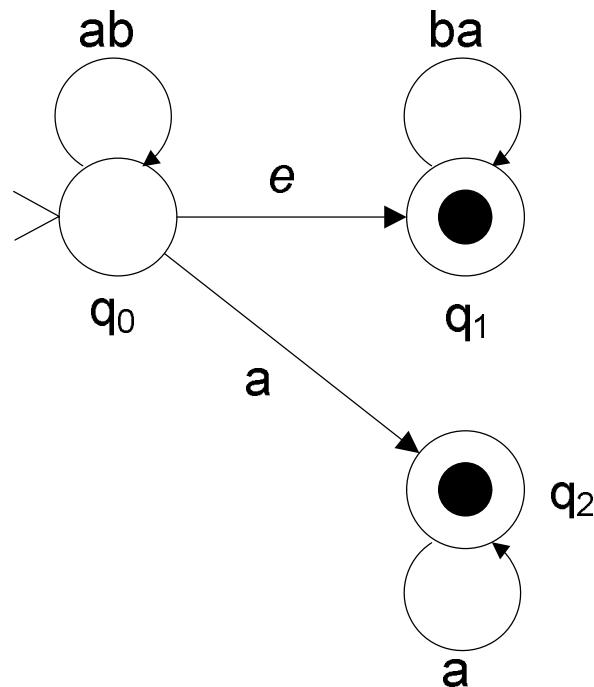
RE \rightarrow NFA: Example 16

- Define NFA M, where $L(M) = L(R)$ and R is given below!
 - $R = (a \cup (ab)^*)(b \cup (ab)^*)$



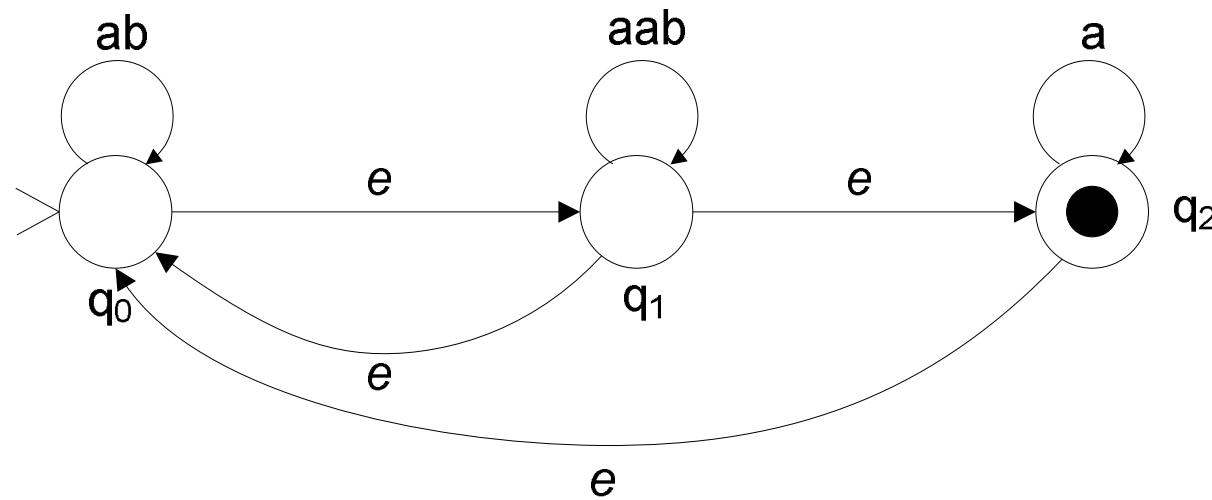
RE \rightarrow NFA: Example 17

- Define NFA M, where $L(M) = L(R)$ and R is given below!
 - $R = (ab)^*((ba)^* \cup aa^*)$



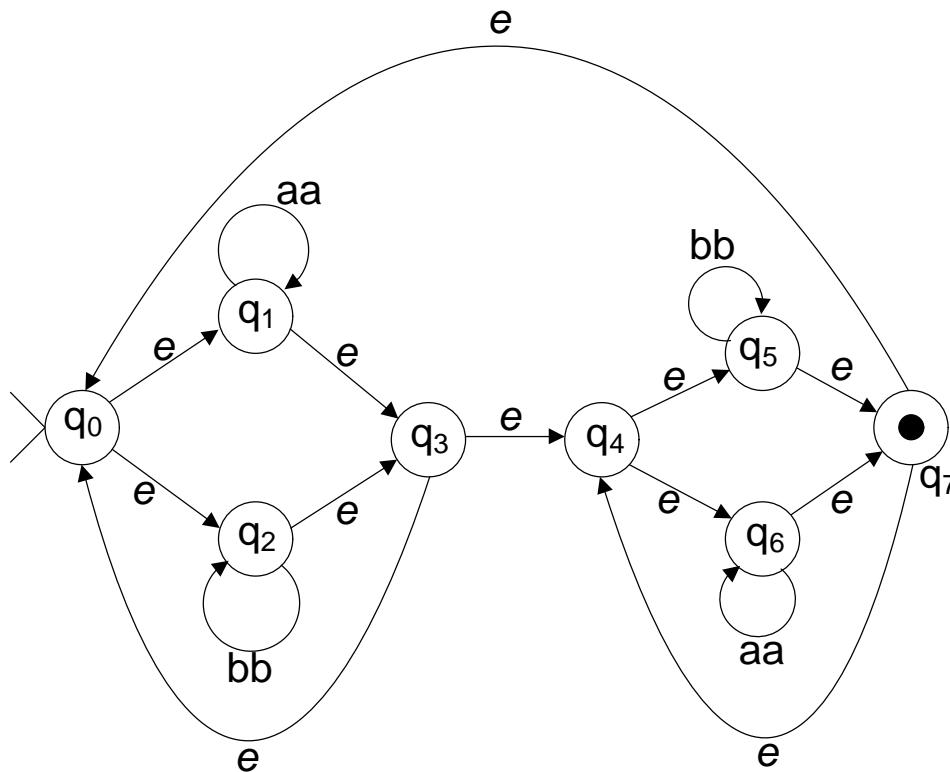
RE → NFA: Example 18

- Define NFA M, where $L(M) = L(R)$ and R is given below!
 - $R = (((ab)^*(aab)^*)^*a^*)^*$



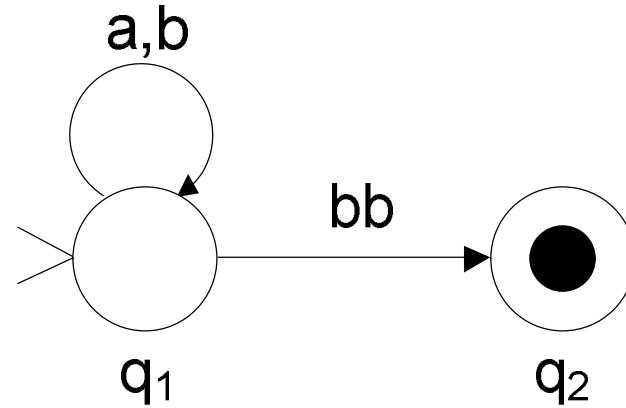
RE → NFA: Example 19

- Define NFA M, where $L(M) = L(R)$ and R is given below!
 - $R = (((aa)^* \cup (bb)^*)^*((bb)^* \cup (aa)^*)^*)^*$



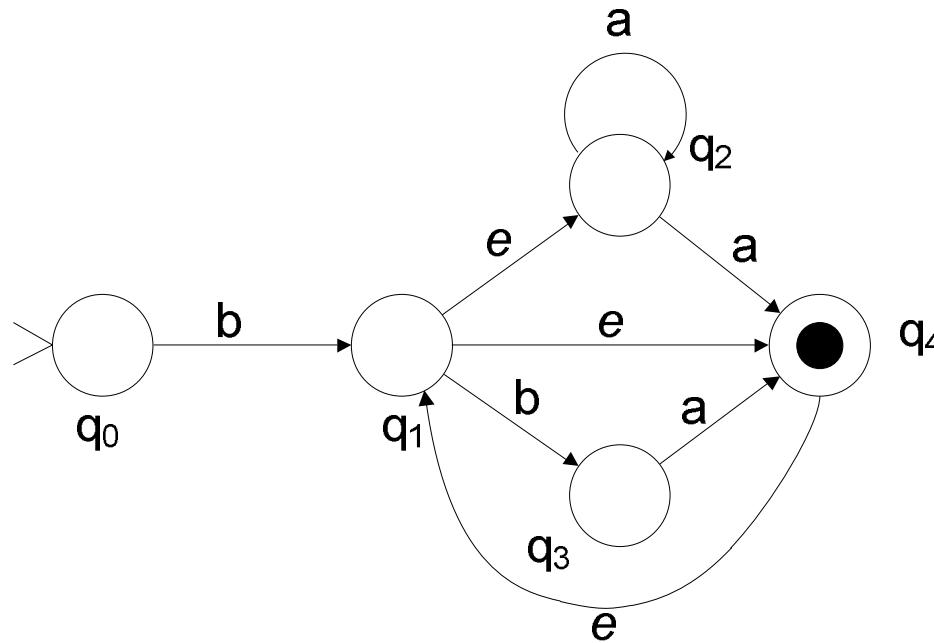
RE \rightarrow NFA: Example 20

- Define NFA M, where $L(M) = L(R)$ and R is given below!
 - $R = (a \cup b)^*bb$



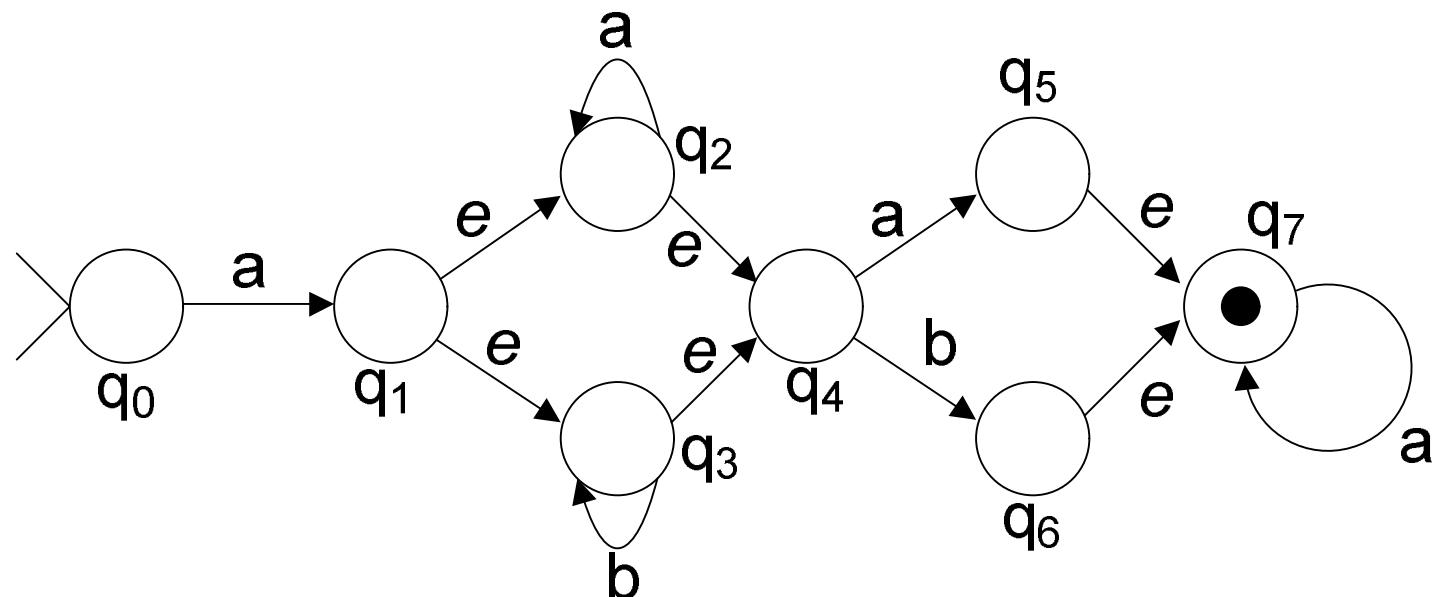
RE \rightarrow NFA: Example 21

- Define NFA M, where $L(M) = L(R)$ and R is given below!
 - $R = b((a^* \cup b)a)^*$



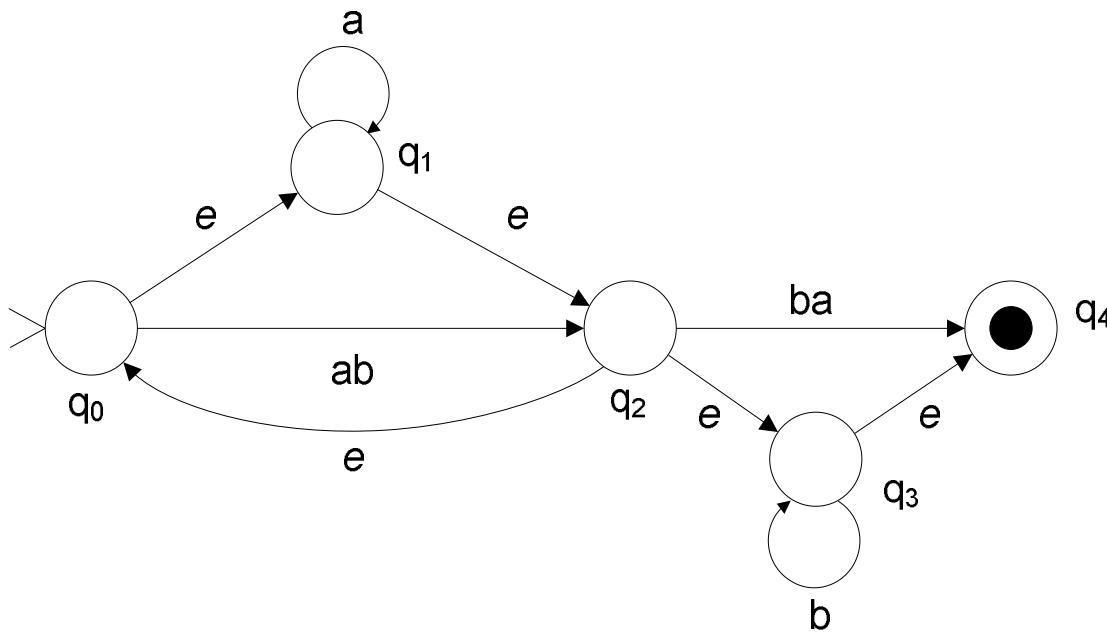
RE \rightarrow NFA: Example 22

- Define NFA M, where $L(M) = L(R)$ and R is given below!
 - $R = a(a^* \cup b^*)(a \cup b)a^*$



RE \rightarrow NFA: Example 23

- Define NFA M, where $L(M) = L(R)$ and R is given below!
 - $R = (a^* \cup ab)^*(b^* \cup ba)$



Element of the Theory of the Computation

Lecture 7

CFG: Example 1

- Which words can be derived at most in 4 steps with $G = \{V, \Sigma, R, S\}$ grammar from S ?
 - $V = \{a, b, A, B, S\}$
 - $\Sigma = \{a, b\}$
 - $R = \{S \rightarrow A, S \rightarrow abA, S \rightarrow aB, A \rightarrow Sa, B \rightarrow b\}$
- Solution:
 - $S \Rightarrow_G A \Rightarrow_G Sa \Rightarrow_G aBa \Rightarrow_G aba$
 - $S \Rightarrow_G abA \Rightarrow_G abSa \Rightarrow_G abaBa \Rightarrow_G ababa$
 - $S \Rightarrow_G aB \Rightarrow_G ab$

CFG: Example 2

- Which words can be derived at most in 4 steps with $G = \{V, \Sigma, R, S\}$ grammar from S ?
 - $V = \{a, b, A, B, S\}$
 - $\Sigma = \{a, b\}$
 - $R = \{S \rightarrow A, S \rightarrow abA, S \rightarrow aB, A \rightarrow a, B \rightarrow Sb\}$
- Solution:
 - $S \Rightarrow_G A \Rightarrow_G a$
 - $S \Rightarrow_G aB \Rightarrow_G aSb \Rightarrow_G aAb \Rightarrow_G aab$
 - $S \Rightarrow_G abA \Rightarrow_G aba$
 - $S \Rightarrow_G aB \Rightarrow_G aSb \Rightarrow_G aabAb \Rightarrow_G aabab$

CFG: Example 3

- Which words can be derived at most in 4 steps with $G = \{V, \Sigma, R, S\}$ grammar from S ?
 - $V = \{a, b, A, B, S\}$
 - $\Sigma = \{a, b\}$
 - $R = \{S \rightarrow ABS, S \rightarrow AB, A \rightarrow aA, B \rightarrow bB, S \rightarrow e, A \rightarrow a, B \rightarrow b\}$
- Solution:
 - $S \Rightarrow_G ABS \Rightarrow_G aBS \Rightarrow_G abS \Rightarrow_G ab$
 - $S \Rightarrow_G AB \Rightarrow_G aB \Rightarrow_G ab$
 - $S \Rightarrow_G AB \Rightarrow_G aAB \Rightarrow_G aaB \Rightarrow_G aab$

CFG: Example 4

- Which words can be derived at most in 4 steps with $G = \{V, \Sigma, R, S\}$ grammar from S ?
 - $V = \{a, b, A, B, S\}$
 - $\Sigma = \{a, b\}$
 - $R = \{S \rightarrow aSB, S \rightarrow bSA, S \rightarrow a, S \rightarrow b, A \rightarrow aS, B \rightarrow bS\}$

CFG: Example 4

- $R = \{S \rightarrow aSB, S \rightarrow bSA, S \rightarrow a, S \rightarrow b, A \rightarrow aS, B \rightarrow bS\}$
- Solution:
 - $S \Rightarrow_G aSB \Rightarrow_G aaB \Rightarrow_G aabS \Rightarrow_G aaba$
 - $S \Rightarrow_G bSA \Rightarrow_G baA \Rightarrow_G baaS \Rightarrow_G baaa$
 - $S \Rightarrow_G a$
 - $S \Rightarrow_G b$
 - $S \Rightarrow_G aSB \Rightarrow_G aSbS \Rightarrow_G aabS \Rightarrow_G aaba$

CFG: Example 5

- Give 3 different deductions to the word "abab"!
 - $V = \{a, b, A, B, S\}$
 - $\Sigma = \{a, b\}$
 - $R = \{S \rightarrow aB, S \rightarrow Ab, B \rightarrow bS, A \rightarrow Sa, S \rightarrow e\}$
- Solution:
 - $S \Rightarrow_G aB \Rightarrow_G abS \Rightarrow_G abaB \Rightarrow_G ababS \Rightarrow_G abab$
 - $S \Rightarrow_G aB \Rightarrow_G abS \Rightarrow_G abAb \Rightarrow_G abSab \Rightarrow_G abab$
 - $S \Rightarrow_G Ab \Rightarrow_G Sab \Rightarrow_G Abab \Rightarrow_G Sabab \Rightarrow_G abab$

CFG: Example 6

- Give 3 different deductions to the word "abba"!
 - $V = \{a, b, S\}$
 - $\Sigma = \{a, b\}$
 - $R = \{S \rightarrow SS, S \rightarrow abS, S \rightarrow Sba, S \rightarrow e\}$
- Solution:
 - $S \Rightarrow_G abS \Rightarrow_G abSba \Rightarrow_G abba$
 - $S \Rightarrow_G SS \Rightarrow_G abSS \Rightarrow_G abSSba \Rightarrow_G abSba \Rightarrow_G abba$
 - $S \Rightarrow_G Sba \Rightarrow_G abSba \Rightarrow_G abba$

CFG: Example 7

- Give context-free grammar G such that $L(G) = L!$
 - $L = \{w \in \{a, b\}^* \mid |w| \text{ is divisible with } 3\}$
- Solution 1:
 - $S \rightarrow AAAS \mid e$
 - $A \rightarrow a \mid b$
- Solution 2:
 - $S \rightarrow aaaS \mid aabS \mid abbS \mid bbbS \mid bbaS \mid baaS \mid babS \mid abaS \mid e$

CFG: Example 8

- Give CFG G such that $L(G) = L!$
 - $\Sigma = \{a\}$
 - $L = \{a^n \in \Sigma^* \mid n \geq 0\}$
- Solution:
 - $V = \{S\}$
 - $R = \{(S, aS), (S, e)\}$
 - $S \rightarrow aS \mid e$
 - e.g.: $S \Rightarrow aS \Rightarrow aaS \Rightarrow aa$

CFG: Example 9

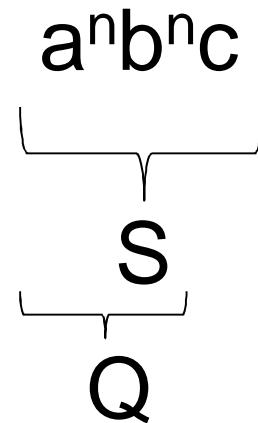
- Give CFG G such that $L(G) = L!$
 - $\Sigma = \{a, b\}$
 - $L = \{a^n b^n \in \Sigma^* \mid n \geq 0\}$
- Solution:
 - $V = \{S\}$
 - $R = \{(S, aSb), (S, e)\}$
 - $S \rightarrow aSb \mid e$
 - e.g.: $S \Rightarrow aSb \Rightarrow aaSbb \Rightarrow aabb$

CFG: Example 10

- Give CFG G such that $L(G) = L!$
 - $\Sigma = \{a, b, c\}$
 - $L = \{a^n cb^n \in \Sigma^* \mid n \geq 0\}$
- Solution:
 - $V = \{S\}$
 - $R = \{(S, aSb), (S, c)\}$
 - $S \rightarrow aSb \mid c$
 - e.g.: $S \Rightarrow aSb \Rightarrow aaSbb \Rightarrow aacbb$

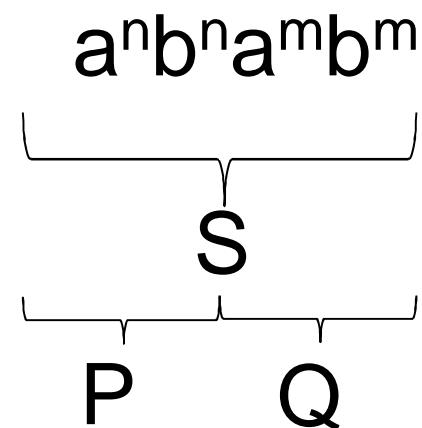
CFG: Example 11

- Give CFG G such that $L(G) = L!$
 - $\Sigma = \{a, b, c\}$
 - $L = \{a^n b^n c \in \Sigma^* \mid n \geq 0\}$
- Solution:
 - $V = \{S, Q\}$
 - $R = \{(S, Qc), (Q, aQb), (Q, e)\}$
 - $S \rightarrow Qc$
 - $Q \rightarrow aQb \mid e$
 - e.g.: $S \Rightarrow Qc \Rightarrow aQbc \Rightarrow aaQbbc \Rightarrow aabbc$



CFG: Example 12

- Give CFG G such that $L(G) = L!$
 - $\Sigma = \{a, b\}$
 - $L = \{a^n b^n a^m b^m \in \Sigma^* \mid n, m \geq 0\}$
- Solution:
 - $S \rightarrow PQ$
 - $Q \rightarrow aQb \mid e$
 - $P \rightarrow aPb \mid e$
 - e.g.: $S \Rightarrow PQ \Rightarrow aQbP \Rightarrow abP \Rightarrow abaPb \Rightarrow abaaPbb \Rightarrow abaabb$



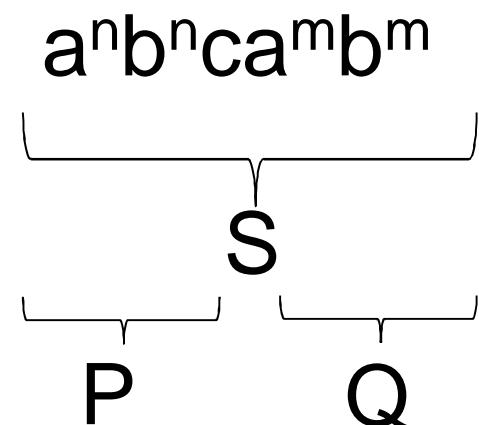
CFG: Example 13

- Give CFG G such that $L(G) = L!$
 - $\Sigma = \{a, b, c\}$
 - $L = \{a^n b^n c a^m b^m \in \Sigma^* \mid n, m \geq 0\}$

- Solution:

- $S \rightarrow P c Q$
- $Q \rightarrow a Q b \mid e$
- $P \rightarrow a P b \mid e$

– e.g.: $S \Rightarrow P c Q \Rightarrow a P b c Q \Rightarrow a a P b b c Q \Rightarrow a a b b c Q \Rightarrow a a b b c$



CFG: Example 14

- Give CFG G such that $L(G) = L!$
 - $\Sigma = \{a, b, c\}$
 - $L = \{a^n b^n a^m c b^m \in \Sigma^* \mid n, m \geq 0\}$
- Solution:
 - $S \rightarrow PQ$
 - $Q \rightarrow aQb \mid c$
 - $P \rightarrow aPb \mid e$
 - e.g.: $S \Rightarrow PQ \Rightarrow PQ \Rightarrow PaQb \Rightarrow Pacb \Rightarrow aPbacb \Rightarrow abacb$

CFG: Example 15

- Give CFG G such that $L(G) = L!$
 - $\Sigma = \{a, b\}$
 - $L = \{a^{n+1}b^n \in \Sigma^* \mid n \geq 0\}$
- Solution:
 - $L = \{a^nab^n \in \Sigma^* \mid n \geq 0\}$
 - $S \rightarrow aSb \mid a$
 - e.g.: $S \Rightarrow aSb \Rightarrow aaSbb \Rightarrow aaabb$

CFG: Example 16

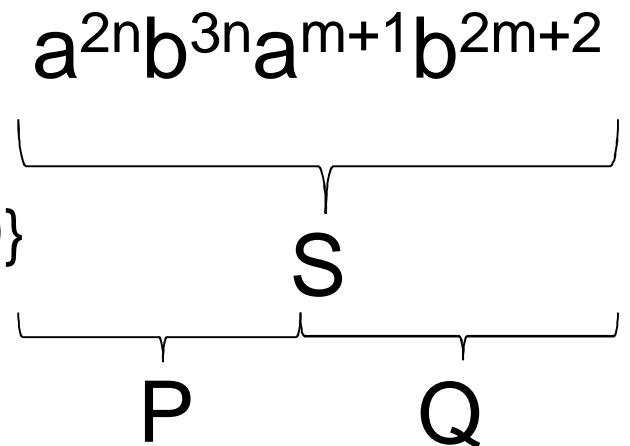
- Give CFG G such that $L(G) = L!$
 - $\Sigma = \{a, b\}$
 - $L = \{a^n b^{3n} \in \Sigma^* \mid n \geq 0\}$
- Solution:
 - $S \rightarrow aSbbb \mid e$
 - e.g.: $S \Rightarrow aSbbb \Rightarrow aaSbbbbbb \Rightarrow aabbbaabbba$

CFG: Example 17

- Give CFG G such that $L(G) = L!$

- $\Sigma = \{a, b\}$

- $L = \{a^{2n}b^{3n}a^{m+1}b^{2m+2} \in \Sigma^* \mid n, m \geq 0\}$



- Solution:

- $S \rightarrow PQ$

- $P \rightarrow aaPbbb \mid e$

- $Q \rightarrow aQbb \mid abb$

- e.g.: $S \Rightarrow PQ \Rightarrow aaPbbbQ \Rightarrow aabbQ \Rightarrow aabbabb$

CFG: Example 18

- Give CFG G such that $L(G) = L!$
 - $\Sigma = \{a, b\}$
 - $L = \{a^n b^m \in \Sigma^* \mid n, m \geq 0 \text{ and } n > m\}$
- Solution:
 - $L = \{a^* a^n b^n \in \Sigma^* \mid n \geq 0\}$
 - $S \rightarrow AB$
 - $A \rightarrow aA \mid a$
 - $B \rightarrow aBb \mid e$
 - e.g.: $S \Rightarrow AB \Rightarrow aAB \Rightarrow aaAB \Rightarrow aaaAB \Rightarrow aaaB \Rightarrow aaaaBb \Rightarrow aaaab$

CFG: Example 19

- Give CFG G such that $L(G) = L!$
 - $\Sigma = \{a, b\}$
 - $L = \{a^n b^m \in \Sigma^* \mid n, m \geq 0 \text{ and } n < m\}$
- Solution:
 - $L = \{a^n b^n b^* \in \Sigma^* \mid n \geq 0\}$
 - $S \rightarrow BA$
 - $A \rightarrow bA \mid b$
 - $B \rightarrow aBb \mid e$
 - e.g.: $S \Rightarrow BA \Rightarrow aBbA \Rightarrow abA \Rightarrow abbA \Rightarrow abbb$

CFG: Example 20

- Give CFG G such that $L(G) = L!$
 - $\Sigma = \{a, b\}$
 - $L = \{a^n b^m \in \Sigma^* \mid n, m \geq 0 \text{ and } n \neq m\}$
- Solution:
 - $L = \{a^n b^m \in \Sigma^* \mid n < m \text{ or } n > m\}$
 - $S \rightarrow CD \mid DE$
 - $C \rightarrow aC \mid a$
 - $D \rightarrow aDb \mid e$
 - $E \rightarrow bE \mid b$
- e.g.: $S \Rightarrow CD \Rightarrow aCD \Rightarrow aaD \Rightarrow aaaDb \Rightarrow aaab$

CFG: Example 21

- Give CFG G such that $L(G) = L!$
 - $\Sigma = \{a, b, c, d\}$
 - $L = \{a^n c^m d^m b^n \in \Sigma^* \mid n, m \geq 0\}$
- Solution:
 - $S \rightarrow aSb \mid A$
 - $A \rightarrow cAd \mid e$
 - e.g.: $S \Rightarrow aSb \Rightarrow aaSbb \Rightarrow aaAbb \Rightarrow aacAdbb \Rightarrow aacdadb$

$$a^n c^m d^m b^n$$
$$\underbrace{ \underbrace{ \underbrace{}_{A}}_{S}}$$

CFG: Example 22

- Give CFG G such that $L(G) = L!$
 - $\Sigma = \{a, b\}$
 - $L = \{(ab)^{2n+1}b^m a^{m+3}b^n \in \Sigma^* \mid n, m \geq 0\}$
- Solution:
 - $L = \{((ab)^2)^n ab[b^m a^m aaa]b^n \in \Sigma^* \mid n, m \geq 0\}$
 - $S \rightarrow (ab)^2 S b \mid ab A$
 - $A \rightarrow b A a \mid aaa$
 - e.g.: $S \Rightarrow abA \Rightarrow abbAa \Rightarrow abbaaaa$

CFG: Example 23

- Give CFG G such that $L(G) = L!$
 - $\Sigma = \{a, b\}$
 - $L = \{(ab)^{2n+1}b^m a^{m+3} a^p b^{p+1} b^n \in \Sigma^* \mid n, m, p \geq 0\}$
- Solution:
 - $L = \{(ab)^{2n}ab[b^m aaaa^m][a^p b b^p]b^n \in \Sigma^* \mid n, m, p \geq 0\}$
 - $S \rightarrow ababSb \mid abAP$
 - $A \rightarrow bAa \mid aaa$
 - $P \rightarrow aPb \mid b$
 - e.g.: $S \Rightarrow ababSb \Rightarrow abababAPb \Rightarrow abababaaaPb \Rightarrow abababaaaabb$

CFG: Example 24

- Give CFG G such that $L(G) = L!$
 - $\Sigma = \{a, b, h, i\}$
 - $L = \{a^n h^k b^{2m} a h a^{m+1} b^n i^p \in \Sigma^* \mid n, m, k, p \geq 0\}$
- Solution:
 - $S \rightarrow JP$
 - $J \rightarrow aJb \mid KM$
 - $K \rightarrow hK \mid e$
 - $M \rightarrow bbMa \mid aha$
 - $P \rightarrow iP \mid e$
 - e.g.: $S \Rightarrow JP \Rightarrow KMP \Rightarrow hKMP \Rightarrow hMP \Rightarrow hbbMaP \Rightarrow hbbahaaP \Rightarrow hbbahaaiP \Rightarrow hbbahaai$

CFG: Example 25

- Give CFG G such that $L(G) = L!$
 - $\Sigma = \{a, b, c\}$
 - $L = \{(ac)^n(bc)^m b^n \in \Sigma^* \mid n, m \geq 0\}$
- Solution:
 - $S \rightarrow N$
 - N the non-terminal corresponds to superscript n
 - $N \rightarrow acNb \mid M$
 - $M \rightarrow bcM \mid e$
 - e.g.: $S \Rightarrow N \Rightarrow acNb \Rightarrow acMb \Rightarrow acbcMb \Rightarrow acbcb$

CFG: Example 26

- Give CFG G such that $L(G) = L!$
 - $\Sigma = \{a, b, c, k\}$
 - $L = \{a^k(ac)^n b^m c^m b^n k^k \in \Sigma^* \mid n, m, k \geq 0\}$
- Solution:
 - $S \rightarrow K$
 - $K \rightarrow aKk \mid N$
 - $N \rightarrow acNb \mid M$
 - $M \rightarrow bMc \mid e$
 - e.g.: $S \Rightarrow K \Rightarrow aKk \Rightarrow aNk \Rightarrow aacNbK \Rightarrow aacMbK \Rightarrow aacbMcbK \Rightarrow aacbcbK$

CFG: Example 27

- Give CFG G such that $L(G) = L!$
 - $\Sigma = \{a, b\}$
 - $L = \{a^{n+1}b^{n+3} \in \Sigma^* \mid n \geq 0\}$
- Solution:
 - $L = \{a^nabbabb^n \in \Sigma^* \mid n \geq 0\}$
 - $S \rightarrow aSb \mid abbb$
 - e.g.: $S \Rightarrow aSb \Rightarrow aabbabb$

CFG: Example 28

- Give CFG G such that $L(G) = L!$
 - $\Sigma = \{a, b\}$
 - $L = \{a^n b^m a^n \in \Sigma^* \mid n, m \geq 0\}$
- Solution:
 - $S \rightarrow N$
 - $N \rightarrow aNa \mid M$
 - $M \rightarrow bM \mid e$
 - e.g.: $S \Rightarrow N \Rightarrow aNa \Rightarrow aaNaa \Rightarrow aaMaa \Rightarrow aabMaa \Rightarrow aabbMaa \Rightarrow aabbaa$

CFG: Example 29

- Give CFG G such that $L(G) = L!$
 - $\Sigma = \{a, b, c\}$
 - $L = \{a^n c b^n c \in \Sigma^* \mid n \geq 0\}$
- Solution:
 - $S \rightarrow Nc$
 - ending c derived
 - $N \rightarrow aNb \mid e$

CFG: Example 30

- Give CFG G such that $L(G) = L!$
 - $\Sigma = \{a, b\}$
 - $L = \{b^n a^n \in \Sigma^* \mid n \geq 0\} \cup \{a^{3n} b^n \in \Sigma^* \mid n \geq 0\}$
- Solution:
 - $S \rightarrow N1 \mid N2$
 - $N1 \rightarrow bN1a \mid e$
 - $N2 \rightarrow aaaN2b \mid e$

CFG: Example 31

- Give CFG G such that $L(G) = L!$
 - $\Sigma = \{a, b, c, d\}$
 - $L = \{a^m b^n c^p d^r \in \Sigma^* \mid m+n = p+r, m, n, p, r \geq 0\}$
- Solution:
 - $S \rightarrow A$
 - $A \rightarrow aAd \mid B \mid C$
 - always one symbol is created from {a, b} and {c, d}
 - $B \rightarrow aBc \mid D$
 - $C \rightarrow bCd \mid D$
 - $D \rightarrow bDc \mid e$
 - e.g.: $S \Rightarrow A \Rightarrow B \Rightarrow aBc \Rightarrow aaBcc \Rightarrow aaDcc \Rightarrow aabDccc \Rightarrow aabccc$

CFG: Example 32

- Give CFG G such that $L(G) = L!$
 - $\Sigma = \{a, b, c, d\}$
 - $L = \{a^{n+2}bc^{2m}d^pa^{n+3}c^{2q}(ad)^m \in \Sigma^* \mid n, m, p, q \geq 0\}$
- Solution:
 - L is not context free because it is a modification of $\{a^n b^m c^n d^m \in \Sigma^* \mid n, m \geq 0\}$ which is not context free
 - the superscripts can be regarded as different kinds of opening and closing parenthesis
 - here the are parenthesis neither nested ([]) nor next to each other ()[] but across ()[]

CFG: Example 32

- Give CFG G such that $L(G) = L!$
 - $\Sigma = \{a, b, c, d\}$
 - $L = \{a^m b^m d^{n+2} b c^4 d^{2p} a^{n+3} c^{2q} (ad)^{n+3} \in \Sigma^* \mid n, m, p, q \geq 0\}$
- Solution:
 - L is not context free because it is a modification of $\{a^n b^n c^n \in \Sigma^* \mid n \geq 0\}$ which is not context free

CFG: Example 33

- Give CFG G such that $L(G) = L!$
 - $\Sigma = \{a, b\}$
 - $L = \{w \in \Sigma^* \mid \#a \neq \#b\}$

CFG: Example 33

- Solution:
 - $S \rightarrow U \mid V$
 - $U \rightarrow TaU \mid TaT$
 - $V \rightarrow TbV \mid TbT$
 - $T \rightarrow aTbT \mid bTaT \mid e$
 - T: strings with the same number of a's as b's
 - U: strings with more a's than b's
 - V: strings with more b's than a's
 - e.g.: $S \Rightarrow U \Rightarrow TaU \Rightarrow bTaTaU \Rightarrow baaU \Rightarrow baaTaT \Rightarrow baaa$

CFG: Example 34

- Give CFG G such that $L(G) = L!$
 - $L = \{b^n a^m b^{2n} : n \geq 0, m > 0\}$
 - beware: $>$ and not \geq

CFG: Example 34

- Solution:
 - $L = \{b^n a a^m b^{2n} : n \geq 0, m \geq 0\}$
 - $S \rightarrow bSbb \mid aA$
 - $A \rightarrow aA \mid e$
 - e.g.: $S \Rightarrow bSbb \Rightarrow bAbb \Rightarrow baAbb \Rightarrow baaAbb \Rightarrow baabb$

$\text{RG} \rightarrow \text{CFG}$: Example 1

- Give CFG G such that $L(G) = L(R)$!
 - $R = ((a \cup b)a)^*$

$\text{RG} \rightarrow \text{CFG}$: Example 1

- Solution 1:
 - $S \rightarrow SXa \mid e$
 - $X \rightarrow a \mid b$
- Solution 2:
 - $S \rightarrow e \mid YS$ (Kleen star)
 - $Y \rightarrow XA$ (concatenation)
 - $X \rightarrow a \mid b$ (union)

$\text{RG} \rightarrow \text{CFG}$: Example 2

- Give CFG G such that $L(G) = L(R)$!
 - $R = (ab)^* \cup a$
- Solution:
 - $S \rightarrow A \mid a$
 - $A \rightarrow abA \mid e$
 - e.g.: $S \Rightarrow a$

$\text{RG} \rightarrow \text{CFG}$: Example 3

- Give CFG G such that $L(G) = L(R)$!
 - $R = (ba \cup b)^*a$

RG → CFG: Example 3

- $R = (ba \cup b)^*a$
- Solution: RE → CFG
 - $S \rightarrow XY$ (concatenation)
 - $Y \rightarrow a$
 - $X \rightarrow ZX \mid e$ (Kleen start)
 - $Z \rightarrow E \mid F$ (union)
 - $E \rightarrow ba$
 - $F \rightarrow b$
 - e.g.: $S \Rightarrow XY \Rightarrow ZX\bar{Y} \Rightarrow EX\bar{Y} \Rightarrow baX\bar{Y} \Rightarrow baY \Rightarrow baa$

$\text{RG} \rightarrow \text{CFG}$: Example 4

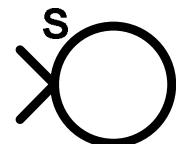
- Give CFG G such that $L(G) = L(R)$!
 - $R = a^*(ba \cup b)$
- Solution:
 - $S \rightarrow Ab \mid Aa$
 - $A \rightarrow aA \mid e$
 - e.g.: $S \Rightarrow Ab \Rightarrow aAb \Rightarrow aaAb \Rightarrow aab$

$\text{RG} \rightarrow \text{NFA}$: Example 1

- Construct NFA M such that $L(M) = L(G)$!
 - $V = \{a, b, A, B, S\}$
 - $\Sigma = \{a, b\}$
 - $S = S$
 - $R = \{S \rightarrow aA \mid bB \mid e, A \rightarrow abS, B \rightarrow baS\}$

RG → NFA: Example 1

- Step a1: introduce a state for non-terminal S (this will be the initial state because S is the starting non-terminal)
 - $V = \{a, b, A, B, S\}$
 - $\Sigma = \{a, b\}$
 - $S = S$
 - $R = \{S \rightarrow aA \mid bB \mid e, A \rightarrow abS, B \rightarrow baS\}$



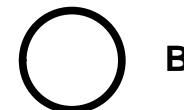
RG → NFA: Example 1

- Step a2: introduce a state for non-terminal A
 - $V = \{a, b, A, B, S\}$
 - $\Sigma = \{a, b\}$
 - $S = S$
 - $R = \{S \rightarrow aA \mid bB \mid e, A \rightarrow abS, B \rightarrow baS\}$



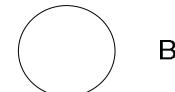
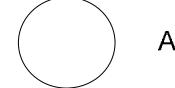
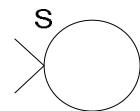
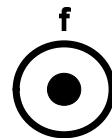
RG → NFA: Example 1

- Step a3: introduce a state for non-terminal B
 - $V = \{a, b, A, B, S\}$
 - $\Sigma = \{a, b\}$
 - $S = S$
 - $R = \{S \rightarrow aA \mid bB \mid e, A \rightarrow abS, B \rightarrow baS\}$



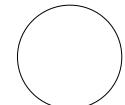
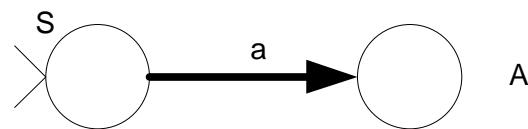
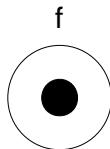
RG → NFA: Example 1

- Step b: introduce a state for non-terminal f (this will be the final state)
 - $V = \{a, b, A, B, S\}$
 - $\Sigma = \{a, b\}$
 - $S = S$
 - $R = \{S \rightarrow aA \mid bB \mid e, A \rightarrow abS, B \rightarrow baS\}$



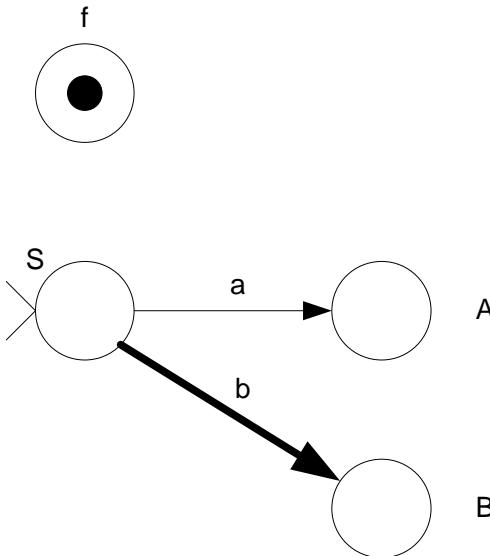
RG → NFA: Example 1

- Step c1: introduce a new arc for $S \rightarrow aA$
 - $V = \{a, b, A, B, S\}$
 - $\Sigma = \{a, b\}$
 - $S = S$
 - $R = \{S \rightarrow aA \mid bB \mid e, A \rightarrow abS, B \rightarrow baS\}$



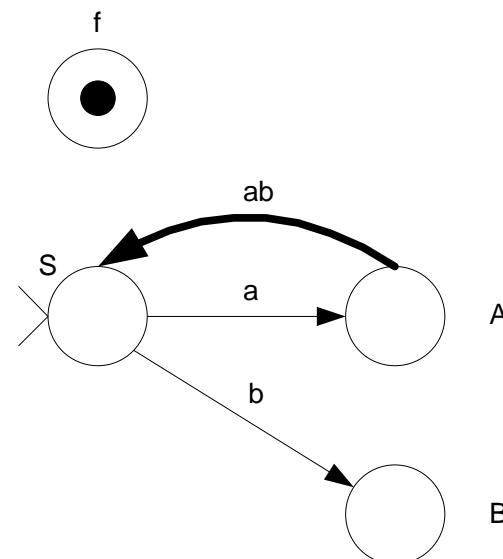
RG → NFA: Example 1

- Step c2: introduce a new arc for $S \rightarrow bB$
 - $V = \{a, b, A, B, S\}$
 - $\Sigma = \{a, b\}$
 - $S = S$
 - $R = \{S \rightarrow aA \mid bB \mid e, A \rightarrow abS, B \rightarrow baS\}$



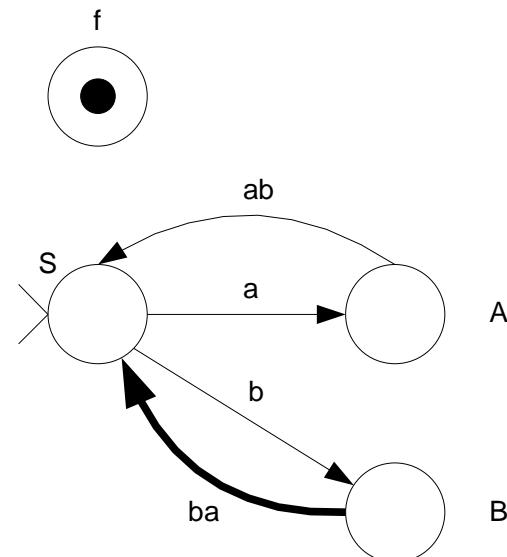
RG → NFA: Example 1

- Step c3: introduce a new arc for $A \rightarrow abS$
 - $V = \{a, b, A, B, S\}$
 - $\Sigma = \{a, b\}$
 - $S = S$
 - $R = \{S \rightarrow aA \mid bB \mid e, A \rightarrow abS, B \rightarrow baS\}$



RG → NFA: Example 1

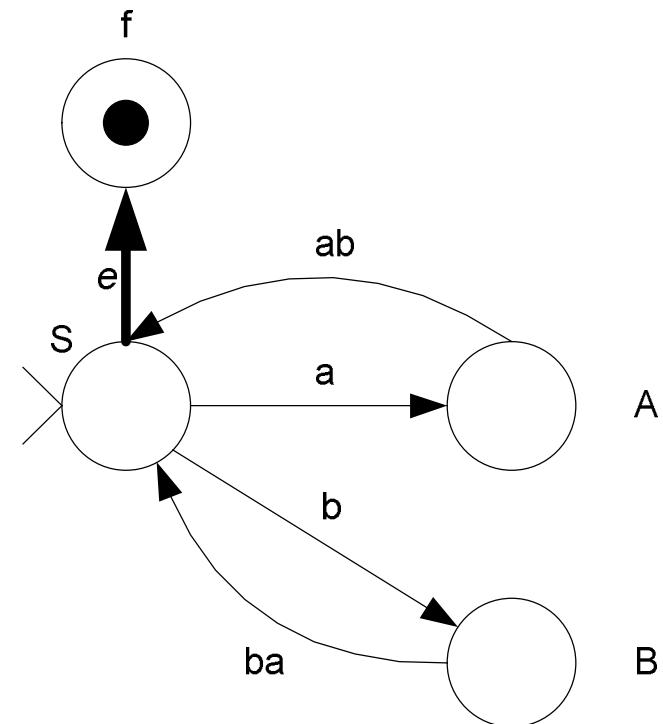
- Step c4: introduce a new arc for $B \rightarrow baS$
 - $V = \{a, b, A, B, S\}$
 - $\Sigma = \{a, b\}$
 - $S = S$
 - $R = \{S \rightarrow aA \mid bB \mid e, A \rightarrow abS, B \rightarrow \mathbf{baS}\}$



RG → NFA: Example 1

- Step d1: introduce a new arc for $S \rightarrow e$

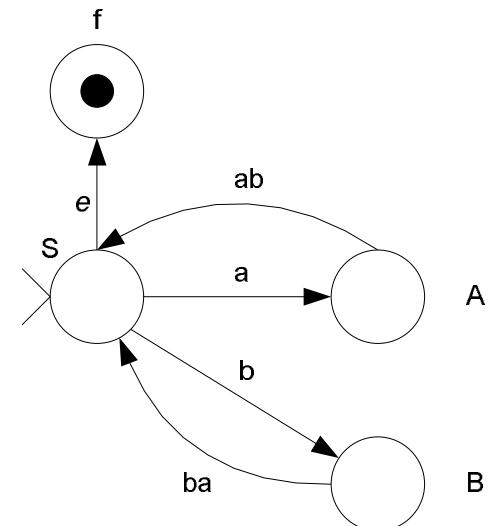
- $V = \{a, b, A, B, S\}$
- $\Sigma = \{a, b\}$
- $S = S$
- $R = \{S \rightarrow aA \mid bB \mid e, A \rightarrow abS, B \rightarrow baS\}$



RG → NFA: Example 1

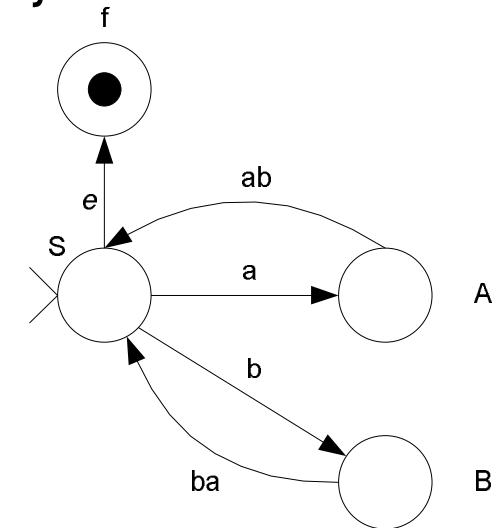
- $R = \{S \rightarrow aA \mid bB \mid e, A \rightarrow abS, B \rightarrow baS\}$

- Operation for aab :
 - $(S, aab) \vdash (A, ab) \vdash (S, e) \vdash (f, e)$
 - $S \Rightarrow aA \Rightarrow aabS \Rightarrow aab$



RG → NFA: Example 1

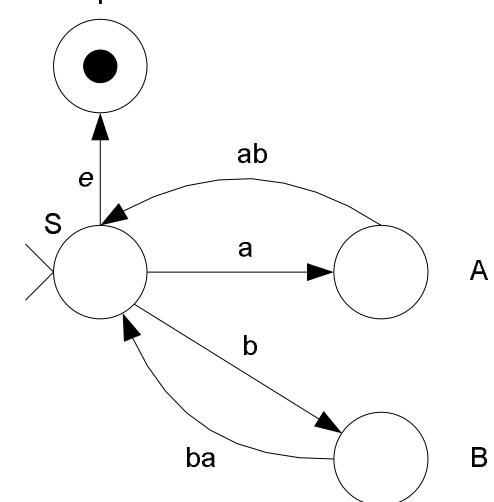
- $R = \{S \rightarrow aA \mid bB \mid e, A \rightarrow abS, B \rightarrow baS\}$



- Operation for aabbbaaab:
 - $(S, aabbbaaab) \vdash (A, abbbaaab) \vdash (S, bbbaaab) \vdash (B, baaab) \vdash (S, aab) \vdash (A, ab) \vdash (S, e) \vdash (f, e)$
 - $S \Rightarrow aA \Rightarrow aabS \Rightarrow aabbB \Rightarrow aabbbaS \Rightarrow aabbbaaA \Rightarrow aabbbaaabS \Rightarrow aabbbaaab$

RG → NFA: Example 1

- $R = \{S \rightarrow aA \mid bB \mid e, A \rightarrow abS, B \rightarrow baS\}$



- Operation for aabbba:
 - $(S, aabbba) \vdash (A, abbba) \vdash (S, bba) \vdash (B, ba) \vdash (S, e) \vdash (f, e)$
 - $S \Rightarrow aA \Rightarrow aabS \Rightarrow aabbB \Rightarrow aabbbaS \Rightarrow aabbba$

RG → NFA: Example 2

- Construct NFA M such that $L(M) = L(RG)$! Describe M with sets!
 - $V = \{a, b, A, B, C, S\}$
 - $\Sigma = \{a, b\}$
 - $S = S$
 - $R = \{S \rightarrow aA \mid bB, A \rightarrow aA \mid aC \mid bb, B \rightarrow e, C \rightarrow a\}$

RG → NFA: Example 2

- Step a1: introduce a state for non-terminal S (this will be the initial state because S is the starting non-terminal)
 - $G(\{a, b, A, B, C, \mathbf{S}\}, \{a, b\}, \{S \rightarrow aA \mid bB, A \rightarrow aA \mid aC \mid bb, B \rightarrow e, C \rightarrow a\}, S)$
 - $M(\{\mathbf{S}\})$

RG → NFA: Example 2

- Step a2: introduce a state for non-terminal A
 - $G(\{a, b, \mathbf{A}, B, C, S\}, \{a, b\}, \{S \rightarrow aA \mid bB, A \rightarrow aA \mid aC \mid bb, B \rightarrow e, C \rightarrow a\}, S)$
 - $M(\{S, \mathbf{A}\})$

RG → NFA: Example 2

- Step a3: introduce a state for non-terminal B
 - $G(\{a, b, A, \mathbf{B}, C, S\}, \{a, b\}, \{S \rightarrow aA \mid bB, A \rightarrow aA \mid aC \mid bb, B \rightarrow e, C \rightarrow a\}, S)$
 - $M(\{S, A, \mathbf{B}\})$

RG → NFA: Example 2

- Step a4: introduce a state for non-terminal C
 - $G(\{a, b, A, B, \mathbf{C}, S\}, \{a, b\}, \{S \rightarrow aA \mid bB, A \rightarrow aA \mid aC \mid bb, B \rightarrow e, C \rightarrow a\}, S)$
 - $M(\{S, A, B, \mathbf{C}\})$

RG \rightarrow NFA: Example 2

- Step b1: introduce the elements of Σ
 - $G(\{a, b, A, B, C, S\}, \{a, b\}, \{S \rightarrow aA \mid bB, A \rightarrow aA \mid aC \mid bb, B \rightarrow e, C \rightarrow a\}, S)$
 - $M(\{S, A, B, C\}, \{a\})$

RG → NFA: Example 2

- Step b2: introduce the elements of Σ
 - $G(\{a, b, A, B, C, S\}, \{a, \mathbf{b}\}, \{S \rightarrow aA \mid bB, A \rightarrow aA \mid aC \mid bb, B \rightarrow e, C \rightarrow a\}, S)$
 - $M(\{S, A, B, C\}, \{a, \mathbf{b}\})$

RG → NFA: Example 2

- Step c1: introduce a new transition for $S \rightarrow aA$
 - $G(\{a, b, A, B, C, S\}, \{a, b\}, \{S \rightarrow aA \mid bB, A \rightarrow aA \mid aC \mid bb, B \rightarrow e, C \rightarrow a\}, S)$
 - $M(\{S, A, B, C\}, \{a, b\}, \Delta)$
 - $\Delta = \{(S, a, A)\}$

RG → NFA: Example 2

- Step c1: introduce a new transition for $S \rightarrow bB$
 - $G(\{a, b, A, B, C, S\}, \{a, b\}, \{S \rightarrow aA \mid bB, A \rightarrow aA \mid aC \mid bb, B \rightarrow e, C \rightarrow a\}, S)$
 - $M(\{S, A, B, C\}, \{a, b\}, \Delta)$
 - $\Delta = \{(S, a, A), (S, b, B)\}$

RG → NFA: Example 2

- Step c1: introduce a new transition for $A \rightarrow aA$
 - $G(\{a, b, A, B, C, S\}, \{a, b\}, \{S \rightarrow aA \mid bB, A \rightarrow aA \mid aC \mid bb, B \rightarrow e, C \rightarrow a\}, S)$
 - $M(\{S, A, B, C\}, \{a, b\}, \Delta)$
 - $\Delta = \{(S, a, A), (S, b, B), (A, a, A)\}$

RG → NFA: Example 2

- Step c1: introduce a new transition for $A \rightarrow aC$
 - $G(\{a, b, A, B, C, S\}, \{a, b\}, \{S \rightarrow aA \mid bB, A \rightarrow aA \mid aC \mid bb, B \rightarrow e, C \rightarrow a\}, S)$
 - $M(\{S, A, B, C\}, \{a, b\}, \Delta)$
 - $\Delta = \{(S, a, A), (S, b, B), (A, a, A), (A, a, C)\}$

RG → NFA: Example 2

- Step d1: introduce a new transition for $A \rightarrow bb$
 - $G(\{a, b, A, B, C, S\}, \{a, b\}, \{S \rightarrow aA \mid bB, A \rightarrow aA \mid aC \mid \mathbf{bb}, B \rightarrow e, C \rightarrow a\}, S)$
 - $M(\{S, A, B, C\}, \{a, b\}, \Delta)$
 - $\Delta = \{(S, a, A), (S, b, B), (A, a, A), (A, a, C), (\mathbf{A}, \mathbf{bb}, f)\}$

RG → NFA: Example 2

- Step d2: introduce a new transition for $B \rightarrow e$
 - $G(\{a, b, A, B, C, S\}, \{a, b\}, \{S \rightarrow aA \mid bB, A \rightarrow aA \mid aC \mid bb, B \rightarrow e, C \rightarrow a\}, S)$
 - $M(\{S, A, B, C\}, \{a, b\}, \Delta)$
 - $\Delta = \{(S, a, A), (S, b, B), (A, a, A), (A, a, C), (A, bb, f), (B, e, f)\}$

RG → NFA: Example 2

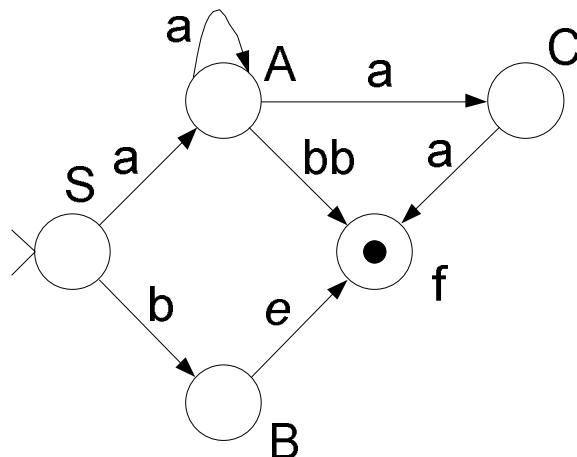
- Step d3: introduce a new transition for $C \rightarrow a$
 - $G(\{a, b, A, B, C, S\}, \{a, b\}, \{S \rightarrow aA \mid bB, A \rightarrow aA \mid aC \mid bb, B \rightarrow e, C \rightarrow a\}, S)$
 - $M(\{S, A, B, C\}, \{a, b\}, \Delta)$
 - $\Delta = \{(S, a, A), (S, b, B), (A, a, A), (A, a, C), (A, bb, f), (B, e, f), (C, a, f)\}$

RG → NFA: Example 2

- Step: introduce the initial state
 - $G(\{a, b, A, B, C, S\}, \{a, b\}, \{S \rightarrow aA \mid bB, A \rightarrow aA \mid aC \mid bb, B \rightarrow e, C \rightarrow a\}, S)$
 - $M(\{S, A, B, C\}, \{a, b\}, \Delta, S)$
 - $\Delta = \{(S, a, A), (S, b, B), (A, a, A), (A, a, C), (A, bb, f), (B, e, f), (C, a, f)\}$

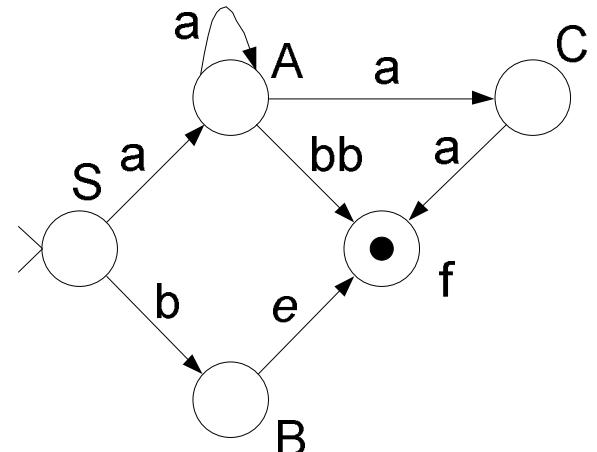
RG → NFA: Example 2

- Step: introduce the final state
 - $G(\{a, b, A, B, C, S\}, \{a, b\}, \{S \rightarrow aA \mid bB, A \rightarrow aA \mid aC \mid bb, B \rightarrow e, C \rightarrow a\}, S)$
 - $M(\{S, A, B, C\}, \{a, b\}, \Delta, S, \{f\})$
 - $\Delta = \{(S, a, A), (S, b, B), (A, a, A), (A, a, C), (A, bb, f), (B, e, f), (C, a, f)\}$



RG → NFA: Example 2

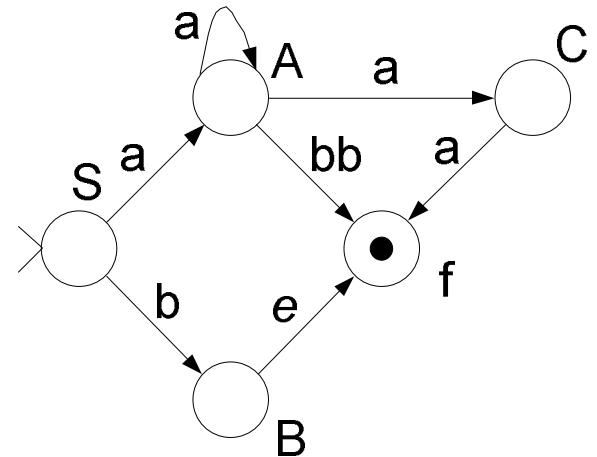
- $R = \{S \rightarrow aA \mid bB, A \rightarrow aA \mid aC \mid bb, B \rightarrow e, C \rightarrow a\}$



- Operation for aabb:
 - $(S, aabb) \xrightarrow{\cdot} (A, abb) \xrightarrow{\cdot} (A, bb) \xrightarrow{\cdot} (f, e)$
 - $S \Rightarrow aA \Rightarrow aaA \Rightarrow aabb$

RG \rightarrow NFA: Example 2

- $R = \{S \rightarrow aA \mid bB, A \rightarrow aA \mid aC \mid bb, B \rightarrow e, C \rightarrow a\}$



- Operation for aaaa:
 - $(S, aaaa) \vdash (A, aaa) \vdash (A, aa) \vdash (C, a) \vdash (f, e)$
 - $S \Rightarrow aA \Rightarrow aaA \Rightarrow aaaC \Rightarrow aaaa$

RG → NFA: Example 3

- Construct NFA M such that $L(M) = L(G)$!
 - $V = \{a, b, A, B, C, D, E, S\}$
 - $\Sigma = \{a, b\}$
 - $S = S$
 - $R = \{S \rightarrow aA, S \rightarrow aB, S \rightarrow aC, S \rightarrow bD, S \rightarrow bA, S \rightarrow bB, A \rightarrow aA, A \rightarrow bA, A \rightarrow aC, A \rightarrow bb, C \rightarrow aE, B \rightarrow aB, B \rightarrow bB, B \rightarrow bD, D \rightarrow bE, D \rightarrow ab, E \rightarrow e\}$

RG → NFA: Example 3

- Steps a: introduce states

- $V = \{a, b, A, B, C, D, E, S\}$

- $\Sigma = \{a, b\}$

- $S = S$

- $R = \{S \rightarrow aA, S \rightarrow aB,$

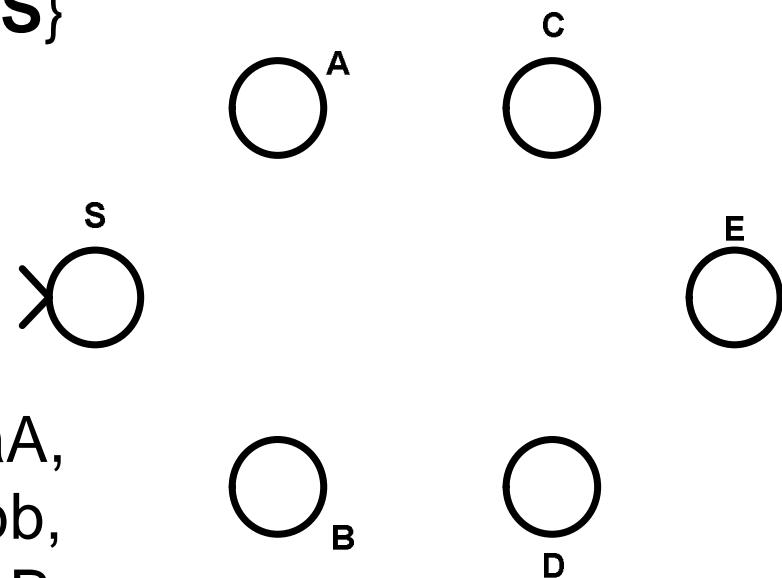
- $S \rightarrow aC, S \rightarrow bD,$

- $S \rightarrow bA, S \rightarrow bB, A \rightarrow aA,$

- $A \rightarrow bA, A \rightarrow aC, A \rightarrow bb,$

- $C \rightarrow aE, B \rightarrow aB, B \rightarrow bB,$

- $B \rightarrow bD, D \rightarrow bE, D \rightarrow ab, E \rightarrow e\}$



RG → NFA: Example 3

- Step b: introduce final state

- $V = \{a, b, A, B, C, D, E, S\}$

- $\Sigma = \{a, b\}$

- $S = S$

- $R = \{S \rightarrow aA, S \rightarrow aB,$

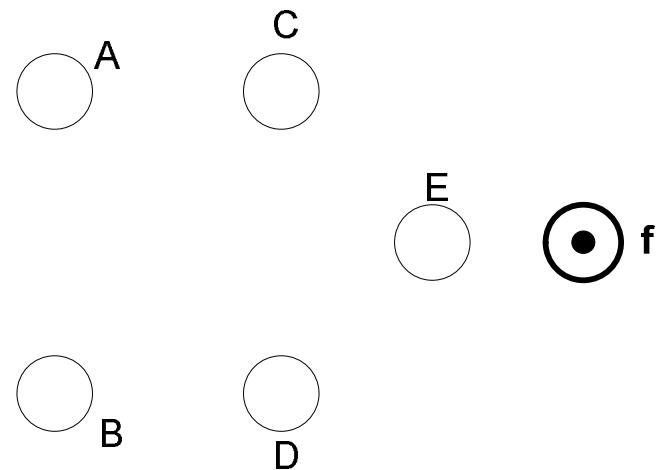
- $S \rightarrow aC, S \rightarrow bD,$

- $S \rightarrow bA, S \rightarrow bB, A \rightarrow aA,$

- $A \rightarrow bA, A \rightarrow aC, A \rightarrow bb,$

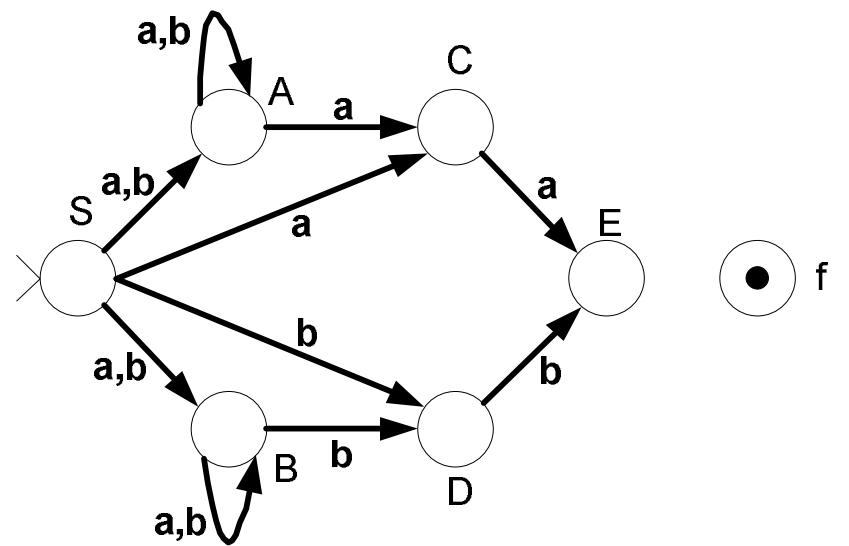
- $C \rightarrow aE, B \rightarrow aB, B \rightarrow bB,$

- $B \rightarrow bD, D \rightarrow bE, D \rightarrow ab, E \rightarrow e\}$



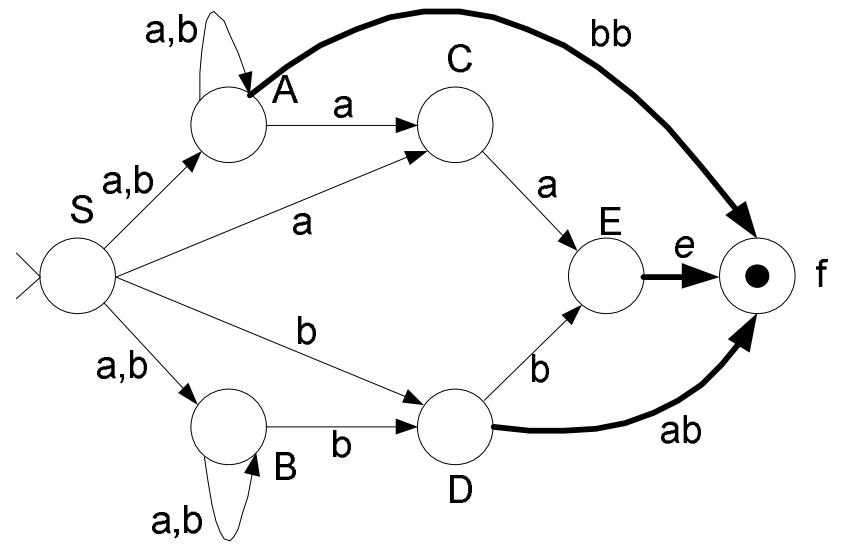
RG → NFA: Example 3

- Step c: introduce arcs
 - $V = \{a, b, A, B, C, D, E, S\}$
 - $\Sigma = \{a, b\}$
 - $S = S$
 - $R = \{S \rightarrow aA, S \rightarrow aB, S \rightarrow aC, S \rightarrow bD, S \rightarrow bA, S \rightarrow bB, A \rightarrow aA, A \rightarrow bA, A \rightarrow aC, A \rightarrow bb, C \rightarrow aE, B \rightarrow aB, B \rightarrow bB, B \rightarrow bD, D \rightarrow bE, D \rightarrow ab, E \rightarrow e\}$



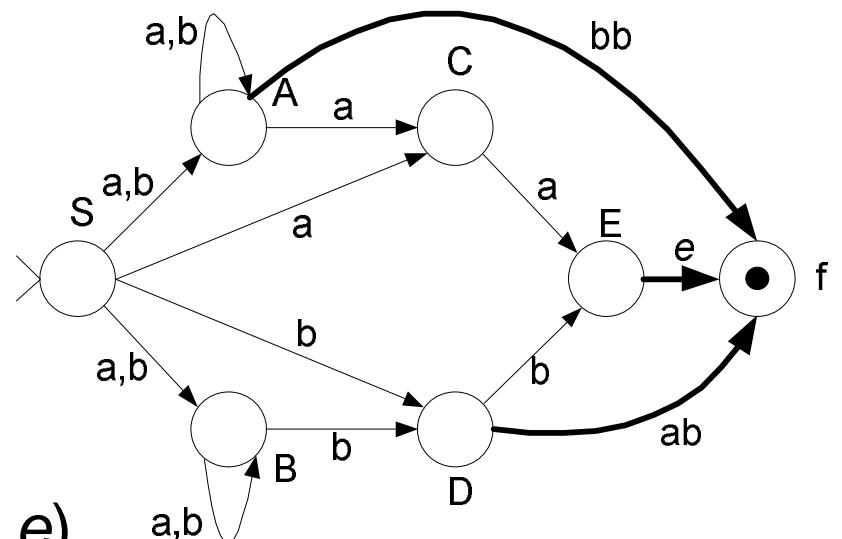
RG → NFA: Example 3

- Step d: introduce arcs
 - $V = \{a, b, A, B, C, D, E, S\}$
 - $\Sigma = \{a, b\}$
 - $S = S$
 - $R = \{S \rightarrow aA, S \rightarrow aB, S \rightarrow aC, S \rightarrow bD, S \rightarrow bA, S \rightarrow bB, A \rightarrow aA, A \rightarrow bA, A \rightarrow aC, \mathbf{A \rightarrow bb}, C \rightarrow aE, B \rightarrow aB, B \rightarrow bB, B \rightarrow bD, D \rightarrow bE, \mathbf{D \rightarrow ab}, \mathbf{E \rightarrow e}\}$



RG → NFA: Example 3

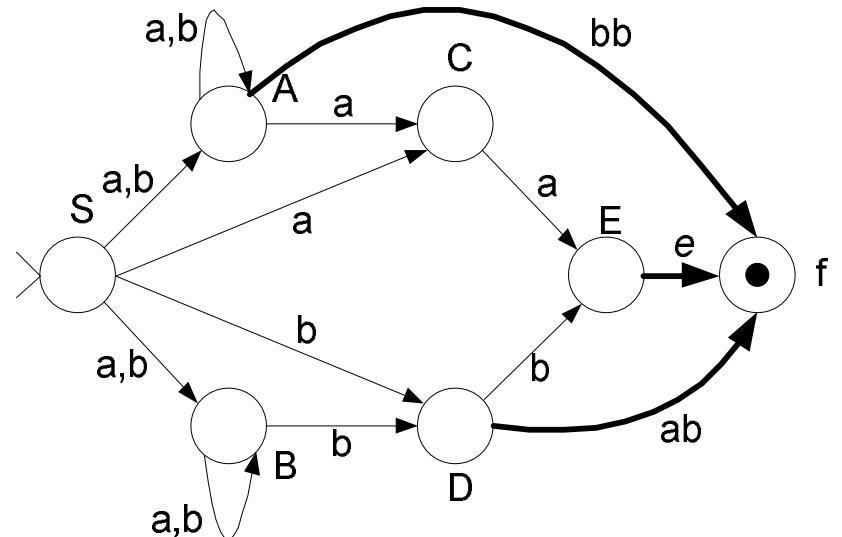
- $R = \{S \rightarrow aA, S \rightarrow aB, S \rightarrow aC, S \rightarrow bD, S \rightarrow bA, S \rightarrow bB, A \rightarrow aA, A \rightarrow bA, A \rightarrow aC, A \rightarrow bb, C \rightarrow aE, B \rightarrow aB, B \rightarrow bB, B \rightarrow bD, D \rightarrow bE, D \rightarrow ab, E \rightarrow e\}$



- Operation for bb:
 - $(S, bb) \vdash (D, b) \vdash (E, e) \vdash (f, e)$
 - $S \Rightarrow bD \Rightarrow bbE \Rightarrow bb$

RG → NFA: Example 3

- $R = \{S \rightarrow aA, S \rightarrow aB, S \rightarrow aC, S \rightarrow bD, S \rightarrow bA, S \rightarrow bB, A \rightarrow aA, A \rightarrow bA, A \rightarrow aC, A \rightarrow bb, C \rightarrow aE, B \rightarrow aB, B \rightarrow bB, B \rightarrow bD, D \rightarrow bE, D \rightarrow ab, E \rightarrow e\}$



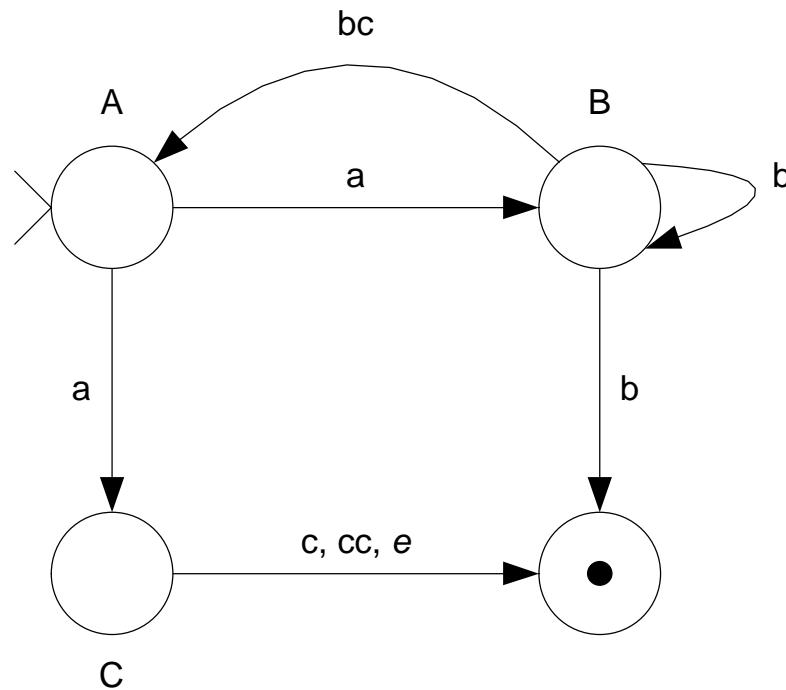
- Operation for abaa:
 - $(S, abaa) \vdash (A, baa) \vdash (A, aa) \vdash (C, a) \vdash (E, e) \vdash (f, e)$
 - $S \Rightarrow aA \Rightarrow abA \Rightarrow abaC \Rightarrow abaaE \Rightarrow abaa$

$\text{RG} \rightarrow \text{NFA}$: Example 4

- Construct NFA M such that $L(M) = L(G)!$
 - $V = \{a, b, c, A, B, C\}$
 - $\Sigma = \{a, b, c\}$
 - $S = A$
 - $R = \{A \rightarrow aB \mid aC, B \rightarrow bcA \mid bB \mid b, C \rightarrow c \mid cc \mid e\}$

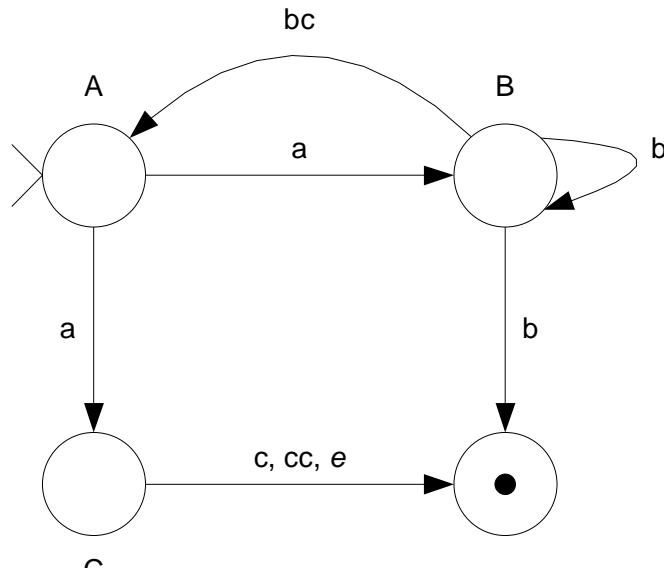
RG → NFA: Example 4

- $R = \{A \rightarrow aB \mid aC, B \rightarrow bcA \mid bB \mid b, C \rightarrow c \mid cc \mid e\}$



RG → NFA: Example 4

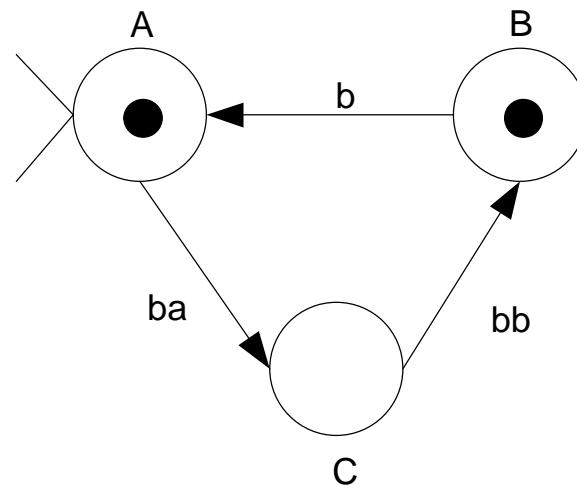
- $R = \{A \rightarrow aB \mid aC, B \rightarrow bcA \mid bB \mid b, C \rightarrow c \mid cc \mid e\}$



- Operation for abaa:
 - $(A, abcabb) \xrightarrow{} (B, bcabb) \xrightarrow{} (A, abb) \xrightarrow{} (B, bb) \xrightarrow{} (B, b) \xrightarrow{} (f, e)$
 - $A \Rightarrow aB \Rightarrow abcA \Rightarrow abcaB \Rightarrow abcabB \Rightarrow abcabb$

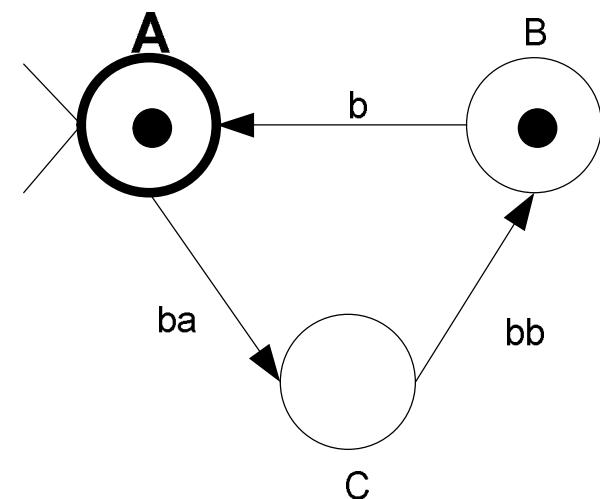
NFA \rightarrow RG: Example 1

- Construct RG G such that $L(G) = L(M)$!



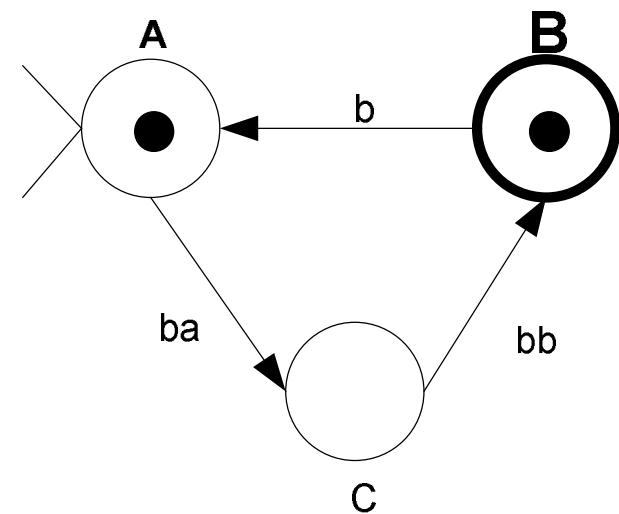
NFA → RG: Example 1

- Step a1: introduce non-terminals
 - $V = \{A\}$



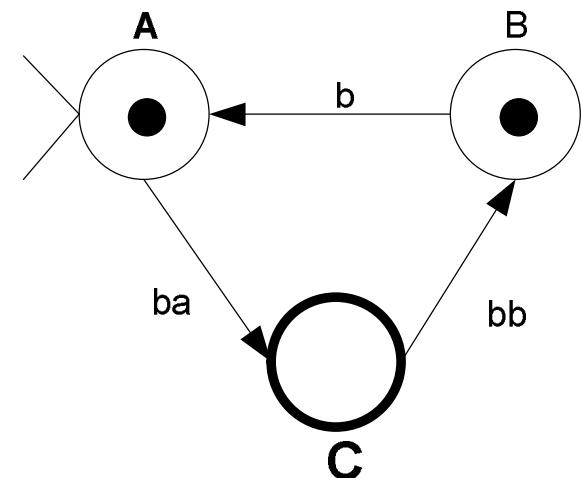
NFA → RG: Example 1

- Step a2: introduce non-terminals
 - $V = \{A, B\}$



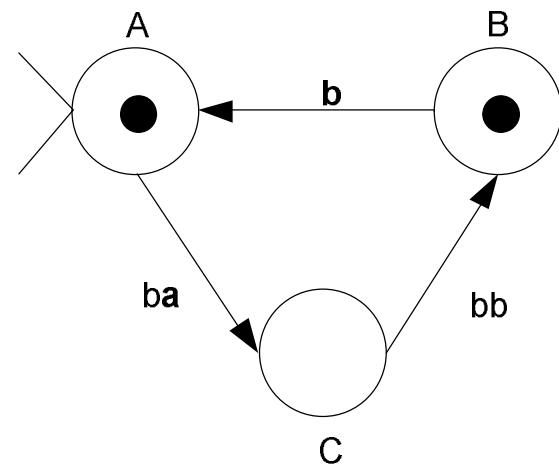
NFA → RG: Example 1

- Step a3: introduce non-terminals
 - $V = \{A, B, C\}$



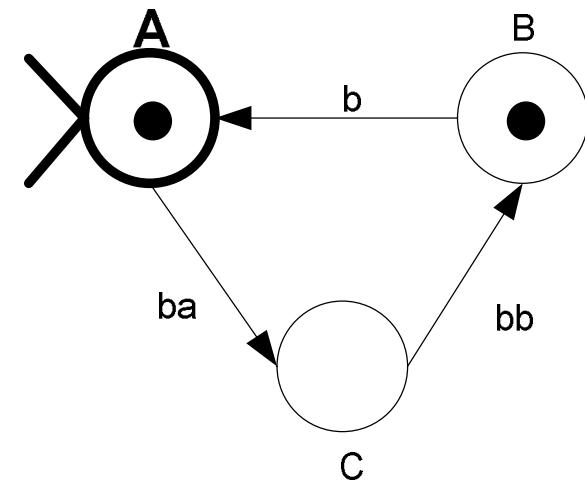
NFA → RG: Example 1

- Step: introduce Σ
 - $V = \{A, B, C\}$
 - $\Sigma = \{a, b\}$



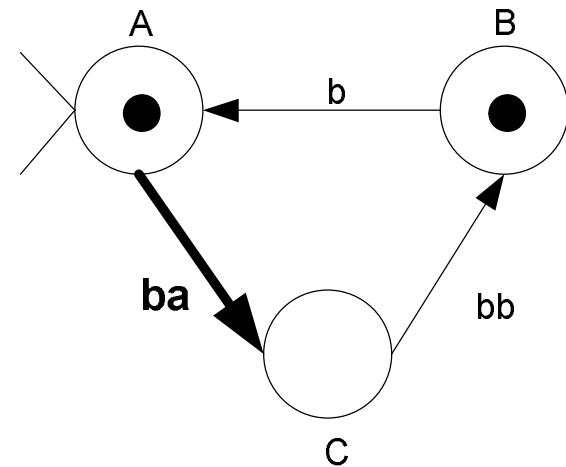
NFA → RG: Example 1

- Step: introduce the starting non-terminal
 - $V = \{A, B, C\}$
 - $\Sigma = \{a, b\}$
 - $S = A$



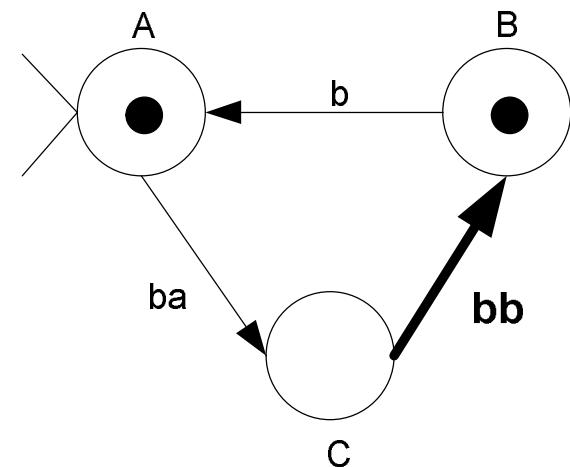
NFA → RG: Example 1

- Step b1: introduce rules for arcs
 - $V = \{A, B, C\}$
 - $\Sigma = \{a, b\}$
 - $S = A$
 - $R = \{A \rightarrow \mathbf{baC}\}$



NFA → RG: Example 1

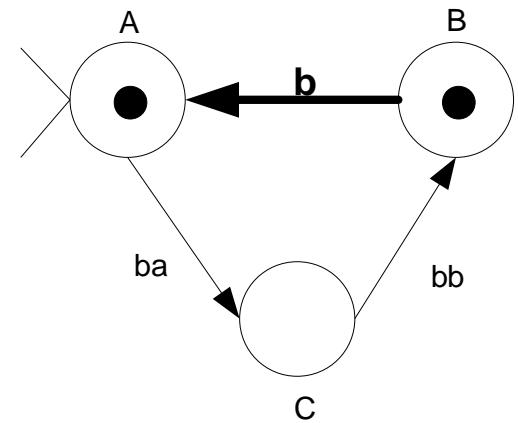
- Step b2: introduce rules for arcs
 - $V = \{A, B, C\}$
 - $\Sigma = \{a, b\}$
 - $S = A$
 - $R = \{A \rightarrow baC, C \rightarrow bbB\}$



NFA → RG: Example 1

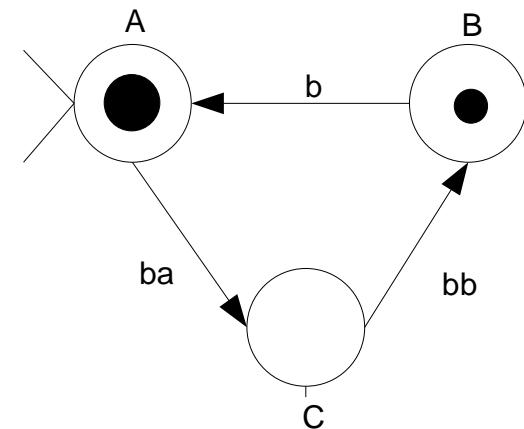
- Step b3: introduce rules for arcs

- $V = \{A, B, C\}$
- $\Sigma = \{a, b\}$
- $S = A$
- $R = \{A \rightarrow baC, C \rightarrow bbB, B \rightarrow bA\}$



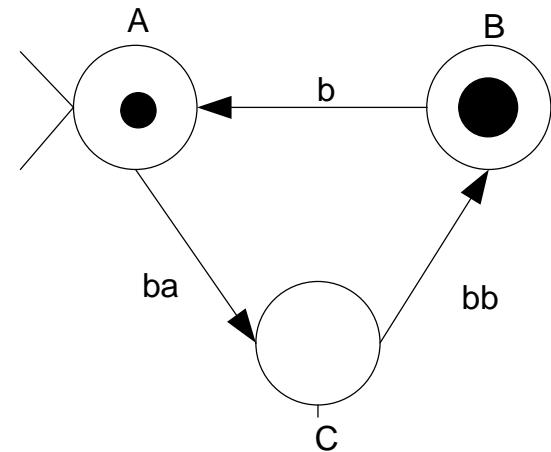
NFA → RG: Example 1

- Step c1: introduce rules for final states
 - $V = \{A, B, C\}$
 - $\Sigma = \{a, b\}$
 - $S = A$
 - $R = \{A \rightarrow baC, C \rightarrow bbB, B \rightarrow bA,$
 $A \rightarrow e\}$



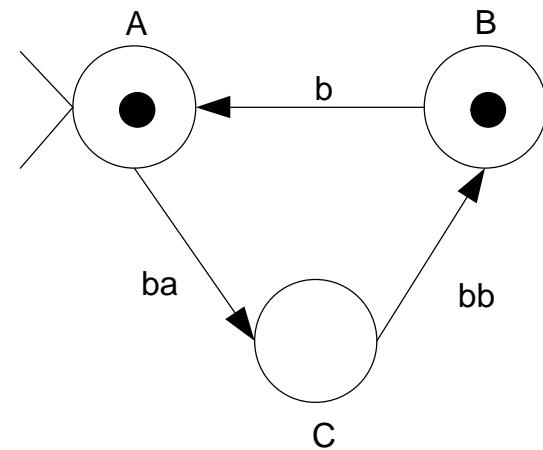
NFA → RG: Example 1

- Step c2: introduce rules for final states
 - $V = \{A, B, C\}$
 - $\Sigma = \{a, b\}$
 - $S = A$
 - $R = \{A \rightarrow baC, C \rightarrow bbB, B \rightarrow bA, A \rightarrow e, B \rightarrow e\}$



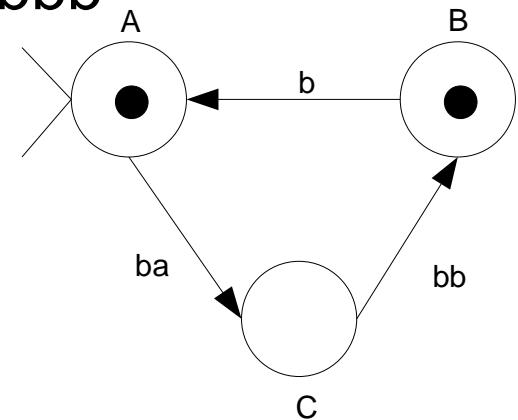
NFA → RG: Example 1

- $R = \{A \rightarrow baC, C \rightarrow bbB, B \rightarrow bA, A \rightarrow e, B \rightarrow e\}$
- Operation for babb:
 - $(A, babb) \vdash (C, bb) \vdash (B, e)$
 - $A \Rightarrow baC \Rightarrow babbB \Rightarrow babb$



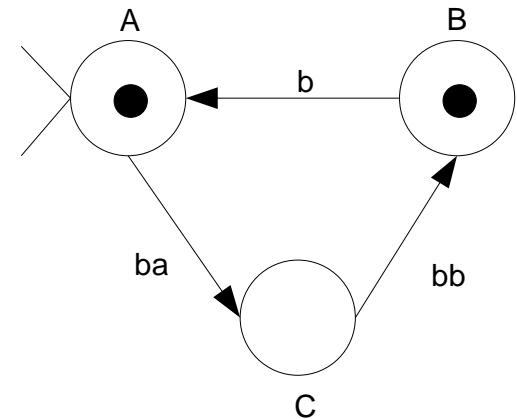
NFA → RG: Example 1

- $R = \{A \rightarrow baC, C \rightarrow bbB, B \rightarrow bA, A \rightarrow e, B \rightarrow e\}$
- Operation for babbb:
 - $(A, babbb) \vdash (C, bbb) \vdash (B, b) \vdash (A, e)$
 - $A \Rightarrow baC \Rightarrow babbB \Rightarrow babbbA \Rightarrow babbb$



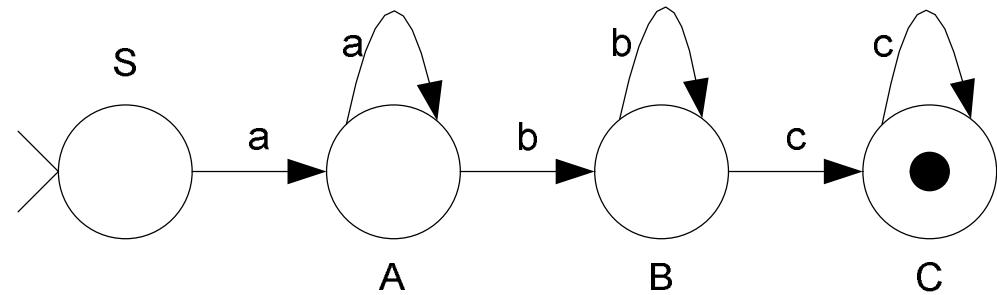
NFA → RG: Example 1

- $R = \{A \rightarrow baC, C \rightarrow bbB, B \rightarrow bA, A \rightarrow e, B \rightarrow e\}$
- Operation for babbabbabb:
 - $(A, babbabbabb) \vdash (C, bbbbabb) \vdash (B, bbabb) \vdash (A, babb) \vdash (C, bb) \vdash (B, e)$
 - $A \Rightarrow baC \Rightarrow babbB \Rightarrow babbA \Rightarrow babbbaC \Rightarrow babbabbabbB \Rightarrow babbabbabb$



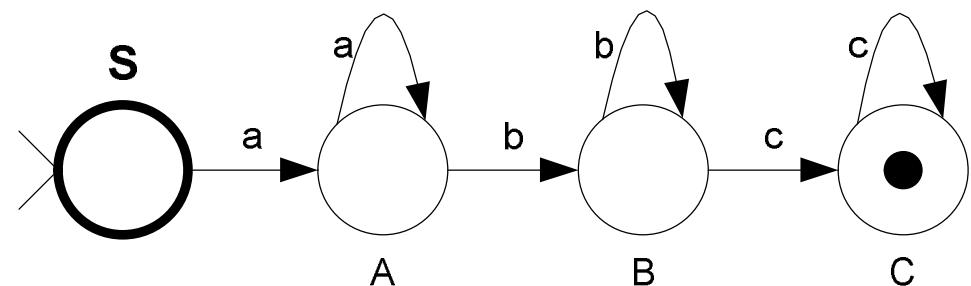
NFA \rightarrow RG: Example 2

- Construct RG G such that $L(G) = L(M)$!



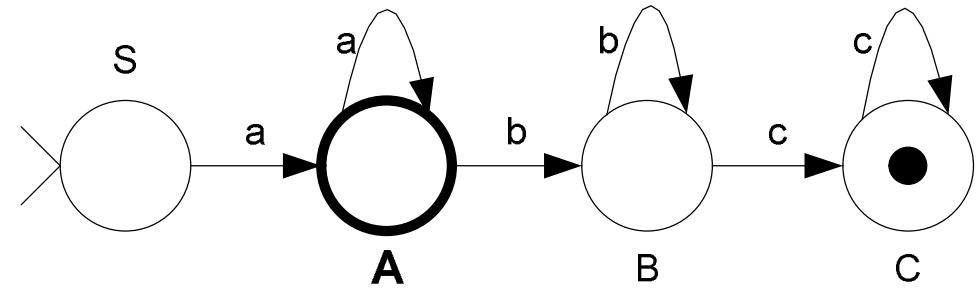
NFA → RG: Example 2

- Step a1: introduce non-terminals
 - $V = \{S\}$



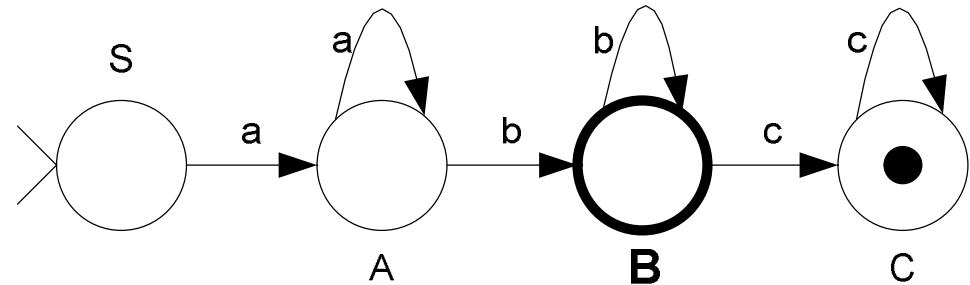
NFA → RG: Example 2

- Step a2: introduce non-terminals
 - $V = \{S, A\}$



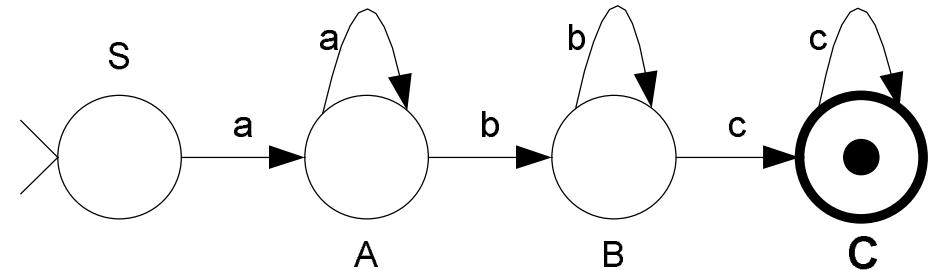
NFA → RG: Example 2

- Step a3: introduce non-terminals
 - $V = \{S, A, B\}$



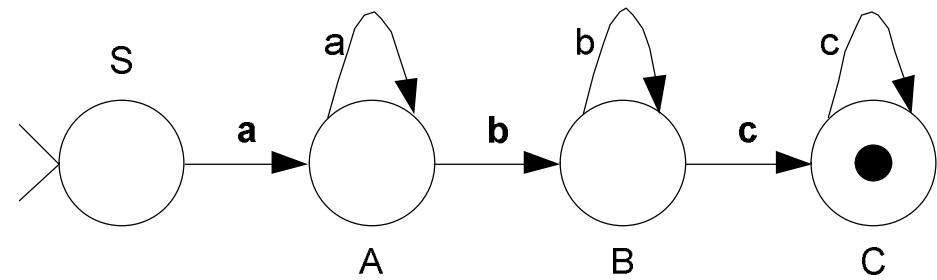
NFA → RG: Example 2

- Step a4: introduce non-terminals
 - $V = \{S, A, B, C\}$



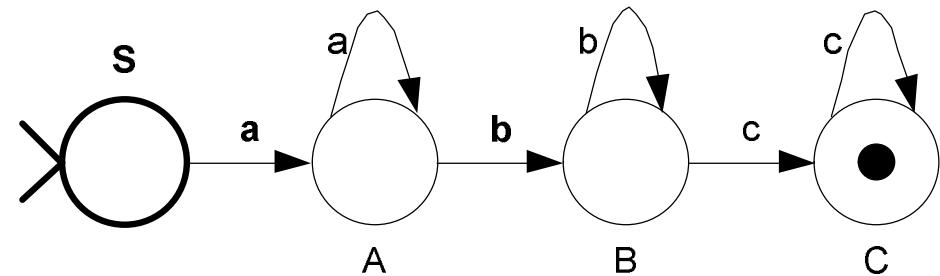
NFA → RG: Example 2

- Step: introduce Σ
 - $V = \{S, A, B, C\}$
 - $\Sigma = \{a, b, c\}$



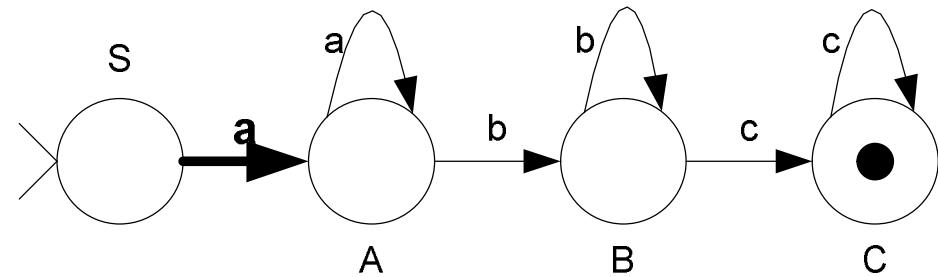
NFA → RG: Example 2

- Step: introduce the starting non-terminal
 - $V = \{S, A, B, C\}$
 - $\Sigma = \{a, b, c\}$
 - $S = S$



NFA → RG: Example 2

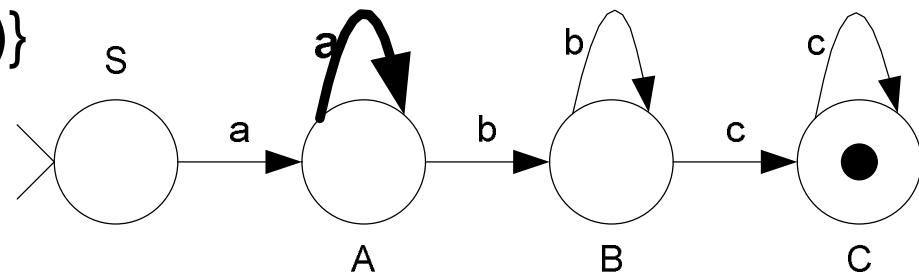
- Step b1: introduce rules for arcs
 - $V = \{S, A, B, C\}$
 - $\Sigma = \{a, b, c\}$
 - $S = S$
 - $R = \{(S, aA)\}$



NFA → RG: Example 2

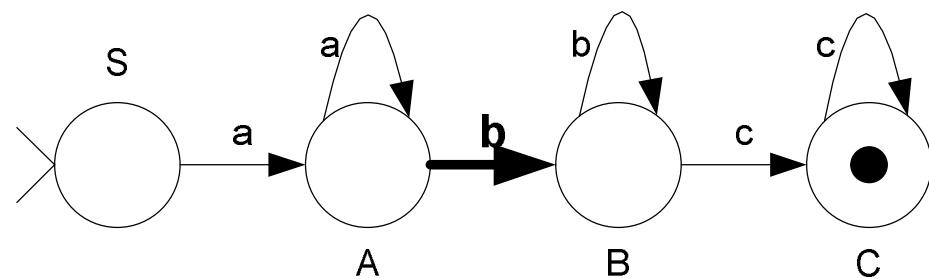
- Step b2: introduce rules for arcs

- $V = \{S, A, B, C\}$
- $\Sigma = \{a, b, c\}$
- $S = S$
- $R = \{(S, aA), (A, aA)\}$



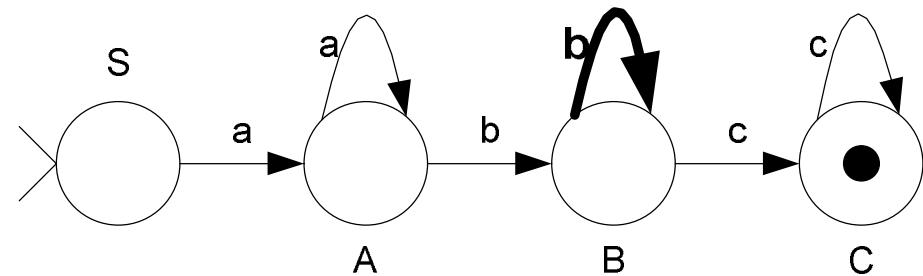
NFA → RG: Example 2

- Step b3: introduce rules for arcs
 - $V = \{S, A, B, C\}$
 - $\Sigma = \{a, b, c\}$
 - $S = S$
 - $R = \{(S, aA), (A, aA), (\mathbf{A}, \mathbf{bB})\}$



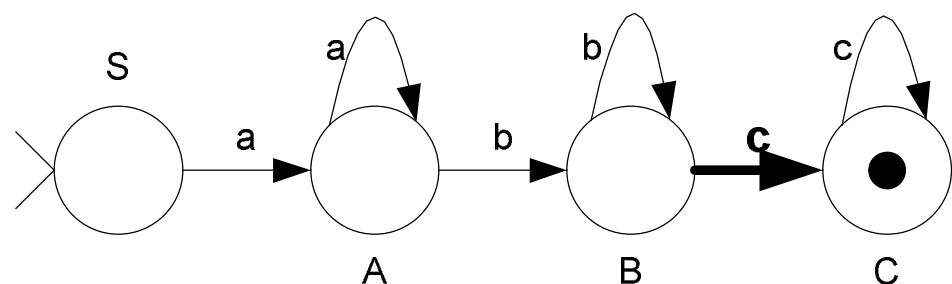
NFA → RG: Example 2

- Step b4: introduce rules for arcs
 - $V = \{S, A, B, C\}$
 - $\Sigma = \{a, b, c\}$
 - $S = S$
 - $R = \{(S, aA), (A, aA), (A, bB), (\mathbf{B}, \mathbf{bB})\}$



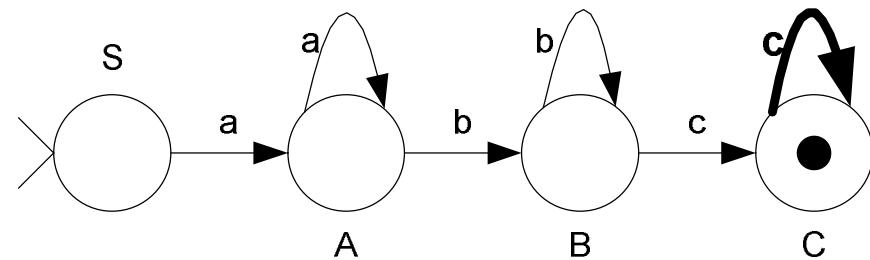
NFA → RG: Example 2

- Step b5: introduce rules for arcs
 - $V = \{S, A, B, C\}$
 - $\Sigma = \{a, b, c\}$
 - $S = S$
 - $R = \{(S, aA), (A, aA), (A, bB), (B, bB), (B, cC)\}$



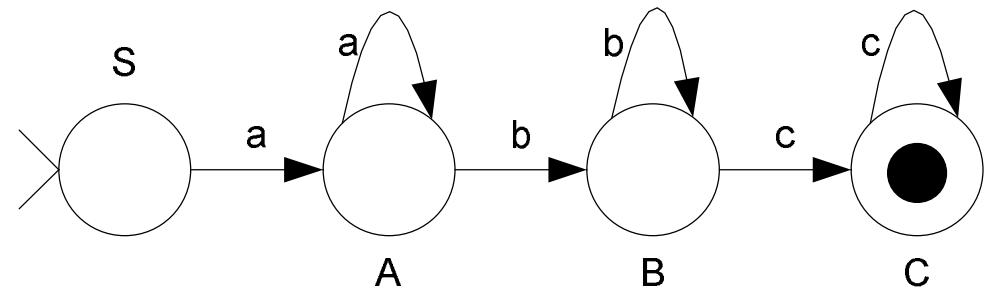
NFA → RG: Example 2

- Step b6: introduce rules for arcs
 - $V = \{S, A, B, C\}$
 - $\Sigma = \{a, b, c\}$
 - $S = S$
 - $R = \{(S, aA), (A, aA), (A, bB), (B, bB), (B, cC), (C, cC)\}$



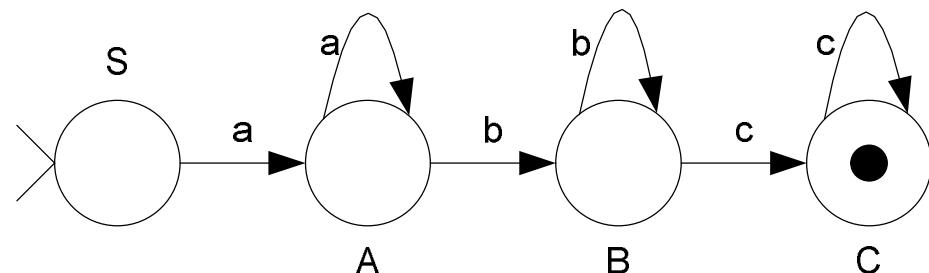
NFA → RG: Example 2

- Step d1: introduce rules for final states
 - $V = \{S, A, B, C\}$
 - $\Sigma = \{a, b, c\}$
 - $S = S$
 - $R = \{(S, aA), (A, aA), (A, bB), (B, bB), (B, cC), (C, cC), (\mathbf{C}, e)\}$



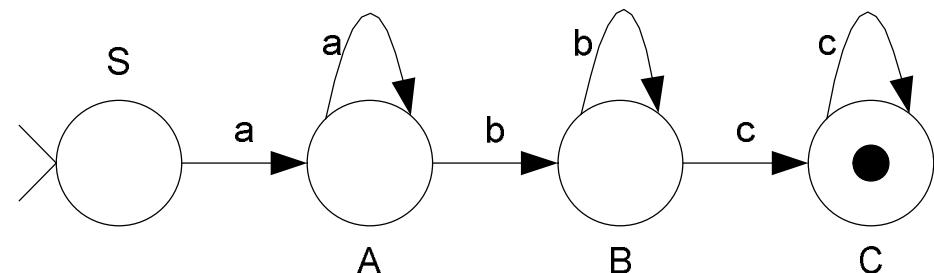
NFA → RG: Example 2

- Operation for abc:
 - $(S, abc) \vdash (A, bc) \vdash (B, c) \vdash (C, e)$
 - $S \Rightarrow aA \Rightarrow abB \Rightarrow abcC \Rightarrow abc$



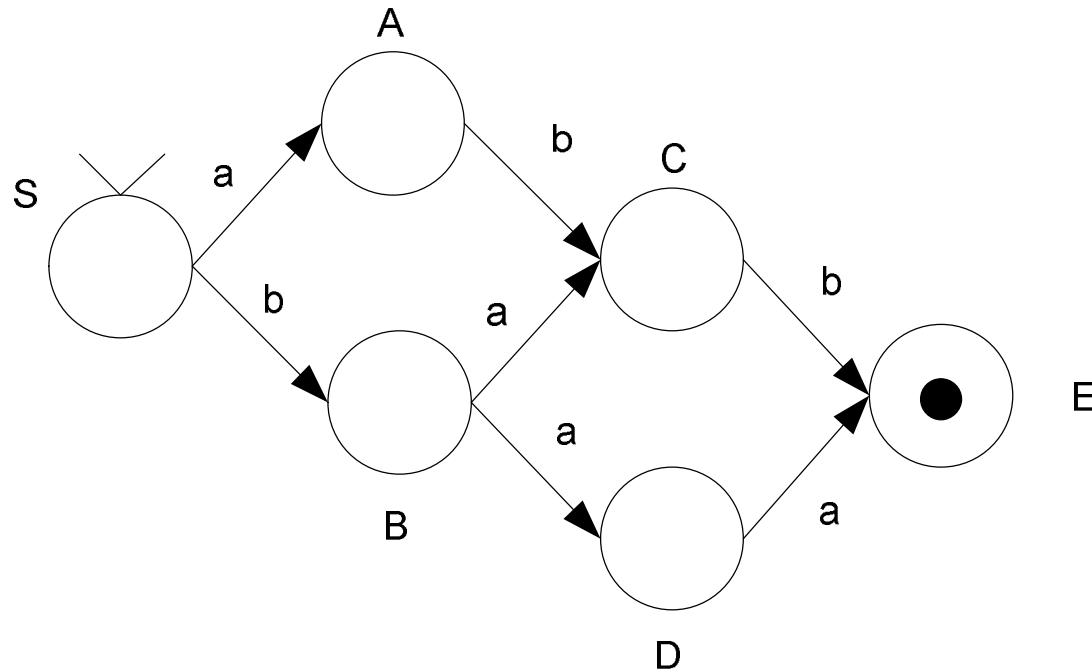
NFA → RG: Example 2

- Operation for aabbcc:
 - $(S, aabbcc) \vdash (A, abbcc) \vdash (A, bbcc) \vdash (B, bcc) \vdash (B, cc) \vdash (C, c) \vdash (C, e)$
 - $S \Rightarrow aA \Rightarrow aaA \Rightarrow aabB \Rightarrow aabbB \Rightarrow aabbcC \Rightarrow aabbccC \Rightarrow aabbcc$



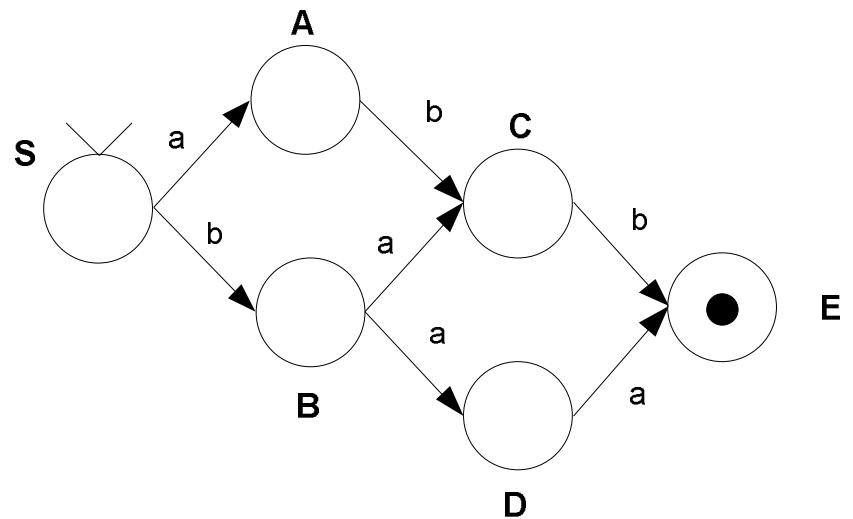
NFA \rightarrow RG: Example 3

- Construct RG G such that $L(G) = L(M)$!



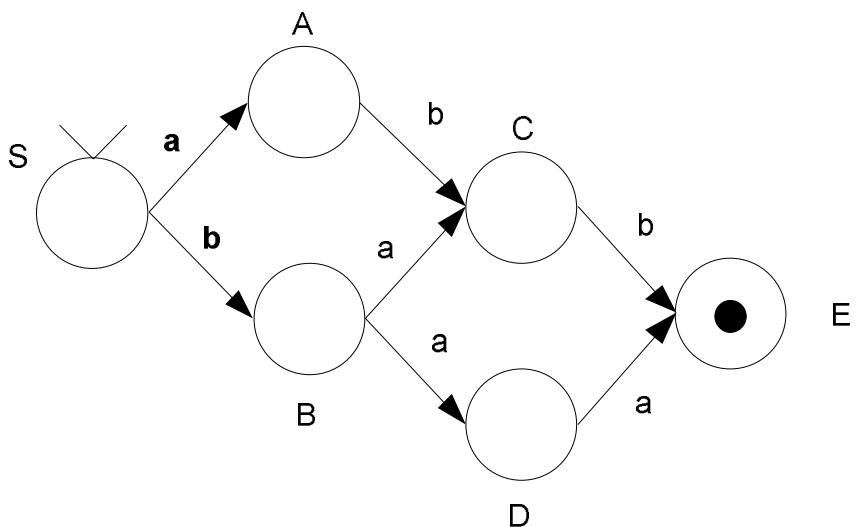
NFA → RG: Example 3

- Steps a: introduce non-terminals
 - $V = \{S, A, B, C, D, E\}$



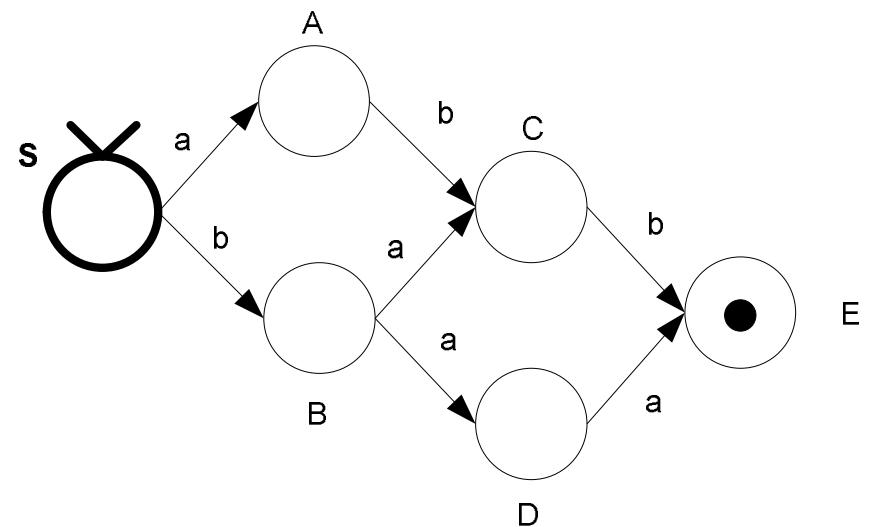
NFA → RG: Example 3

- Step: introduce Σ
 - $V = \{S, A, B, C, D, E\}$
 - $\Sigma = \{a, b\}$



NFA → RG: Example 3

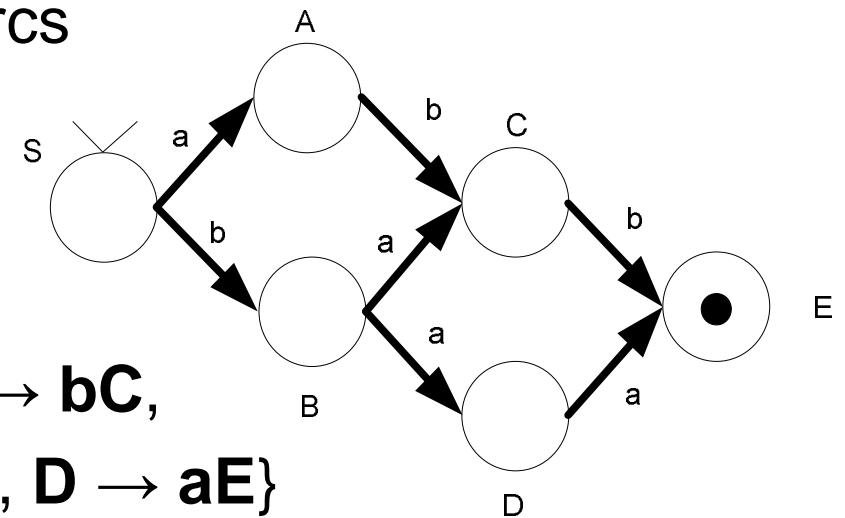
- Step: introduce starting non-terminal
 - $V = \{S, A, B, C, D, E\}$
 - $\Sigma = \{a, b\}$
 - $S = S$



NFA → RG: Example 3

- Steps b: introduce rules for arcs

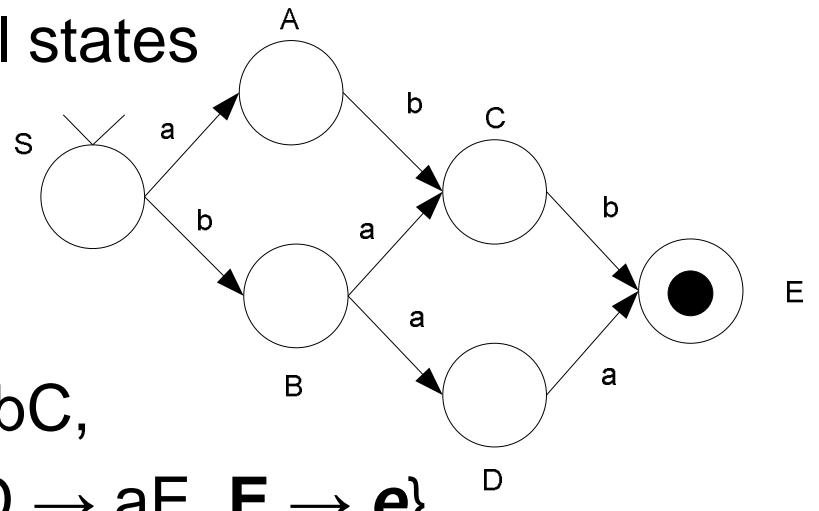
- $V = \{S, A, B, C, D, E\}$
 - $\Sigma = \{a, b\}$
 - $S = S$
 - $R = \{S \rightarrow aA, S \rightarrow bB, A \rightarrow bC, B \rightarrow aC, B \rightarrow aD, C \rightarrow bE, D \rightarrow aE\}$



NFA → RG: Example 3

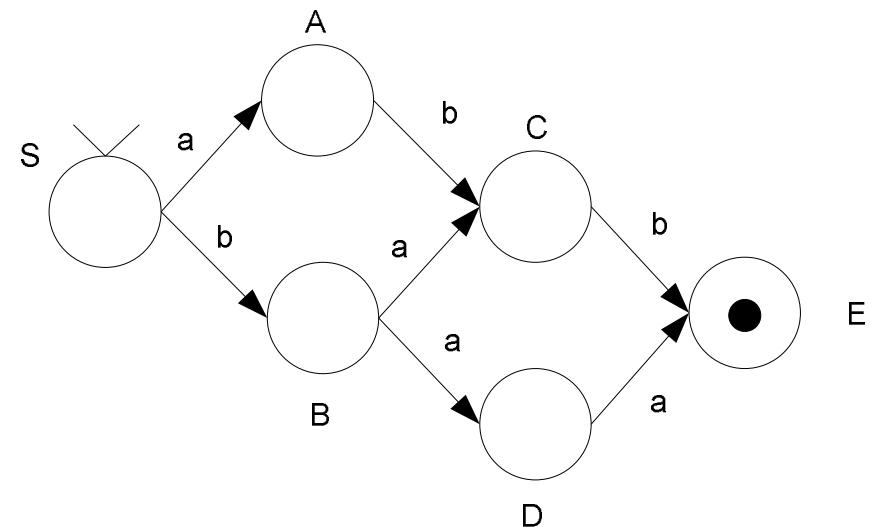
- Steps c: introduce rules for final states

- $V = \{A, B, C, D, E\}$
 - $\Sigma = \{a, b\}$
 - $S = S$
 - $R = \{S \rightarrow aA, S \rightarrow bB, A \rightarrow bC,$
 $B \rightarrow aC, B \rightarrow aD, C \rightarrow bE, D \rightarrow aE, E \rightarrow e\}$



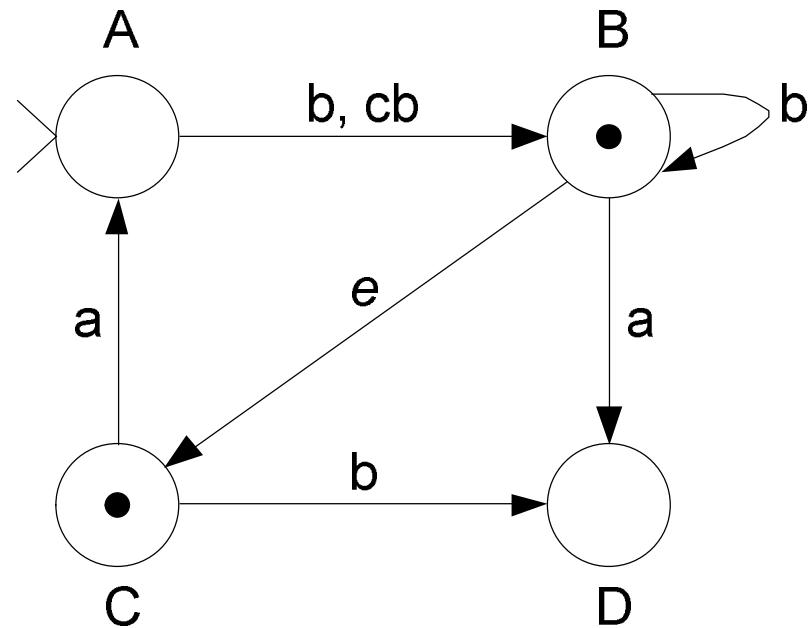
NFA → RG: Example 3

- $R = \{S \rightarrow aA, S \rightarrow bB, A \rightarrow bC, B \rightarrow aC, B \rightarrow aD, C \rightarrow bE, D \rightarrow aE, E \rightarrow e\}$
- Operation for abb:
 - $(S, \text{abb}) \vdash (A, \text{bb}) \vdash (C, \text{b}) \vdash (E, \text{e})$
 - $S \Rightarrow aA \Rightarrow abC \Rightarrow abbE \Rightarrow \text{abb}$



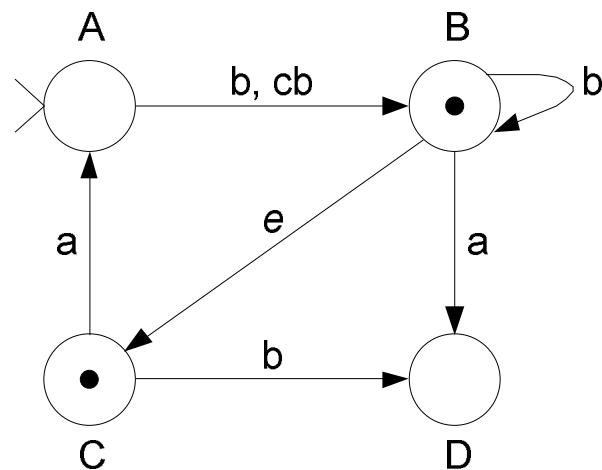
NFA \rightarrow RG: Example 4

- Construct RG G such that $L(G) = L(M)$!



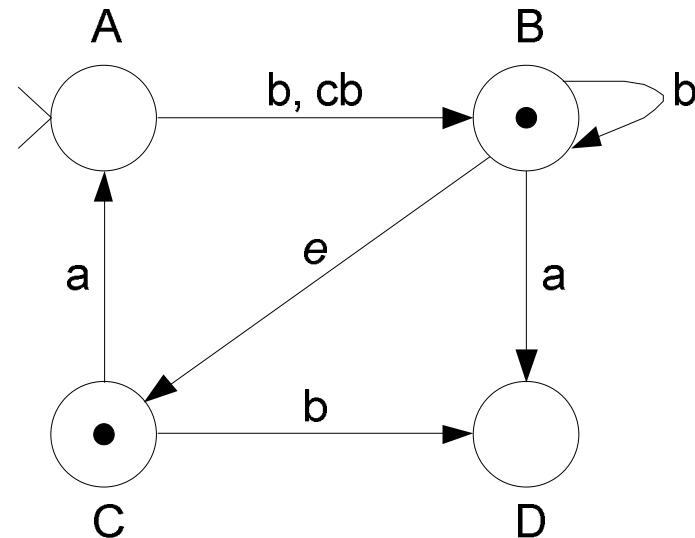
NFA → RG: Example 4

- Solution:
 - $V = \{A, B, C, D\}$
 - $\Sigma = \{a, b, c\}$
 - $S = A$
 - $R = \{A \rightarrow bB \mid cbB, B \rightarrow bB \mid C \mid aD \mid e,$
 $C \rightarrow aA \mid bD \mid e\}$



NFA → RG: Example 4

- $R = \{A \rightarrow bB \mid cbB, B \rightarrow bB \mid C \mid aD \mid e, C \rightarrow aA \mid bD \mid e\}$
- Operation for cbb:
 - $(A, cbb) \vdash (B, b) \vdash (B, e)$
 - $A \Rightarrow cbB \Rightarrow cbbB \Rightarrow cbb$



$L \rightarrow RG$: Example 1

- Give RG G such that $L(G) = \{a^n \in \Sigma^*, n \geq 0\}$!
- $G = (\{a, N\}, \{a\}, R, S)$
 - $R = \{S \rightarrow aS \mid e\}$
 - derivation: $S \Rightarrow aS \Rightarrow aaS \Rightarrow aa$

$L \rightarrow RG$: Example 2

- Give RG G such that $L(G) = \{ba^n \in \Sigma^*, n \geq 0\}!$
- $G = (\{N, a, b\}, \{a, b\}, R, S)$
 - $R = \{S \rightarrow bN, N \rightarrow aN \mid e\}$
 - derivation: $S \Rightarrow bN \Rightarrow baN \Rightarrow baa$

$L \rightarrow RG$: Example 3

- Give RG G such that $L(G) = \{cba^n b^2 \in \Sigma^*, n \geq 0\}!$
- $G = (\{N, a, b, c\}, \{a, b, c\}, R, S)$
 - $L(G) = \{cb(a^n)b^2 \in \Sigma^*, n \geq 0\}$
 - $R = \{S \rightarrow cbNbB, N \rightarrow aN \mid e\}$
 - derivation: $S \Rightarrow cbNbB \Rightarrow cbaNbB \Rightarrow cbaaNbb \Rightarrow cbaabb$

$L \rightarrow RG$: Example 4

- Give RG G such that $L(G) = \{ac^{m+3}a^{n+1}c^2 \in \Sigma^*, n, m \geq 0\}$!
- $G = (\{N, M, a, b, c\}, \{a, b, c\}, R, S)$
 - $L(G) = \{acc(c^m)a(a^n)c^2 \in \Sigma^*, n, m \geq 0\}$
 - $R = \{S \rightarrow aMcccNacc, M \rightarrow cM \mid e, N \rightarrow aN \mid e\}$
 - derivation: $S \Rightarrow aMcccNacc \Rightarrow acMcccNacc \Rightarrow accccNacc \Rightarrow accccaNacc \Rightarrow acccccaacc$

Element of the Theory of the Computation

Lecture 8

PDA computation

- Give the state diagram for M!

- $L(M) = \{ww^R : w \in \{a, b\}^*\}$

- $K = \{s, f\}$

- $\Sigma = \{a, b\}$

- $F = \{f\}$

- Δ :

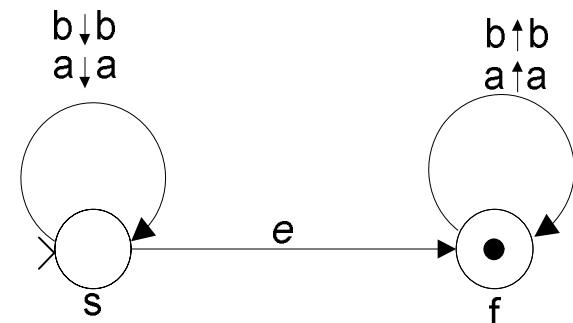
- $((s, a, e), (s, a))$

- $((s, b, e), (s, b))$

- $((s, e, e), (f, e))$

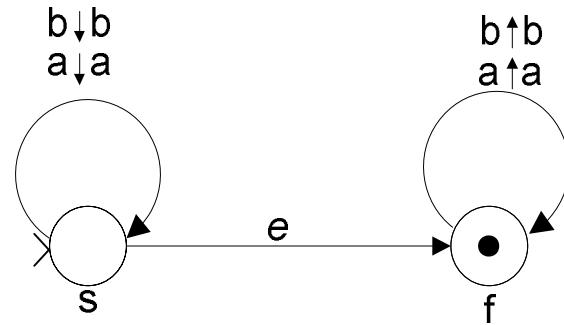
- $((f, a, a), (f, e))$

- $((f, b, b), (f, e))$



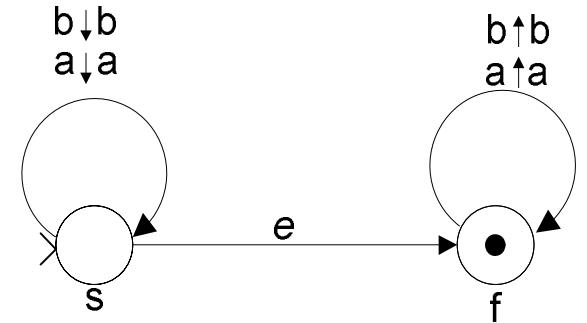
PDA computation

- Give the computation of M for string "abaaba"!



- Result:
 - $(s, abaaba, e) \vdash (s, baaba, a) \vdash (s, aaba, ba) \vdash (s, aba, aba) \vdash (f, aba, aba) \vdash (f, ba, ba) \vdash (f, a, a) \vdash (f, e, e)$

PDA computation



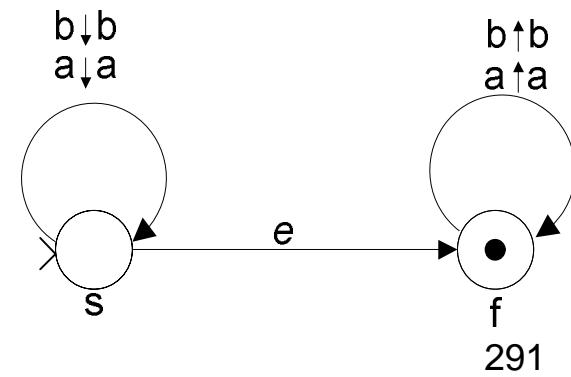
- Give the computation of M for string "bbbb" using the same M as before!

State	Unread input	Stack
s	bbbb	e
s	bbb	b
s	bb	bb
f	bb	bb
f	b	b
f	e	e

PDA computation

State	Unread input	Stack
s	bbbb	e
s	bbb	b
s	b	bbb
f	b	bbb
f	e	bb
f	e	e

- Does the above table represent a computation of M?
 - yes
- Is bbbb accepted or rejected?
 - accepted



PDA: Example 1

- Give PDA M such that $L(M) = L!$
 - $L = \{w \in \{a, b\}^* \mid |a| = |b|\}$

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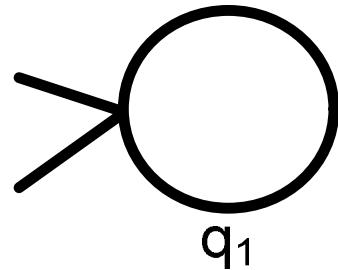
q_1 — 'a' reader state

- Solution:

q_2 — 'b' reader state

'+' — 'a' excess, 'b' lack

'-' — 'b' excess, 'a' lack



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- Give PDA M such that $L(M) = L!$
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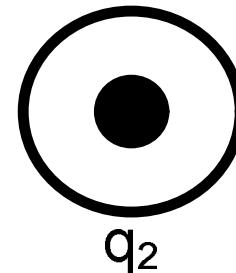
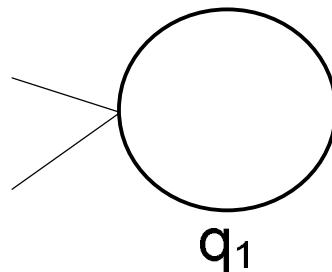
q_1 — 'a' reader state

- Solution:

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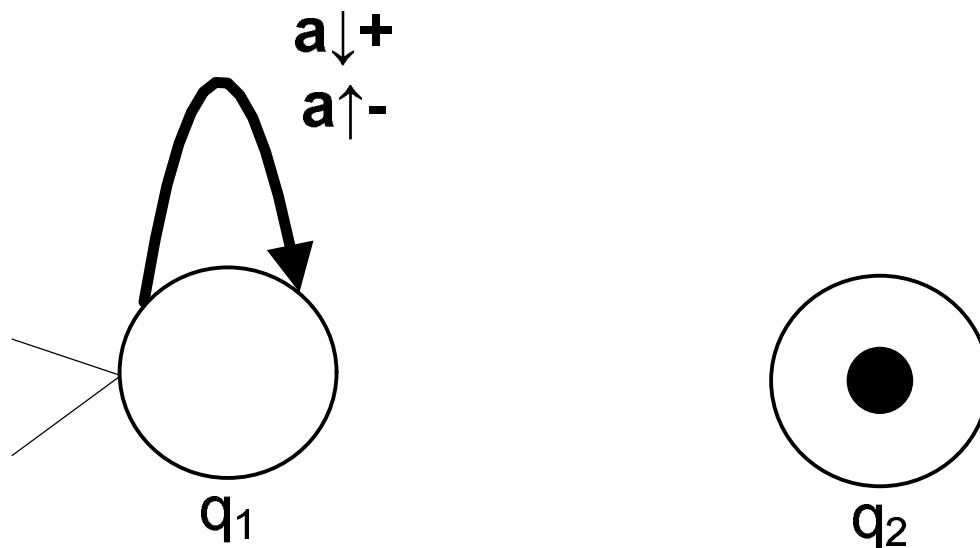
q_1 — 'a' reader state

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 - $L = \{w \in \{a, b\}^* \mid |a| = |b|\}$

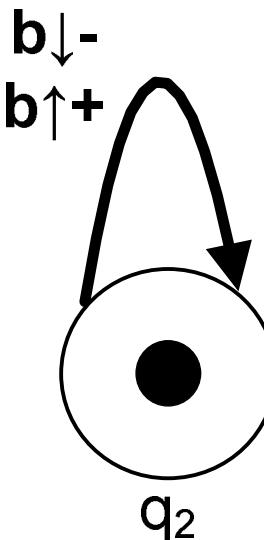
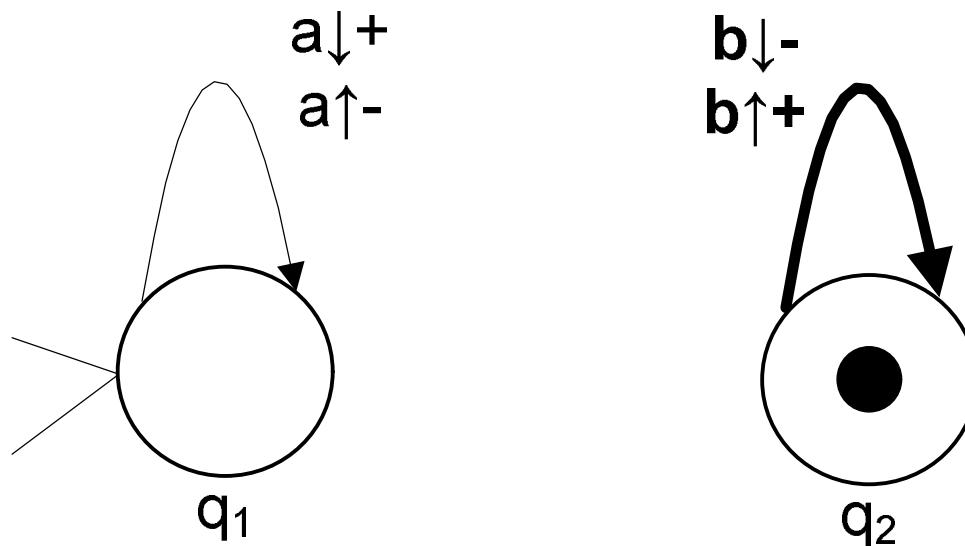
q_1 — 'a' reader state

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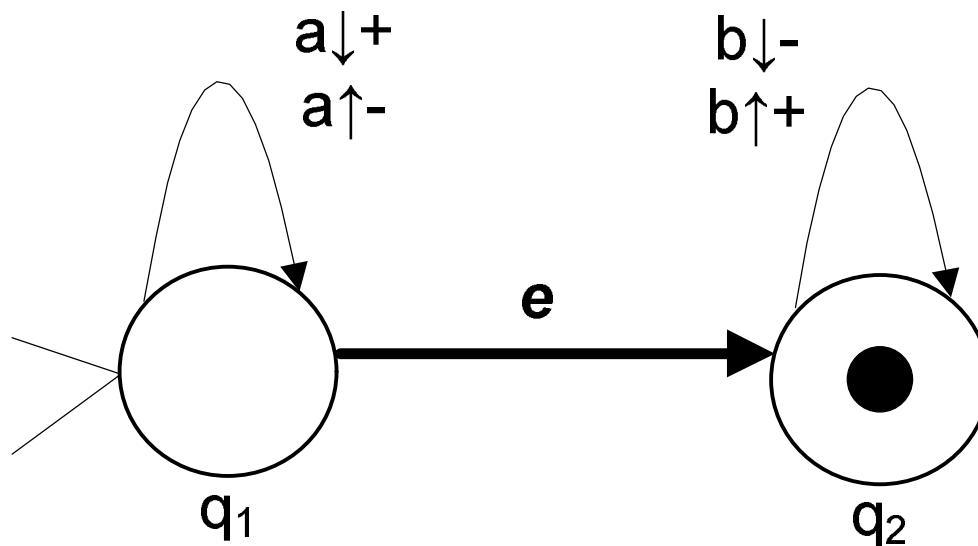
- Solution:

q_1 — 'a' reader state

q_2 — 'b' reader state

'+' — 1 'a' excess, 1 'b' lack

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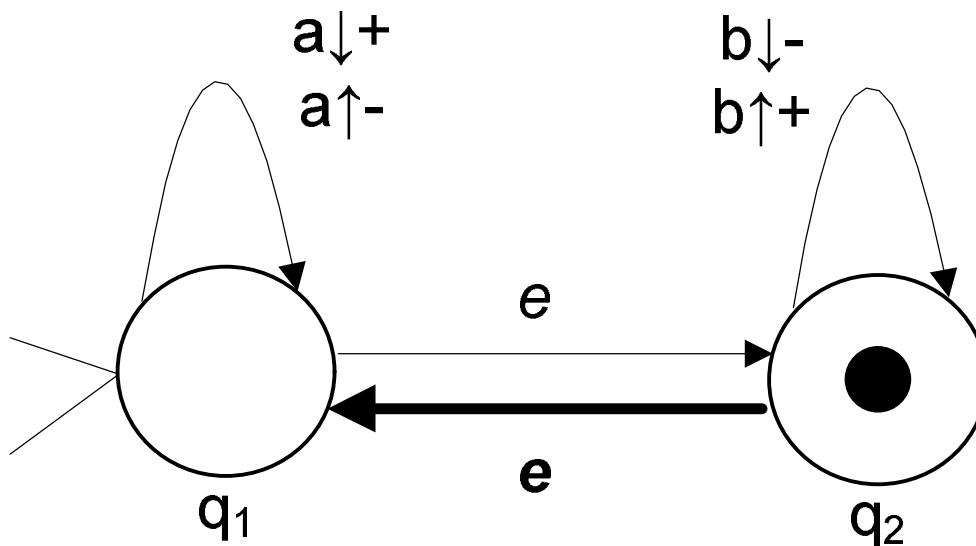
- Solution:

q_1 — 'a' reader state

q_2 — 'b' reader state

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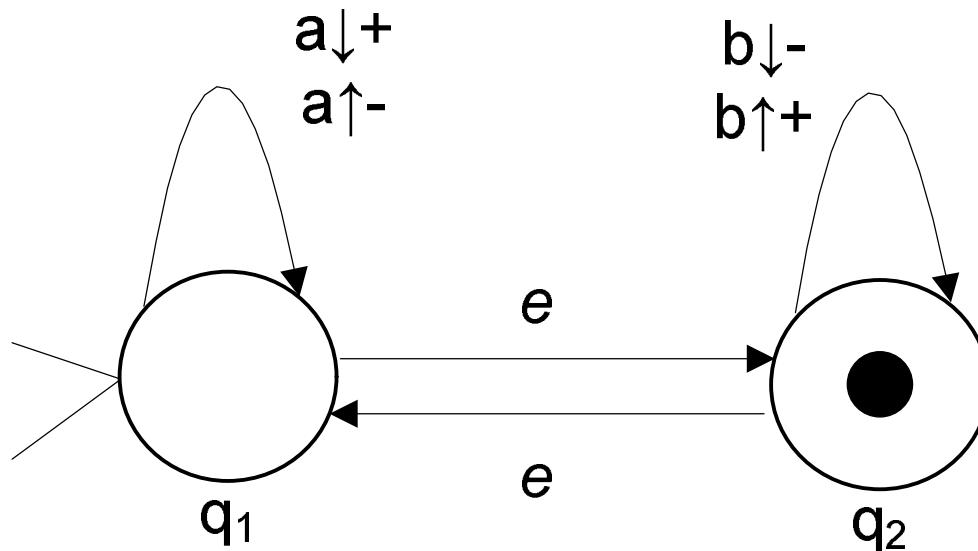
q_1 — 'a' reader state

- Solution:

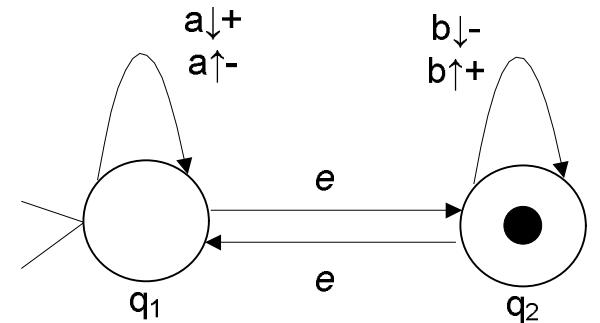
q_2 — 'b' reader state

'+' — 1 'a' excess, 1 'b' lack

'-' — 1 'b' excess, 1 'a' lack



PDA: Example 1



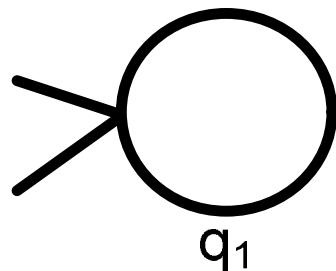
State	Unread input	Stack
q_1	abbaba	e
q_1	bbaba	$+$
q_2	bbaba	$+$
q_2	baba	e
q_2	aba	$-$
q_1	aba	$-$
q_1	ba	e
q_2	ba	e
q_2	a	$-$
q_1	a	$-$
q_1	e	e
q_2	e	e

PDA: Example 2

- Give PDA M such that $L(M) = L!$
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PDA: Example 2

- Give PDA M such that $L(M) = L!$
 - $L = \{w \in \{a, b\}^* \mid |a| = 3|b|\}$
- Remark:
 - '+' in the stack means $1/3$ 'b' lack
 - $1/3$ 'b' cannot be read, but you can read two additional 'a'; then "+++" means $3/3$ 'b' lack
- Solution:



PDA: Example 2

- Give PDA M such that $L(M) = L!$
 - $L = \{w \in \{a, b\}^* \mid |a| = 3|b|\}$

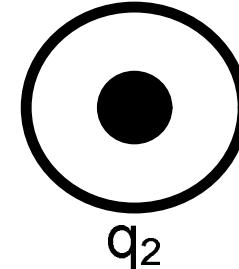
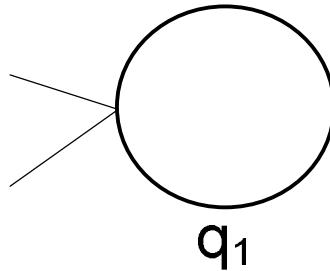
- Solution:

q_1 — 'a' reader state

q_2 — 'b' reader state

'+' — 1 'a' excess, $1/3$ 'b' lack

'-' — $1/3$ 'b' excess, 1 'a' lack



PDA: Example 2

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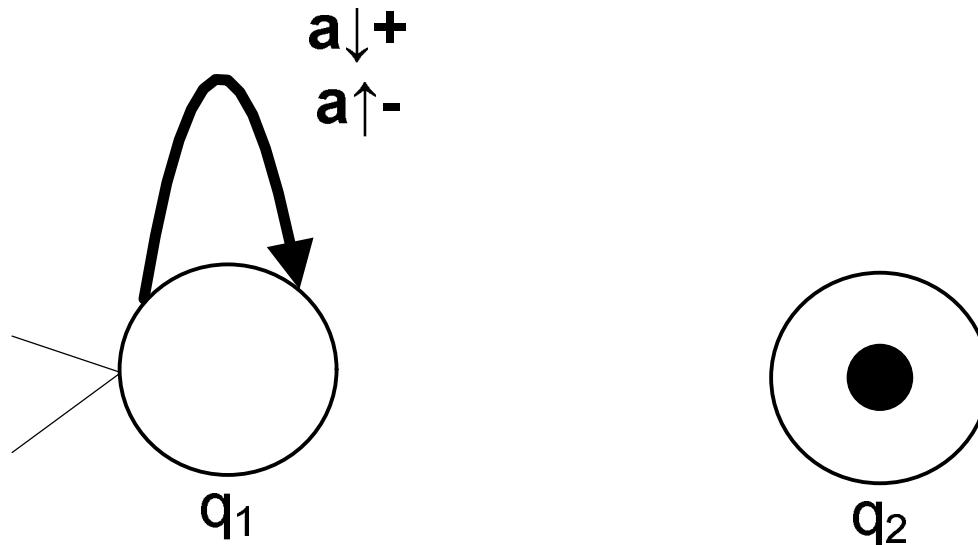
- Solution:

q_1 — 'a' reader state

q_2 — 'b' reader state

'+' — 1 'a' excess, $1/3$ 'b' lack

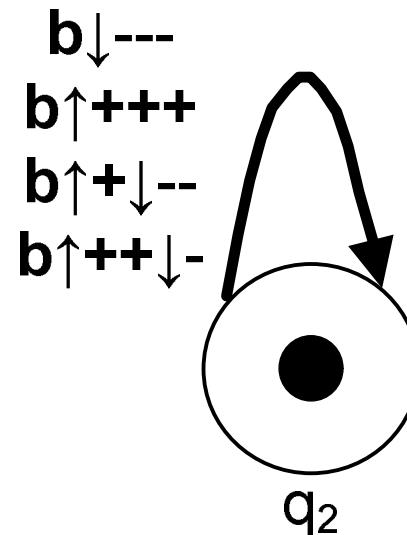
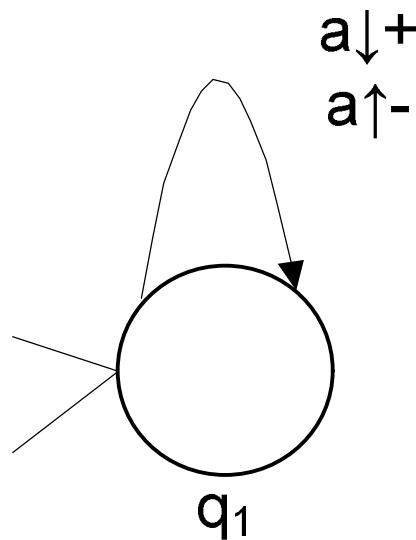
'-' — $1/3$ 'b' excess, 1 'a' lack



PDA: Example 2

- Give PDA M such that $L(M) = L!$
 - $L = \{w \in \{a, b\}^* \mid |a| = 3|b|\}$

- Solution:



q_1 — 'a' reader state

q_2 — 'b' reader state

'+' — 1 'a' excess, $1/3$ 'b' lack

'-' — $1/3$ 'b' excess, 1 'a' lack

push $3 * 1/3$ 'b' excess

pop $3 * 1/3$ 'b' lack

pop $1 * 1/3$ 'b' lack and push $2 * 1/3$ 'b' excess

pop $2 * 1/3$ 'b' lack and push $1 * 1/3$ 'b' excess

PDA: Example 2

- Give PDA M such that $L(M) = L!$
 - $L = \{w \in \{a, b\}^* \mid |a| = 3|b|\}$

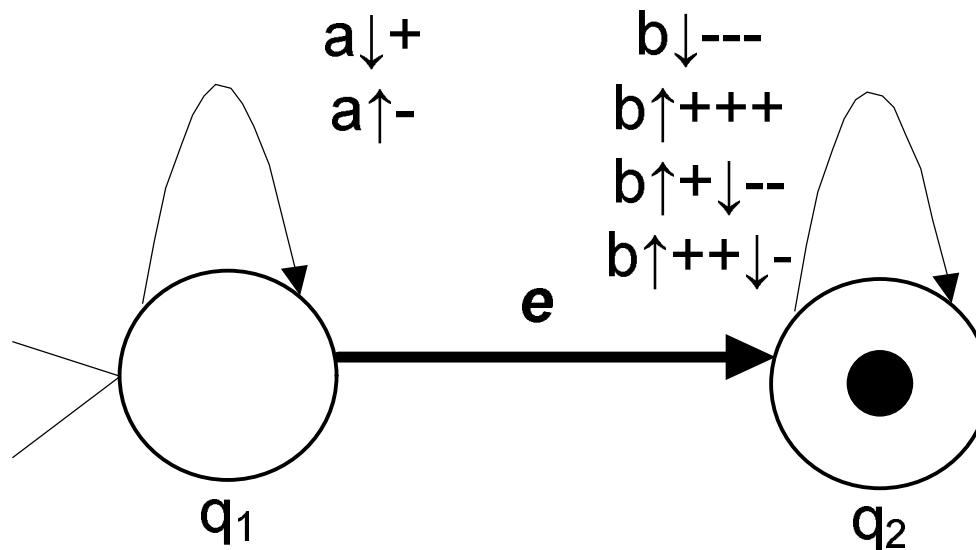
- Solution:

q_1 — 'a' reader state

q_2 — 'b' reader state

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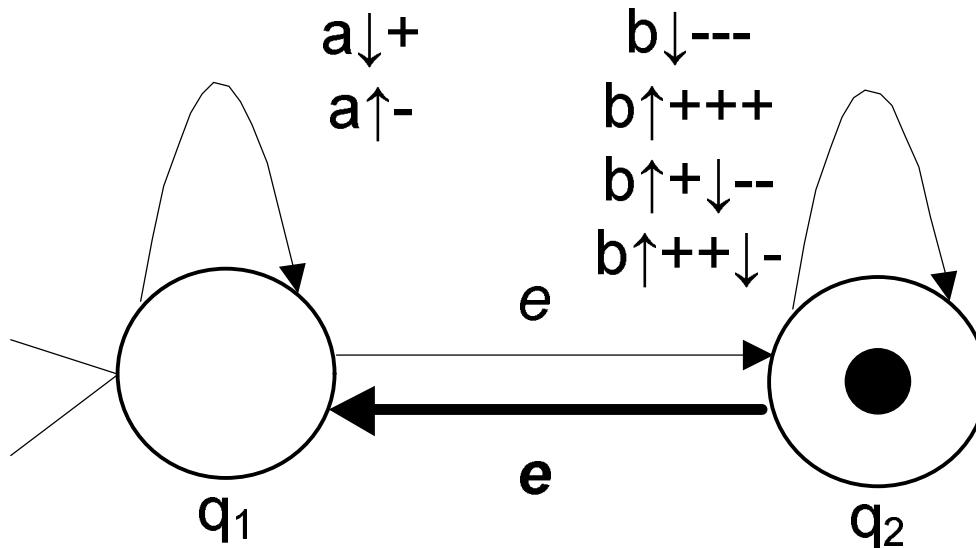
- Solution:

q_1 — 'a' reader state

q_2 — 'b' reader state

'+' — 1 'a' excess, 1/3 'b' lack

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PDA: Example 2

- Give PDA M such that $L(M) = L!$

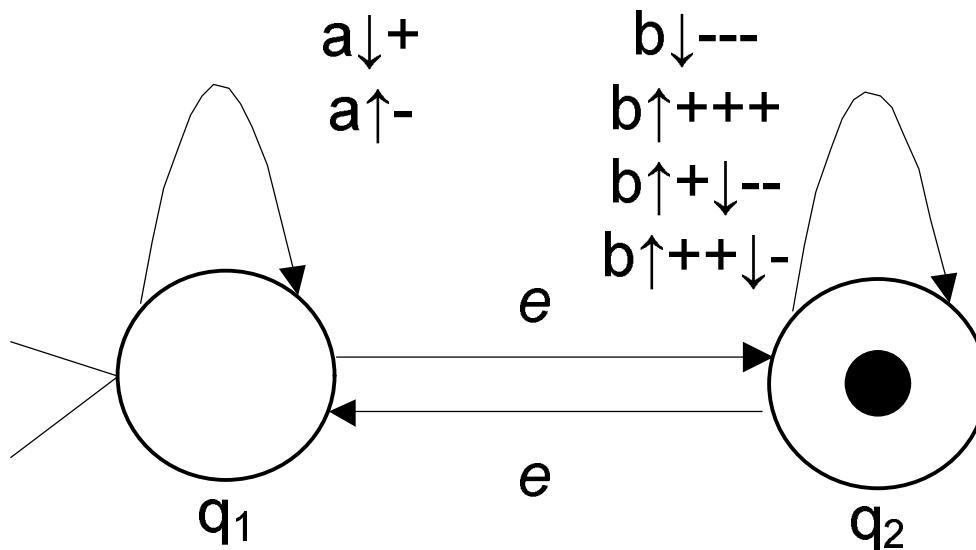
- $L = \{w \in \{a, b\}^* \mid |a| = 3|b|\}$

q_1 — 'a' reader state

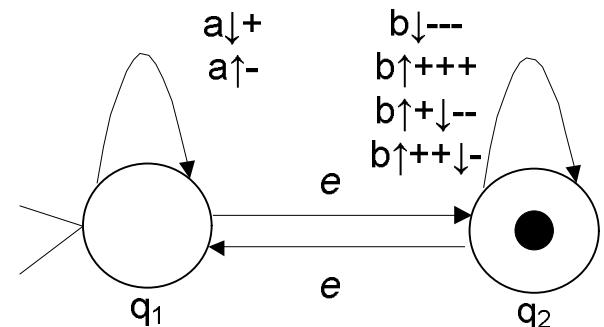
q_2 — 'b' reader state

'+' — 1 'a' excess, 1/3 'b' lack

'-' — 1/3 'b' excess, 1 'a' lack



PDA: Example 2



State	Unread input	Stack
q_1	aaba	e
q_1	aba	+
q_1	ba	++
q_2	ba	++
q_2	a	-
q_1	a	-
q_1	e	e
q_2	e	e

PDA: Example 3

- Give PDA M such that $L(M) = L!$
 - $L = \{w \in \{a, b\}^* \mid 2|a| = 3|b|\}$
- Let ' $+$ ' = 1/2 'a' excess
 - if ' $+$ ' is in the stack then the $2|a| = 3|b|$ equation does not hold
 - instead $2(|a| - 1/2) = 3|b|$ is true
 - $2|a| - 1 = 3|b|$
 - $2|a| = 3|b| + 1$
 - $2|a| = 3(|b| + 1/3)$
 - ' $+$ ' also means 1/3 'b' lack
- Let ' $-$ ' = 1/3 'b' excess
 - ' $-$ ' also means 1/2 'a' lack

PDA: Example 3

- Give PDA M such that $L(M) = L!$

- $L = \{w \in \{a, b\}^* \mid 2|a| = 3|b|\}$

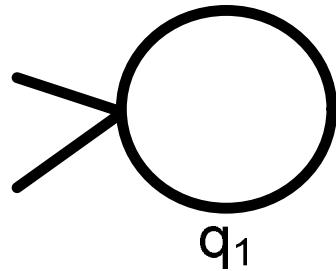
q_1 — 'a' reader state

q_2 — 'b' reader state

- Solution:

'+' — 1/2 'a' excess, 1/3 'b' lack

'-' — 1/3 'b' excess, 1/2 'a' lack



PDA: Example 3

- Give PDA M such that $L(M) = L!$

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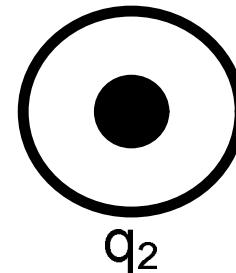
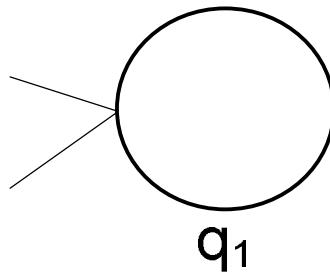
q_1 — 'a' reader state

q_2 — 'b' reader state

- Solution:

'+' — 1/2 'a' excess, 1/3 'b' lack

'-' — 1/3 'b' excess, 1/2 'a' lack



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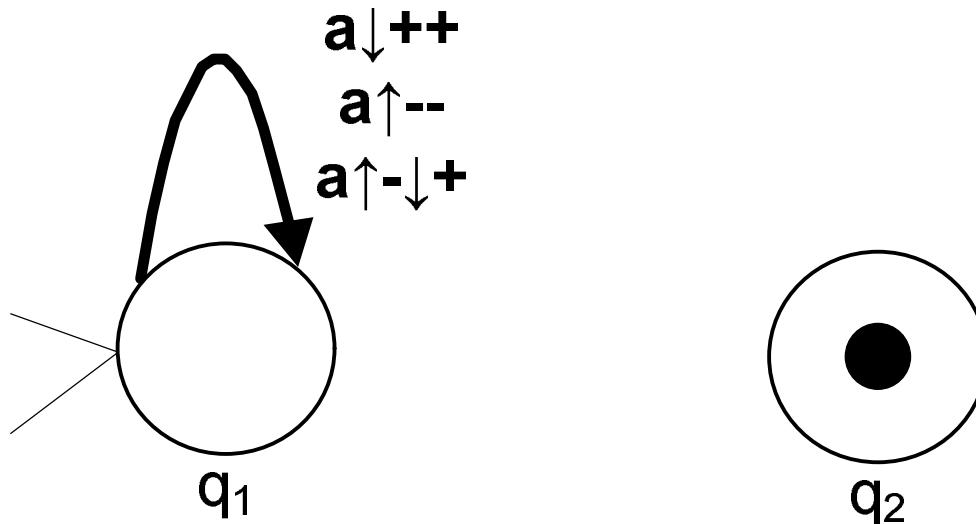
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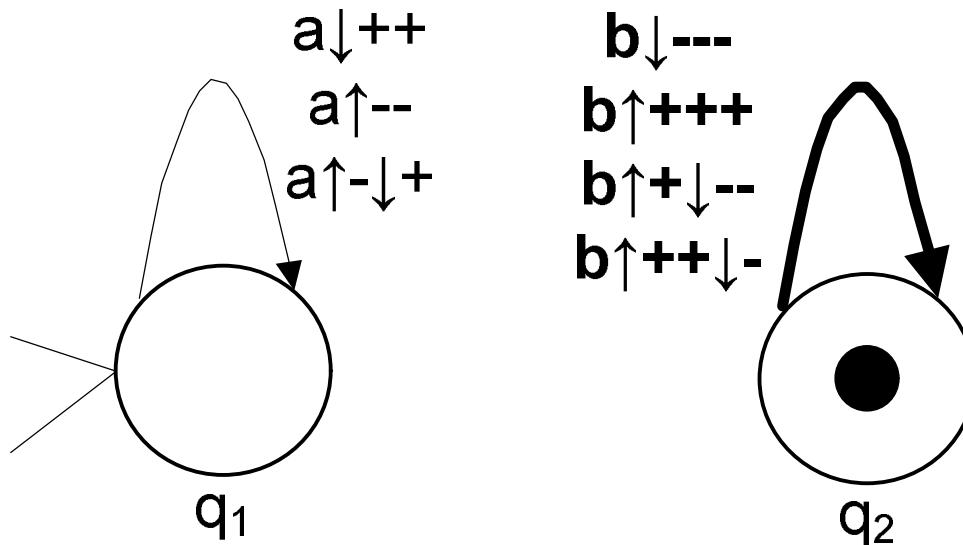
- $L = \{w \in \{a, b\}^* \mid 2|a| = 3|b|\}$

q_1 — 'a' reader state

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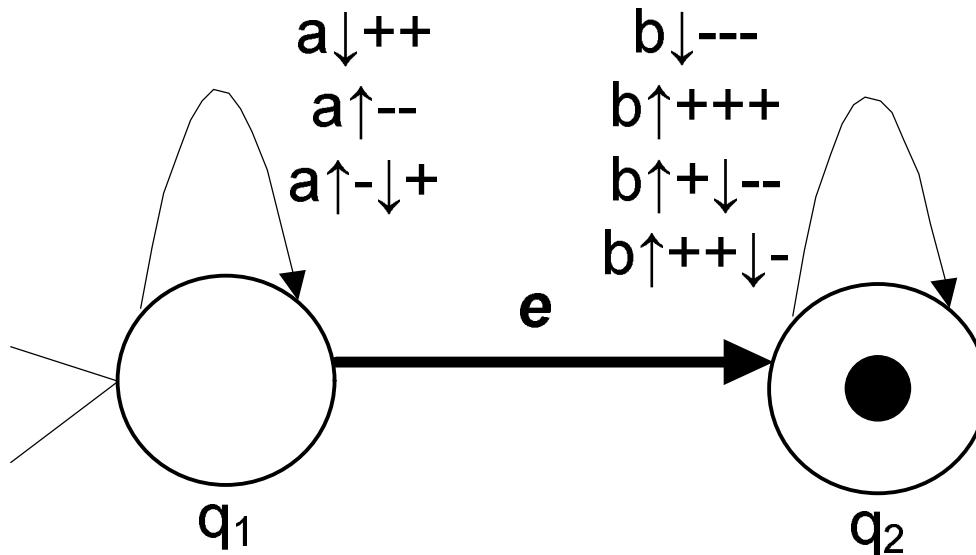
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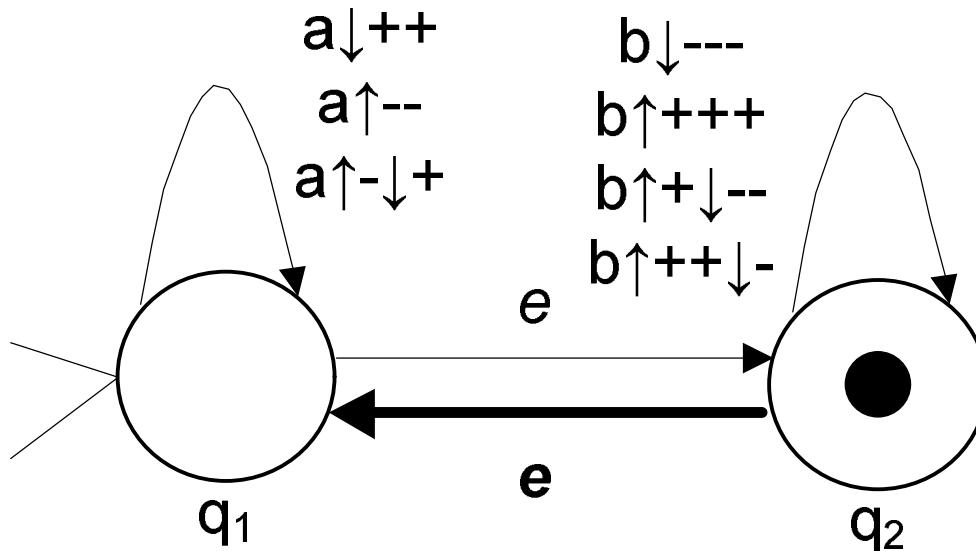
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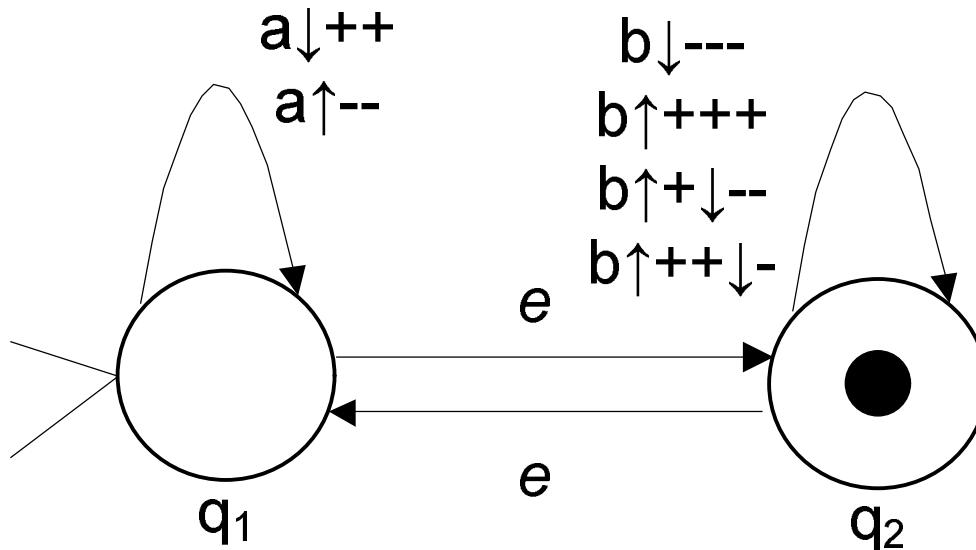
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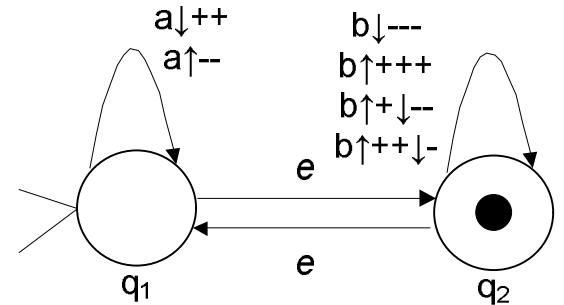
- Solution:

'+' — 1/2 'a' excess, 1/3 'b' lack

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PDA: Example 3



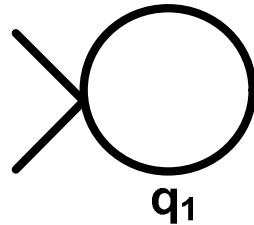
State	Unread input	Stack
q_1	aabab	e
q_1	abab	$++$
q_1	bab	$++++$
q_2	bab	$++++$
q_2	ab	$+$
q_1	ab	$+$
q_1	b	$+++$
q_2	b	$+++$
q_2	e	e

PDA: Example 4

- Give PDA M such that $L(M) = L!$
 - $L = \{w \in \{a, b\}^* \mid 3|a| - 2|b| = 4\}$

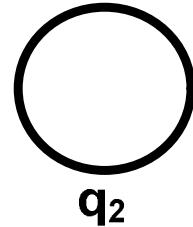
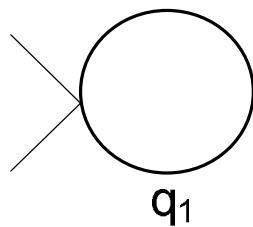
PDA: Example 4

- Give PDA M such that $L(M) = L!$
 - $L = \{w \in \{a, b\}^* \mid 3|a| - 2|b| = 4\}$
 - $L = \{w \in \{a, b\}^* \mid 3|a| = 2|b| + 4\}$
- Solution:
 - '+' — 1/3 'a' excess, 1/2 'b' lack
 - '-' — 1/2 'b' excess, 1/3 'a' lack
 - '+' and '-' is interpreted without the +4



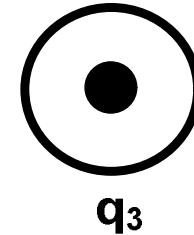
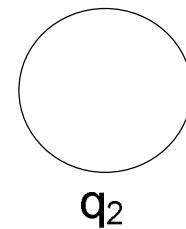
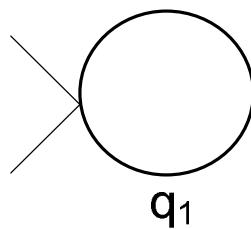
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- Solution:
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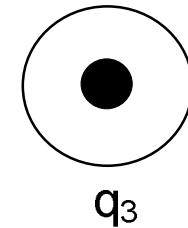
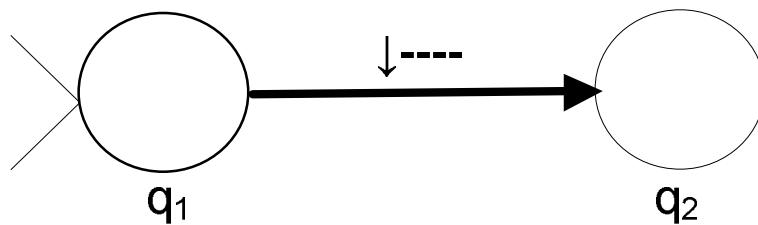


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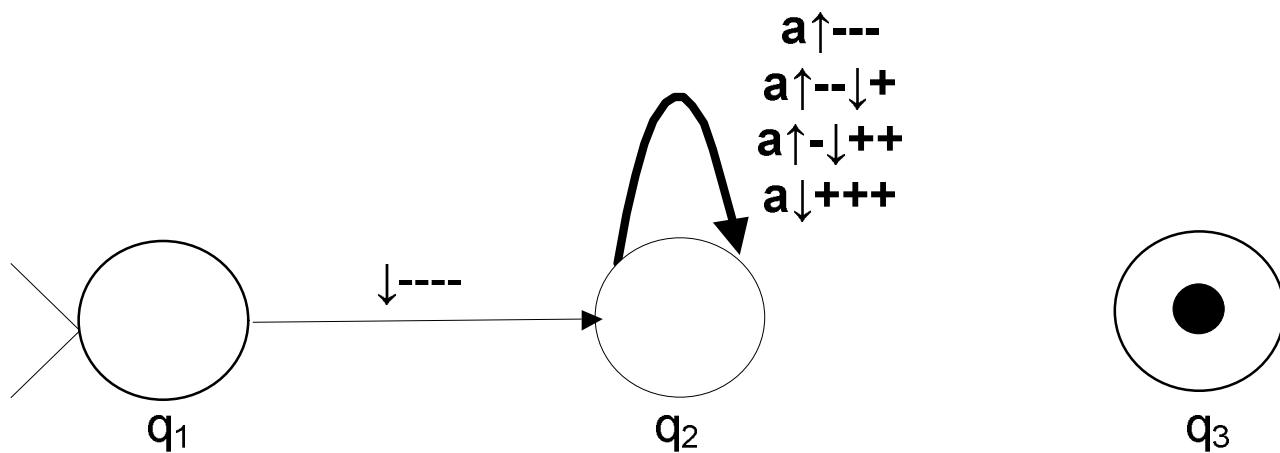


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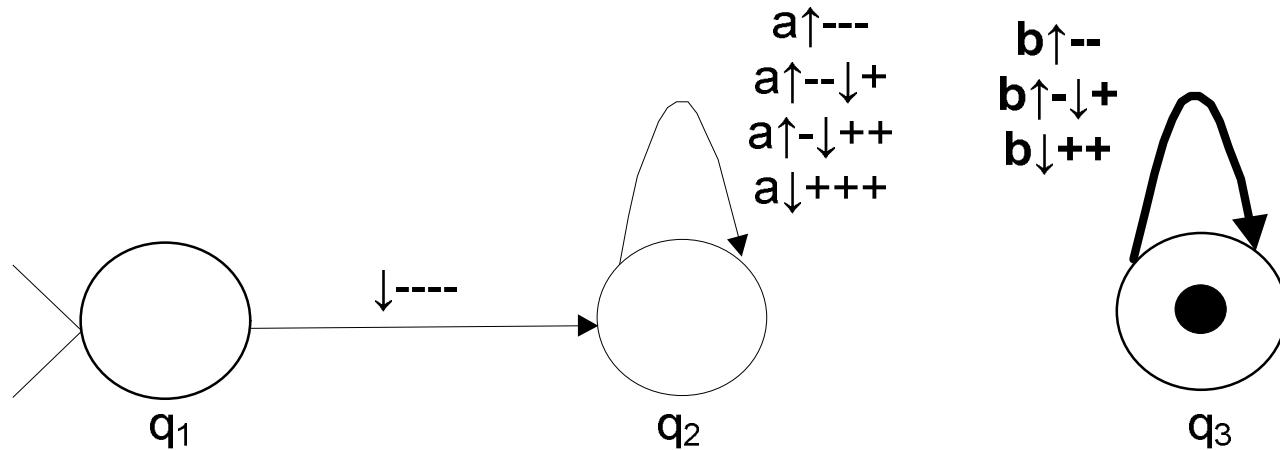
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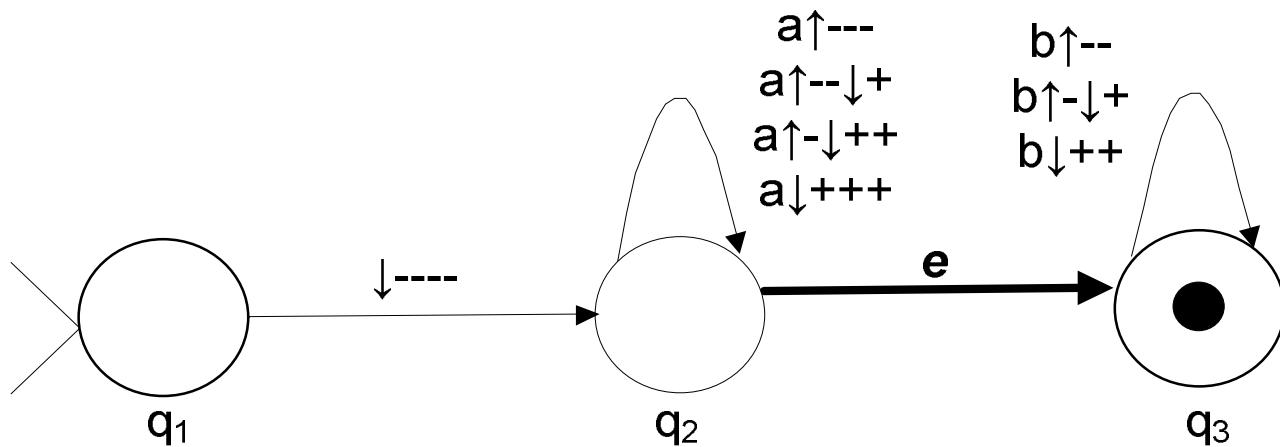


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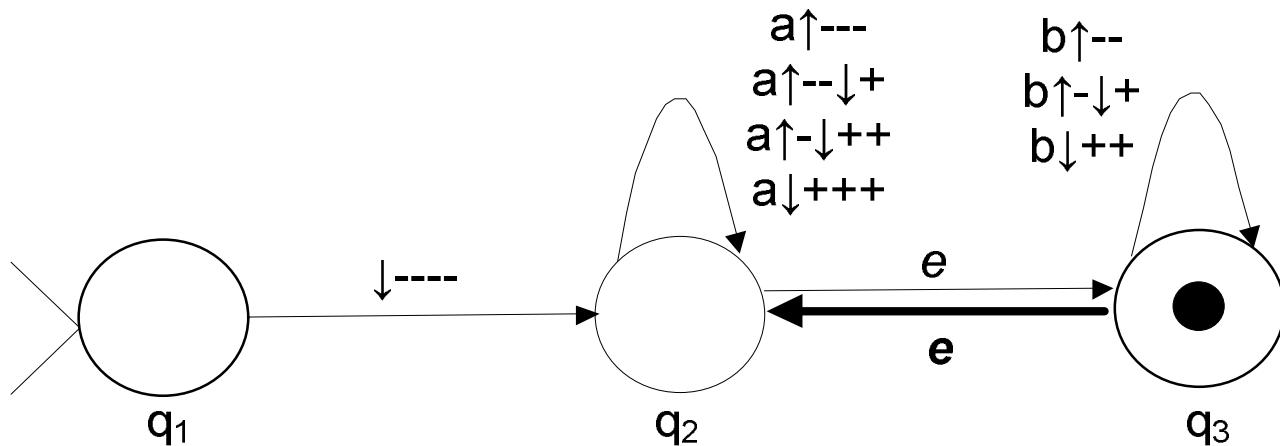


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- Solution:

'+' — 1/3 'a' excess, 1/2 'b' lack
'-' — 1/2 'b' excess, 1/3 'a' lack

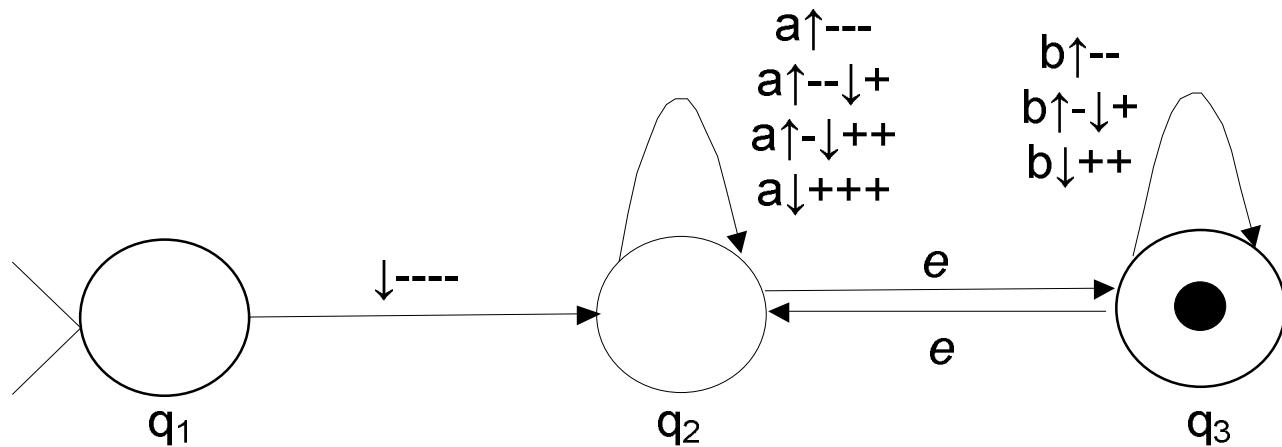


PDA: Example 4

- Give PDA M such that $L(M) = L!$
 - $L = \{w \in \{a, b\}^* \mid 3|a| - 2|b| = 4\}$

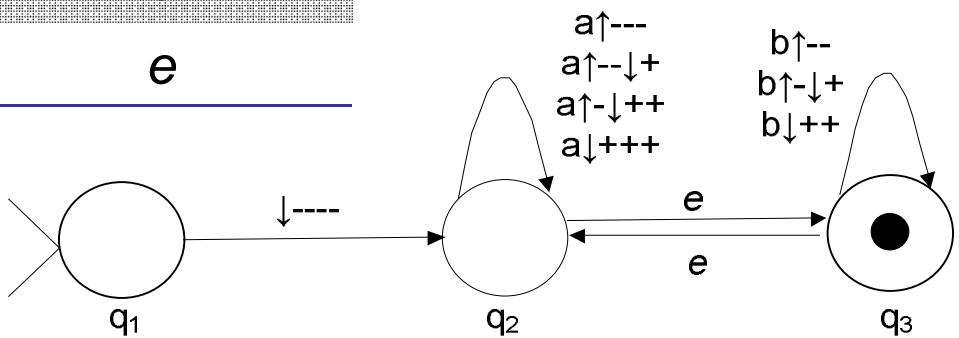
- Solution:

'+' — 1/3 'a' excess, 1/2 'b' lack
'-' — 1/2 'b' excess, 1/3 'a' lack



PDA: Example 4

State	Unread input	Stack
q_1	aba	e
q_2	aba	---
q_2	ba	-
q_3	ba	-
q_3	a	---
q_2	a	---
q_2	e	e
q_3	e	e



PDA: Example 5

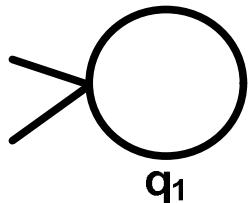
- Give PDA M such that $L(M) = L!$
 - $L = \{w \in a^n b^m c^p d^q \mid 2n + m = 3p + q\}$

PDA: Example 5

- Give PDA M such that $L(M) = L!$
 - $L = \{w \in a^n b^m c^p d^q \mid 2n + m = 3p + q\}$
- Solution:

q_1 — 'a' reader state	q_2 — 'b' reader state
q_3 — 'c' reader state	q_4 — 'd' reader state

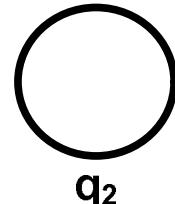
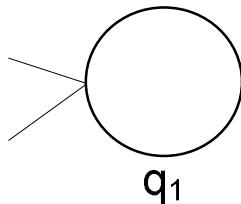
'+' — 1/2 "a" excess, 1 "b" excess, 1/3 "c" lack and 1 "d" lack
'-' is not used because the order of the characters is fixed here



PDA: Example 5

- Give PDA M such that $L(M) = L!$
 - $L = \{w \in a^n b^m c^p d^q \mid 2n + m = 3p + q\}$
- Solution:

q_1 — 'a' reader state	q_2 — 'b' reader state
q_3 — 'c' reader state	q_4 — 'd' reader state
'+' — 1/2 "a" excess, 1 "b" excess, 1/3 "c" lack and 1 "d" lack	



PDA: Example 5

- Give PDA M such that $L(M) = L!$
 - $L = \{w \in a^n b^m c^p d^q \mid 2n + m = 3p + q\}$

- Solution:

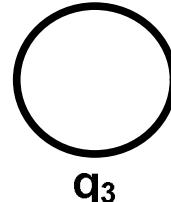
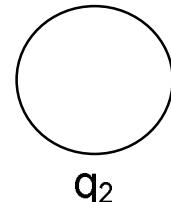
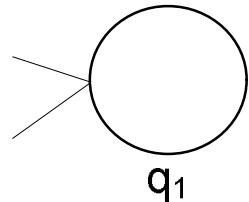
q_1 — 'a' reader state

q_2 — 'b' reader state

q_3 — 'c' reader state

q_4 — 'd' reader state

'+' — 1/2 "a" excess, 1 "b" excess, 1/3 "c" lack and 1 "d" lack



PDA: Example 5

- Give PDA M such that $L(M) = L!$
 - $L = \{w \in a^n b^m c^p d^q \mid 2n + m = 3p + q\}$

- Solution:

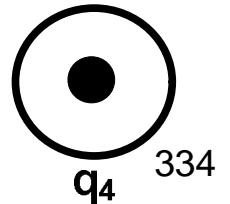
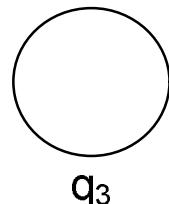
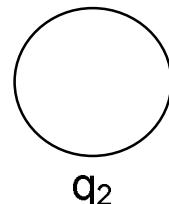
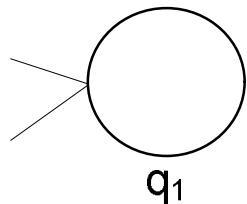
q_1 — 'a' reader state

q_2 — 'b' reader state

q_3 — 'c' reader state

q_4 — 'd' reader state

'+' — 1/2 "a" excess, 1 "b" excess, 1/3 "c" lack, 1 "d" lack



PDA: Example 5

- Give PDA M such that $L(M) = L!$
 - $L = \{w \in a^n b^m c^p d^q \mid 2n + m = 3p + q\}$

- Solution:

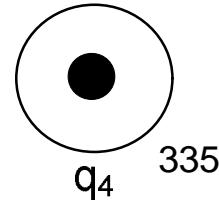
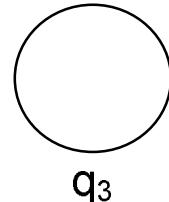
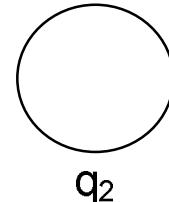
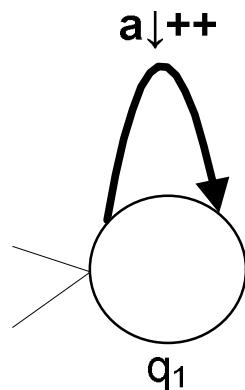
q_1 — 'a' reader state

q_2 — 'b' reader state

q_3 — 'c' reader state

q_4 — 'd' reader state

'+' — 1/2 "a" excess, 1 "b" excess, 1/3 "c" lack, 1 "d" lack



PDA: Example 5

- Give PDA M such that $L(M) = L!$
 - $L = \{w \in a^n b^m c^p d^q \mid 2n + m = 3p + q\}$

- Solution:

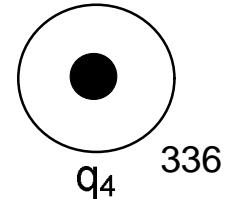
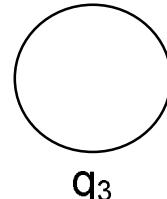
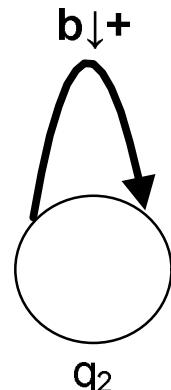
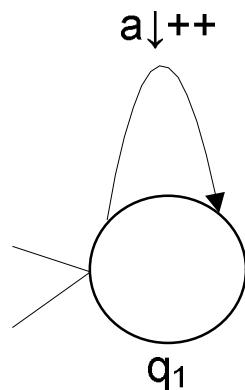
q_1 — 'a' reader state

q_2 — 'b' reader state

q_3 — 'c' reader state

q_4 — 'd' reader state

'+' — 1/2 "a" excess, 1 "b" excess, 1/3 "c" lack, 1 "d" lack



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PDA: Example 5

- Give PDA M such that $L(M) = L!$
 - $L = \{w \in a^n b^m c^p d^q \mid 2n + m = 3p + q\}$

- Solution:

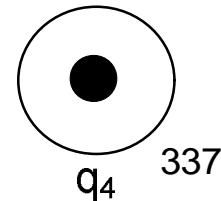
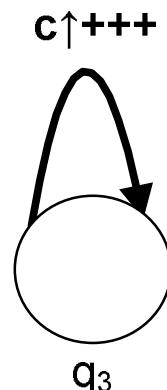
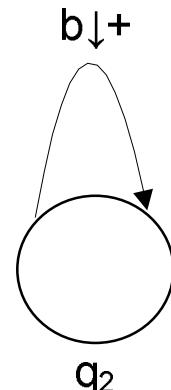
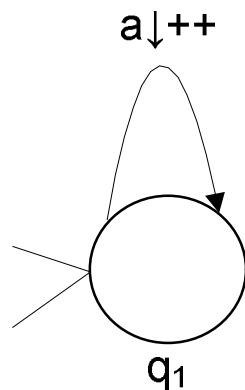
q_1 — 'a' reader state

q_2 — 'b' reader state

q_3 — 'c' reader state

q_4 — 'd' reader state

'+' — 1/2 "a" excess, 1 "b" excess, 1/3 "c" lack, 1 "d" lack



337

PDA: Example 5

- Give PDA M such that $L(M) = L!$
 - $L = \{w \in a^n b^m c^p d^q \mid 2n + m = 3p + q\}$

- Solution:

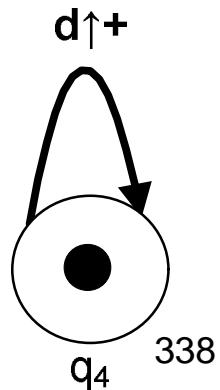
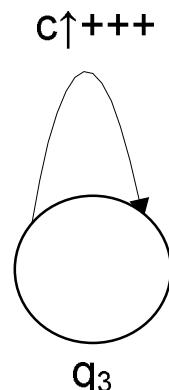
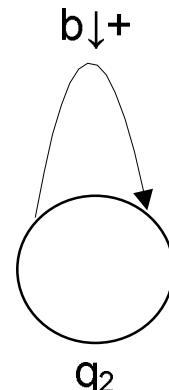
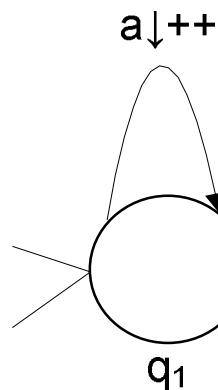
q_1 — 'a' reader state

q_2 — 'b' reader state

q_3 — 'c' reader state

q_4 — 'd' reader state

'+' — 1/2 "a" excess, 1 "b" excess, 1/3 "c" lack, 1 "d" lack



PDA: Example 5

- Give PDA M such that $L(M) = L!$
 - $L = \{w \in a^n b^m c^p d^q \mid 2n + m = 3p + q\}$

- Solution:

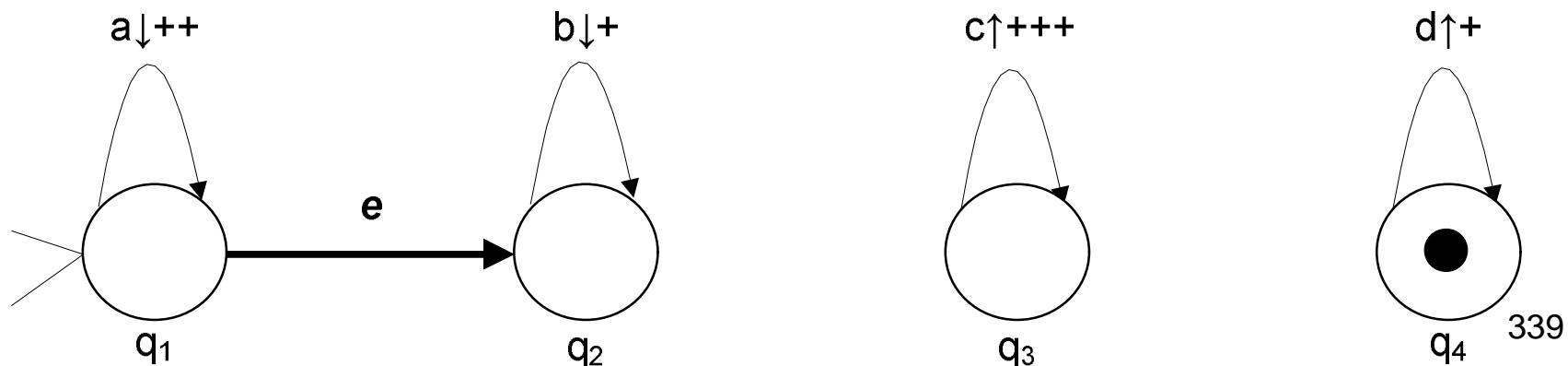
q_1 — 'a' reader state

q_2 — 'b' reader state

q_3 — 'c' reader state

q_4 — 'd' reader state

'+' — 1/2 "a" excess, 1 "b" excess, 1/3 "c" lack, 1 "d" lack



PDA: Example 5

- Give PDA M such that $L(M) = L!$
 - $L = \{w \in a^n b^m c^p d^q \mid 2n + m = 3p + q\}$

- Solution:

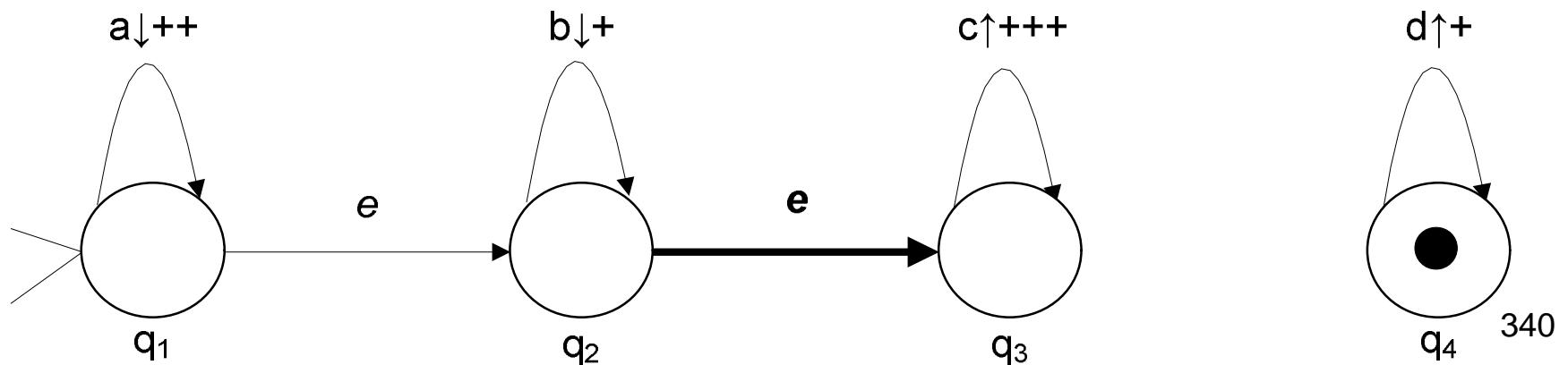
q_1 — 'a' reader state

q_2 — 'b' reader state

q_3 — 'c' reader state

q_4 — 'd' reader state

'+' — 1/2 "a" excess, 1 "b" excess, 1/3 "c" lack, 1 "d" lack



PDA: Example 5

- Give PDA M such that $L(M) = L!$
 - $L = \{w \in a^n b^m c^p d^q \mid 2n + m = 3p + q\}$

- Solution:

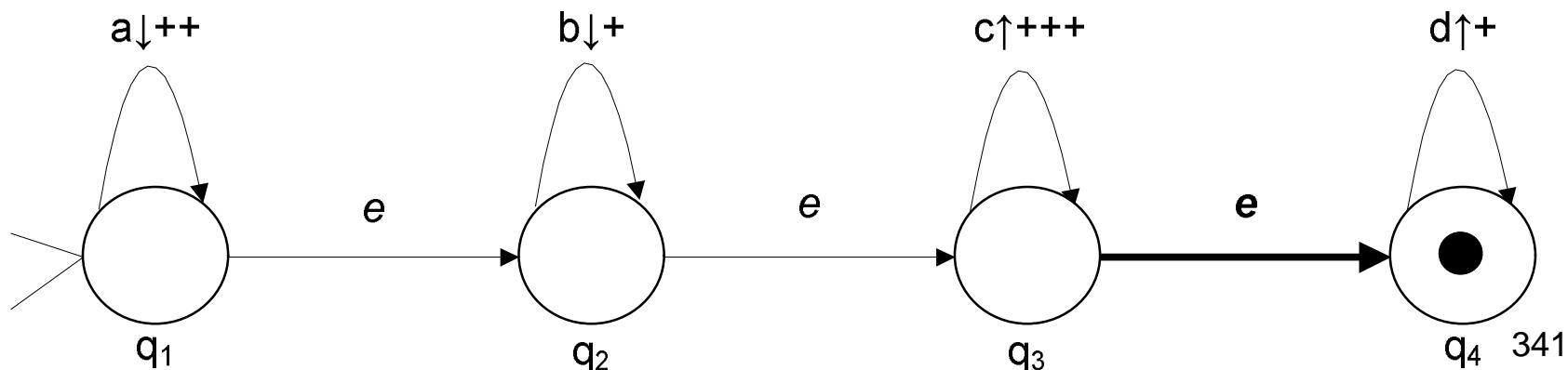
q_1 — 'a' reader state

q_2 — 'b' reader state

q_3 — 'c' reader state

q_4 — 'd' reader state

'+' — 1/2 "a" excess, 1 "b" excess, 1/3 "c" lack, 1 "d" lack



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PDA: Example 5

- Give PDA M such that $L(M) = L!$
 - $L = \{w \in a^n b^m c^p d^q \mid 2n + m = 3p + q\}$

- Solution:

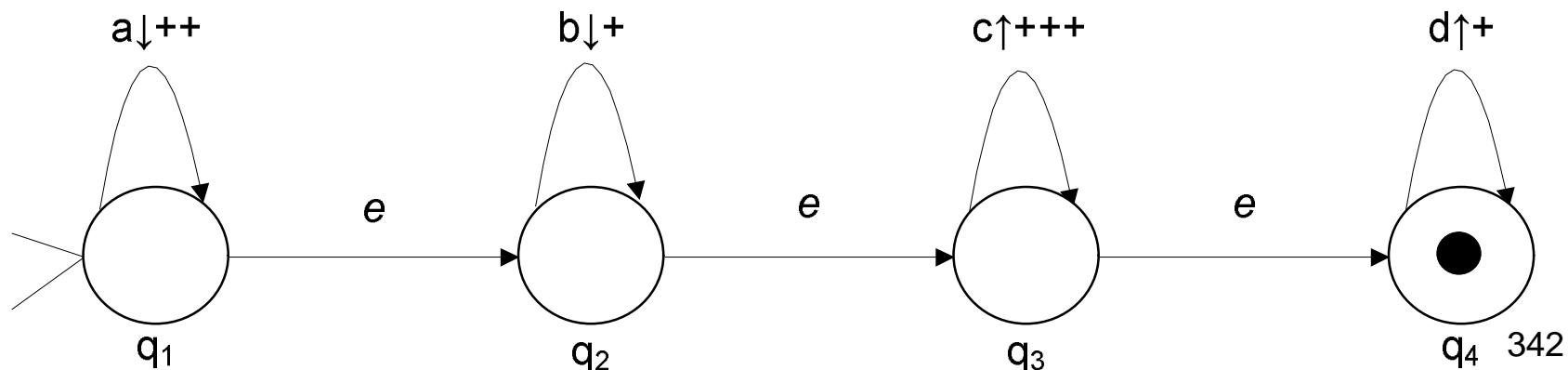
q_1 — 'a' reader state

q_2 — 'b' reader state

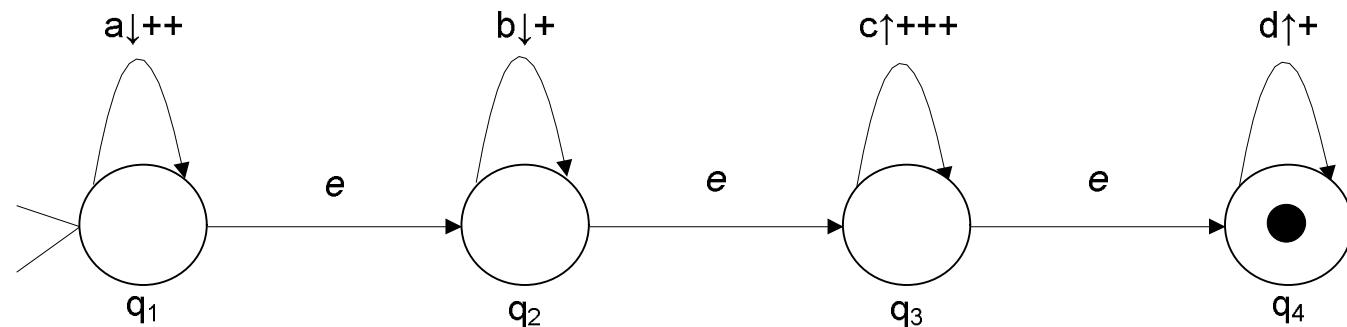
q_3 — 'c' reader state

q_4 — 'd' reader state

'+' — 1/2 "a" excess, 1 "b" excess, 1/3 "c" lack, 1 "d" lack



State	Unread input	Stack
q_1	aaabcccd	e
q_1	aabcccd	$+^2$
q_1	abcccd	$+^4$
q_1	bcccd	$+^6$
q_2	bcccd	$+^6$
q_2	ccd	$+^7$
q_3	ccd	$+^7$
q_3	cd	$+^4$
q_3	d	$+^1$
q_4	d	$+^1$
q_4	e	e



PDA: Example 6

- Give PDA M such that $L(M) = L!$
 - $L = \{w \in a^n b^m c^p d^q \mid 2n + 5m = 4p + 3q + 2\}$
- Solution:

q_1 — 'a' reader state

q_2 — 'b' reader state

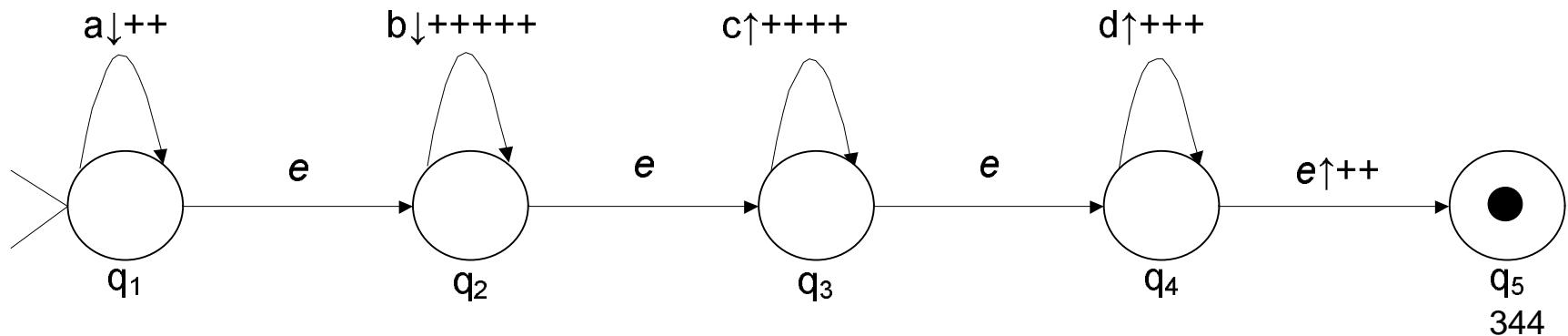
q_3 — 'c' reader state

q_4 — 'd' reader state

q_5 — state for +2

'+' — 1/2 "a" excess, 1/5 "b" excess, 1/4 "c" lack, 1/3 "d" lack

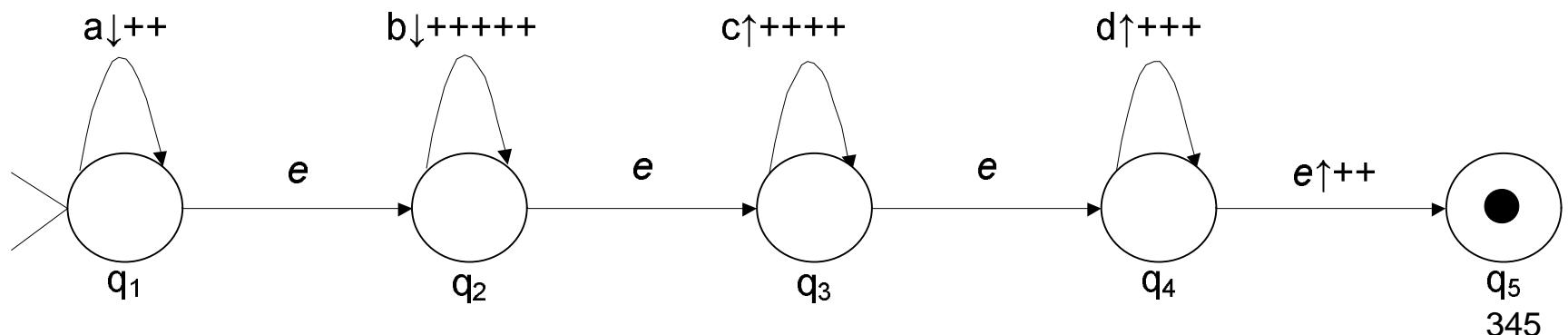
'+' is interpreted without the +2 constant



PDA: Example 6

- Computation for "aaaabcccd":

$(q_1, \text{aaaabcccd}, e) \vdash (q_1, \text{aaabcccd}, +^2) \vdash$
 $(q_1, \text{aabcccd}, +^4) \vdash (q_1, \text{abcccd}, +^6) \vdash (q_1, \text{bcccd}, +^8) \vdash$
 $(q_2, \text{bcccd}, +^8) \vdash (q_2, \text{ccd}, +^{13}) \vdash (q_3, \text{ccd}, +^{13}) \vdash$
 $(q_3, \text{cd}, +^9) \vdash (q_3, \text{d}, +^5) \vdash (q_4, \text{d}, +^5) \vdash (q_4, \text{e}, +^2) \vdash$
 $(q_5, \text{e}, \text{e})$



PDA: Example 7

- Give PDA M such that $L(M) = L!$
 - $L = \{w \in \{a, b\}^* \mid w = a^n b^n\}$

PDA: Example 7

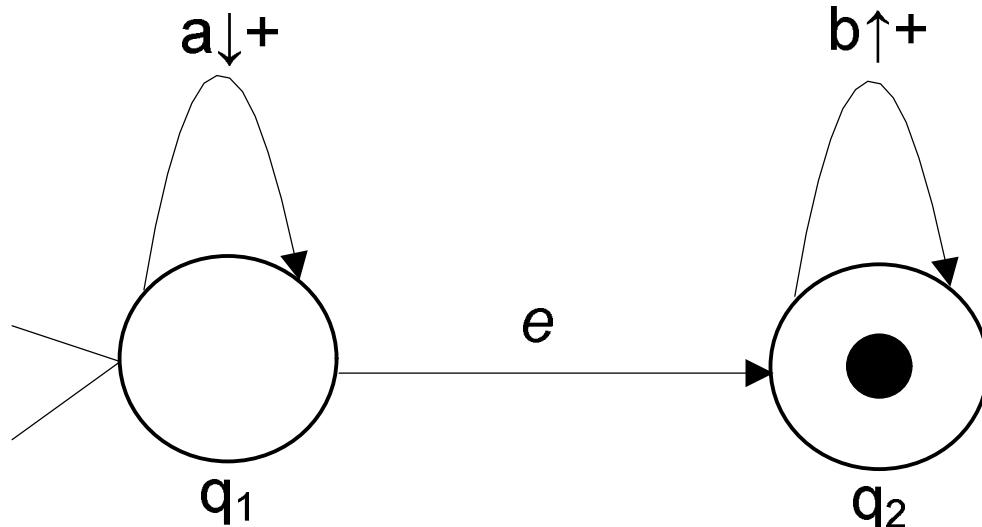
- Give PDA M such that $L(M) = L!$
 - $L = \{w \in \{a, b\}^* \mid w = a^n b^n\}$

- Solution:

q_1 — 'a' reader state

q_2 — 'b' reader state

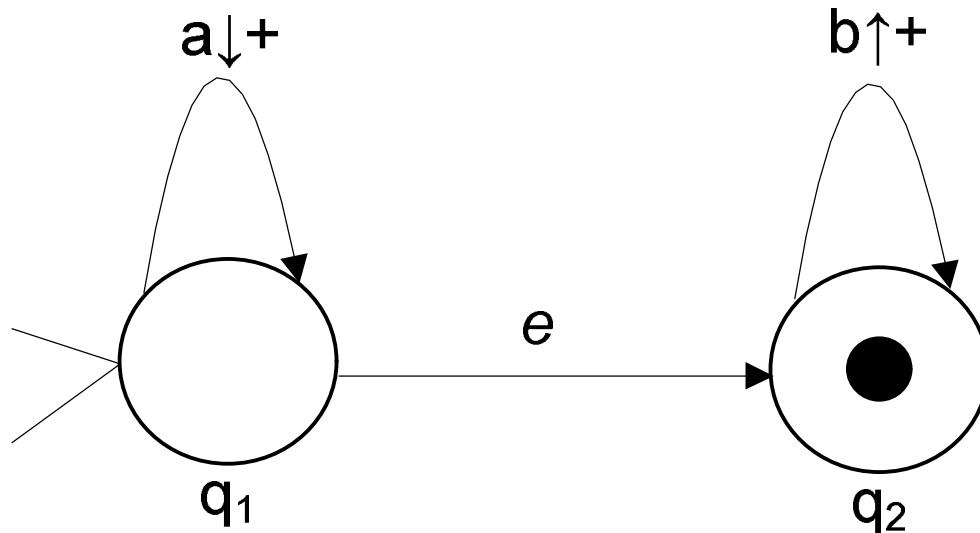
'+' — 1 "a" excess and 1 "b" lack



PDA: Example 7

- Computation for "aaaabbbb":

$(q_1, \text{aaaabbbb}, e) \vdash (q_1, \text{aaabbbb}, +^1) \vdash$
 $(q_1, \text{aabbbb}, +^2) \vdash (q_1, \text{abbbb}, +^3) \vdash (q_1, \text{bbbb}, +^4) \vdash$
 $(q_2, \text{bbbb}, +^4) \vdash (q_2, \text{bbb}, +^3) \vdash (q_2, \text{bb}, +^2) \vdash$
 $(q_2, \text{b}, +) \vdash (q_2, \text{e}, \text{e})$



PDA: Example 8

- Give PDA M such that $L(M) = L!$
 - $L = \{w \in \{a, b\}^* \mid w = a^n b^{2n}\}$

PDA: Example 8

- Give PDA M such that $L(M) = L!$

– $L = \{w \in \{a, b\}^* \mid w = a^n b^{2n}\}$

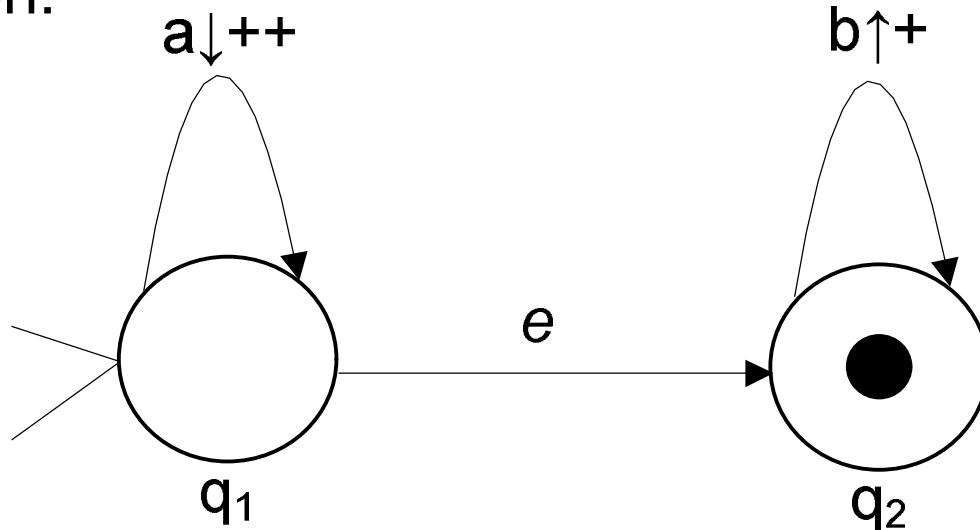
– $L = \{w \in a^x b^y \mid 2x = y\}$

q_1 — 'a' reader state

q_2 — 'b' reader state

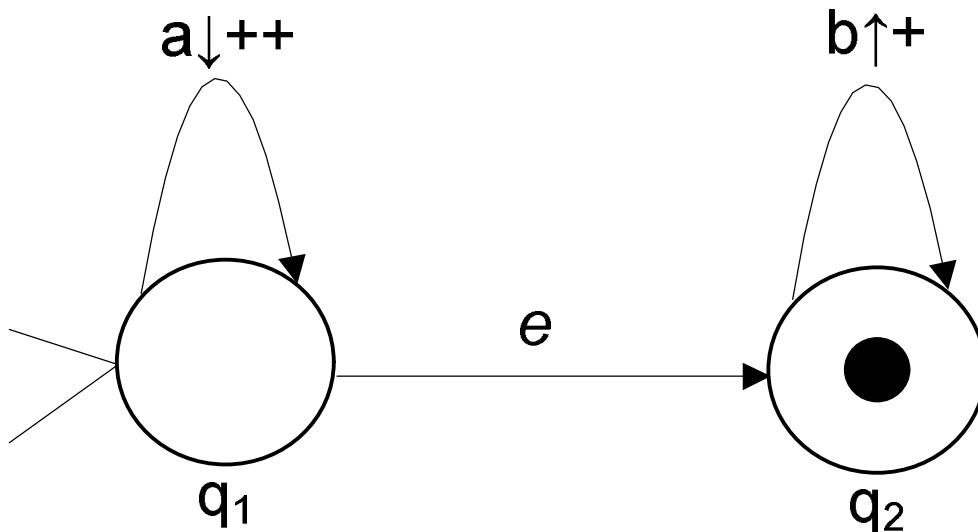
'+' — 1/2 "a" excess and 1 "b" lack

- Solution:



PDA: Example 8

- Computation for "aabbbb":

$$\begin{aligned} & (q_1, aabbbb, e) \xrightarrow{-} (q_1, abbbb, +^2) \xrightarrow{-} (q_1, bbbb, +^4) \xrightarrow{-} \\ & (q_2, bbbb, +^4) \xrightarrow{-} (q_2, bbb, +^3) \xrightarrow{-} (q_2, bb, +^2) \xrightarrow{-} \\ & (q_2, b, +) \xrightarrow{-} (q_2, e, e) \end{aligned}$$


PDA: Example 9

- Give PDA M such that $L(M) = L!$
 - $L = \{w \in \{a, b\}^* \mid w = a^{3n}b^{2n}\}$

PDA: Example 9

- Give PDA M such that $L(M) = L!$

- $L = \{w \in \{a, b\}^* \mid w = a^{3n}b^{2n}\}$

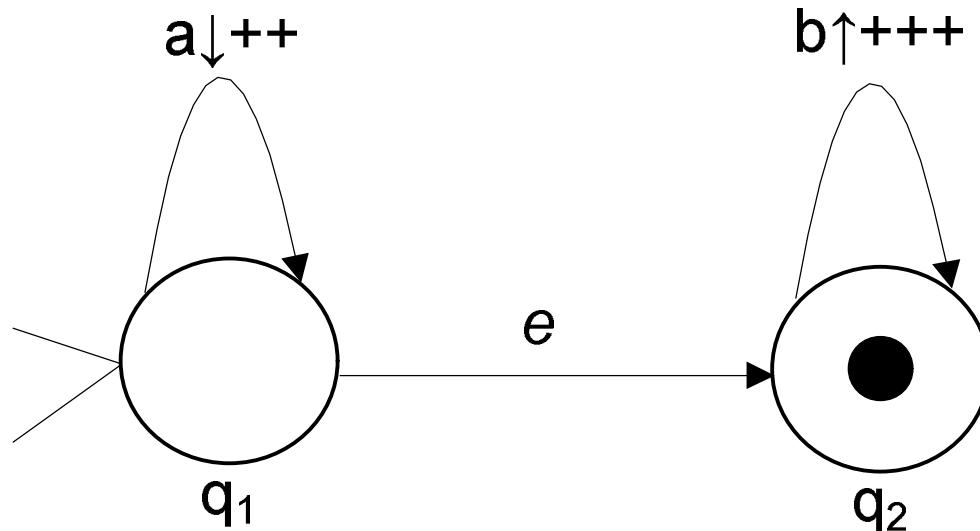
- $L = \{w \in a^x b^y \mid 2x = 3y\}$

q_1 — 'a' reader state

q_2 — 'b' reader state

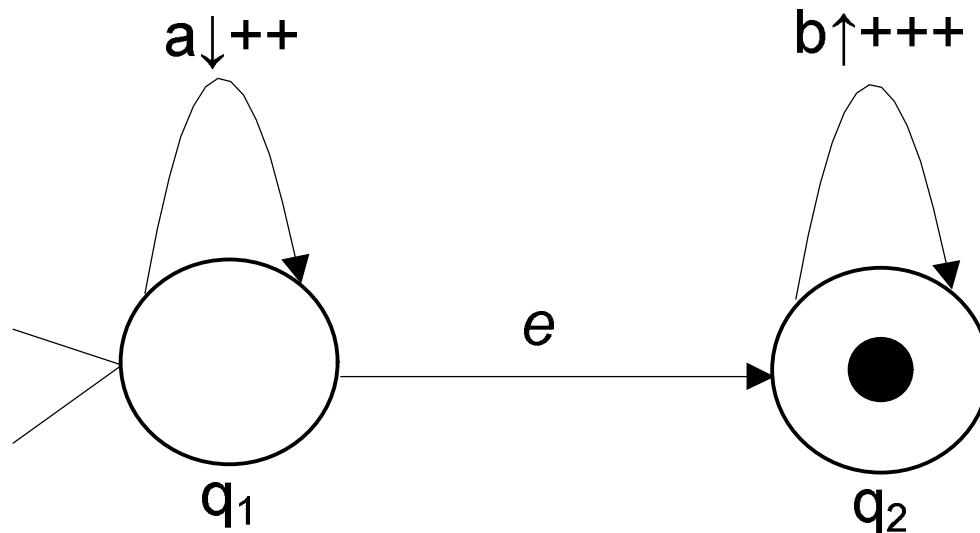
'+' — 1/2 "a" excess, 1/3 "b" lack

- Solution:



PDA: Example 9

- Computation for "aaabb":

$$(q_1, \text{aaabb}, e) \xrightarrow{-} (q_1, \text{aabb}, +^2) \xrightarrow{-} (q_1, \text{abb}, +^4) \xrightarrow{-} (q_1, \text{bb}, +^6) \xrightarrow{-} (q_2, \text{bb}, +^6) \xrightarrow{-} (q_2, \text{b}, +^3) \xrightarrow{-} (q_2, e, e)$$


PDA: Example 10

- Give PDA M such that $L(M) = L!$
 - $L = \{w \in \{a, b\}^* \mid w = a^{3n+2}b^{2n}\}$
 - $L = \{w \in a^x b^y \mid 2(x - 2) = 3y\}$
 - 2 'a' will not change the stack
 - $L = \{w \in \{a, b\}^* \mid w = a^{3n}aab^{2n}\}$
 - nested problem:
 - inner problem: generate aa
 - outer problem: generate $a^{3n}b^{2n}$
 - $L = \{w \in a^x b^y \mid 2x = 3y + 4\}$
 - it would require the removal of $+^4$ at the end

PDA: Example 10

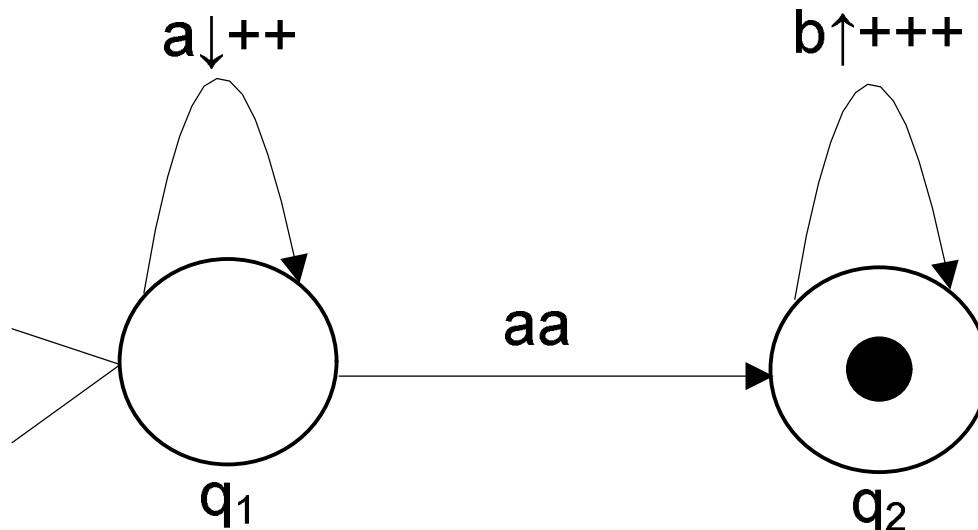
- Give PDA M such that $L(M) = L!$
 - $L = \{w \in \{a, b\}^* \mid w = a^{3n+2}b^{2n}\}$

- Solution:

q_1 — 'a' reader state

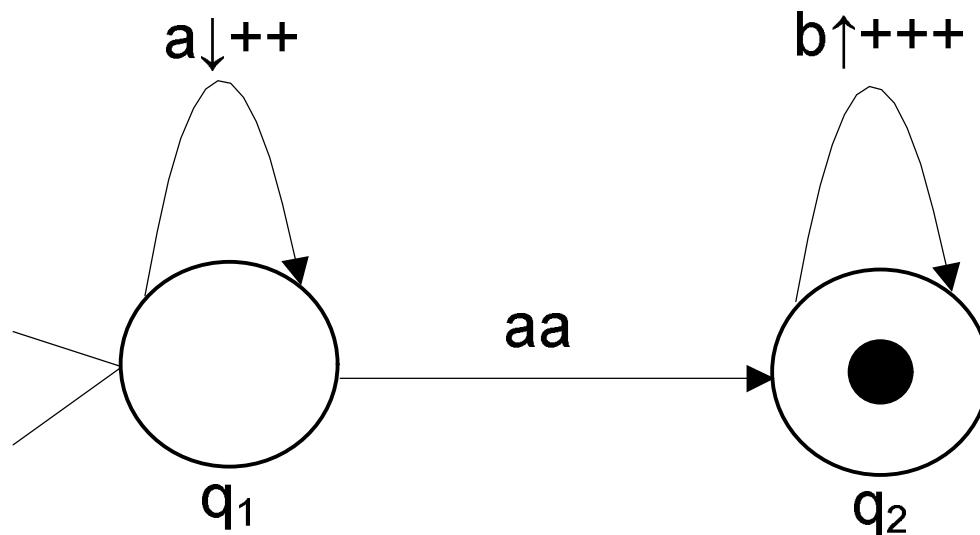
q_2 — 'b' reader state

'+' — 1/2 "a" excess, 1/3 "b" lack



PDA: Example 10

- Computation for "aaaaabb":

$$(q_1, \text{aaaaabb}, e) \xrightarrow{-} (q_1, \text{aaaabb}, +^2) \xrightarrow{-} (q_1, \text{aaabb}, +^4) \xrightarrow{-} (q_1, \text{aabb}, +^6) \xrightarrow{-} (q_2, \text{bb}, +^6) \xrightarrow{-} (q_2, \text{b}, +^3) \xrightarrow{-} (q_2, e, e)$$


PDA: Example 11

- Give PDA M such that $L(M) = L!$
 - $L = \{w \in \{a, b\}^* \mid w = a^n b^{n+2}\}$

PDA: Example 11

- Give PDA M such that $L(M) = L!$

- $L = \{w \in \{a, b\}^* \mid w = a^n b^{n+2}\}$

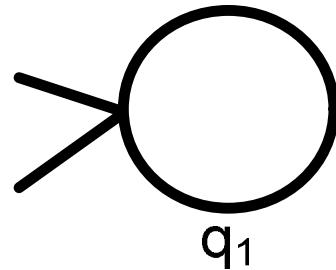
- $L = \{w \in \{a, b\}^* \mid w = a^n bbb^n\}$

q_1 — 'a' reader state

q_2 — 'b' reader state

'+' — 1 "a" excess, 1 "b" lack

- Solution:



PDA: Example 11

- Give PDA M such that $L(M) = L!$

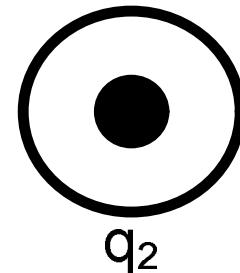
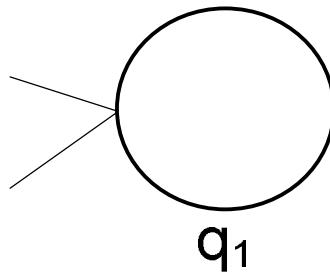
- $L = \{w \in \{a, b\}^* \mid w = a^n b^{n+2}\}$

- Solution:

q_1 — 'a' reader state

q_2 — 'b' reader state

'+' — 1 "a" excess, 1 "b" lack



PDA: Example 11

- Give PDA M such that $L(M) = L!$

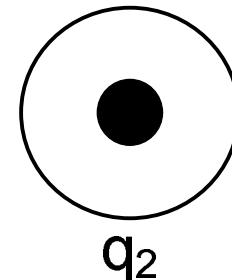
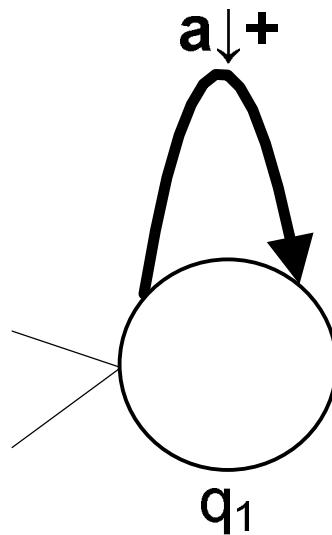
– $L = \{w \in \{a, b\}^* \mid w = a^n b^{n+2}\}$

- Solution:

q_1 — 'a' reader state

q_2 — 'b' reader state

'+' — 1 "a" excess, 1 "b" lack



PDA: Example 11

- Give PDA M such that $L(M) = L!$

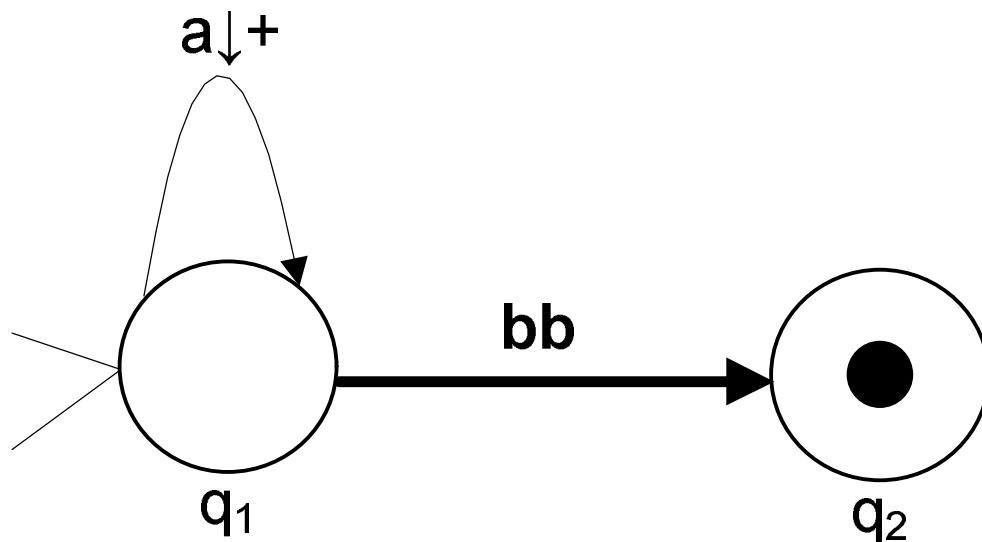
– $L = \{w \in \{a, b\}^* \mid w = a^n b^{n+2}\}$

- Solution:

q_1 — 'a' reader state

q_2 — 'b' reader state

'+' — 1 "a" excess, 1 "b" lack



PDA: Example 11

- Give PDA M such that $L(M) = L!$

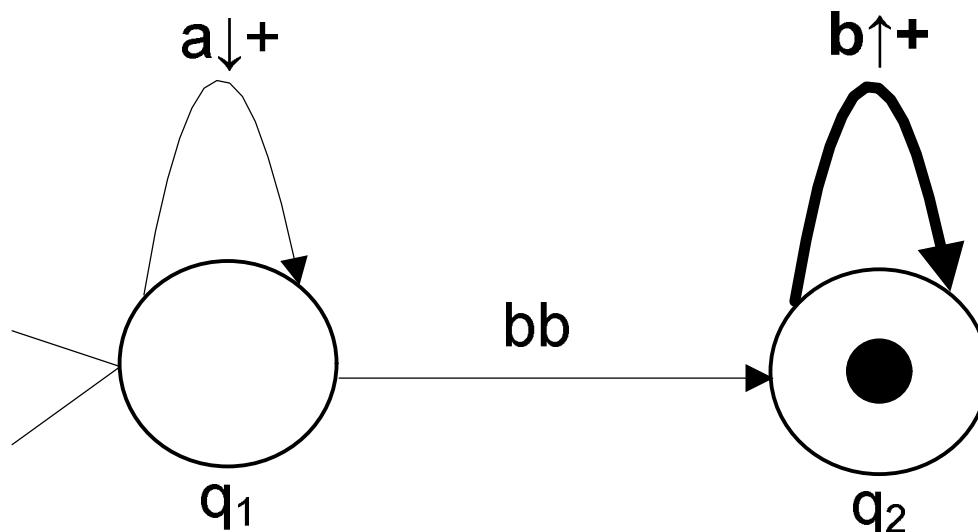
- $L = \{w \in \{a, b\}^* \mid w = a^n b^{n+2}\}$

- Solution:

q_1 — 'a' reader state

q_2 — 'b' reader state

'+' — 1 "a" excess, 1 "b" lack



PDA: Example 11

- Give PDA M such that $L(M) = L!$

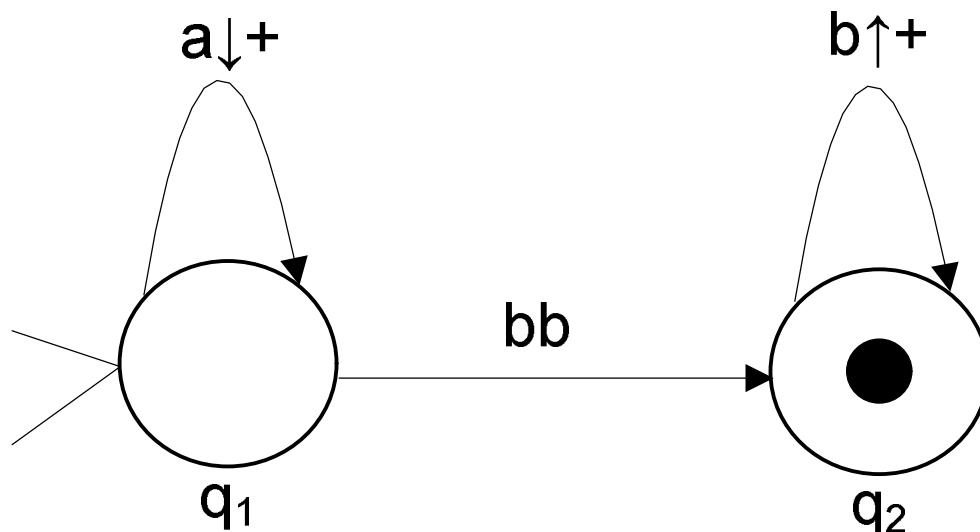
- $L = \{w \in \{a, b\}^* \mid w = a^n b^{n+2}\}$

- Solution:

q_1 — 'a' reader state

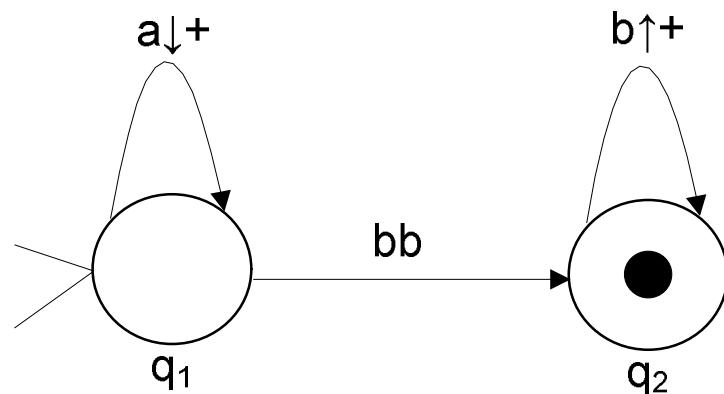
q_2 — 'b' reader state

'+' — 1 "a" excess, 1 "b" lack



PDA: Example 11

State	Unread input	Stack
q_1	abbb	e
q_1	bbb	+
q_2	b	+
q_2	e	e



PDA: Example 12

- Give PDA M such that $L(M) = L!$

- $L = \{w \in \{a, b\}^* \mid w = a^{n+2}b^{n+3}\}$

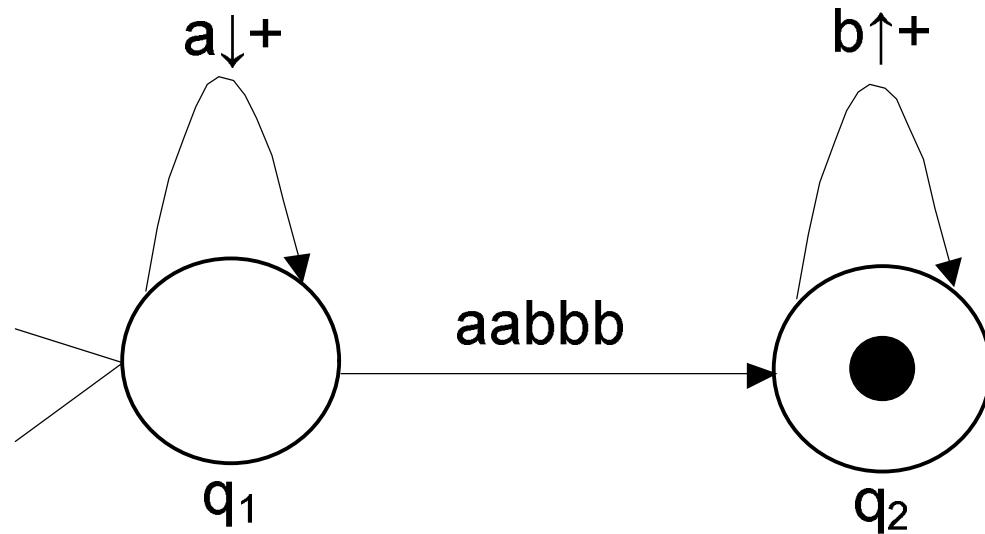
- $L = \{w \in \{a, b\}^* \mid w = a^n a b b b b^n\}$

q_1 — 'a' reader state

q_2 — 'b' reader state

'+' — 1 "a" excess, 1 "b" lack

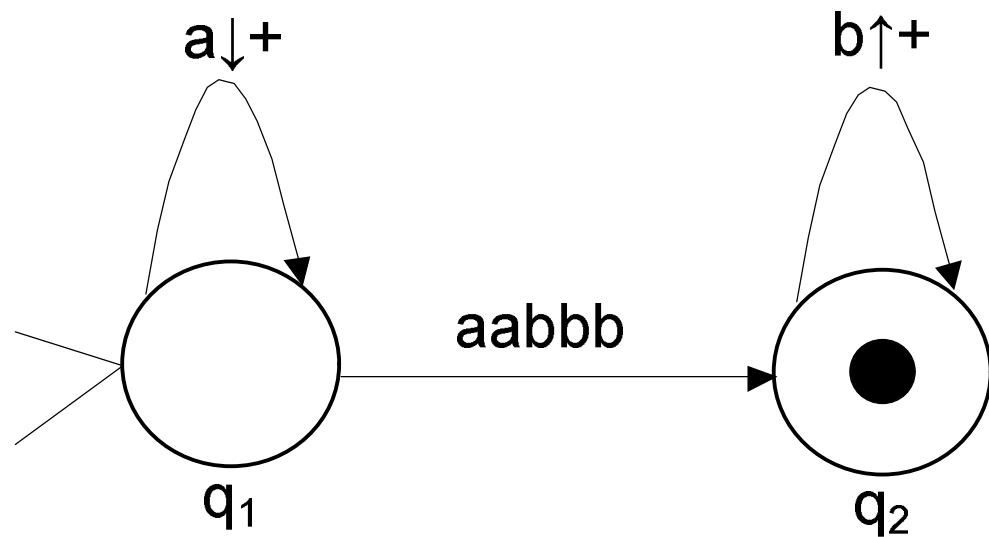
- Solution:



PDA: Example 12

- Computation for "aaaabbbbb":

$(q_1, \text{aaaabbbbb}, e) \xrightarrow{-} (q_1, \text{aaabbbbb}, +) \xrightarrow{-}$
 $(q_1, \text{aabbbbb}, +^2) \xrightarrow{-} (q_2, \text{bb}, +^2) \xrightarrow{-} (q_2, \text{b}, +) \xrightarrow{-} (q_2, e, e)$

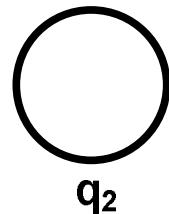


PDA: Example 13

- Give PDA M such that $L(M) = L!$
 - $L = \{w \in b^m a^m \cup a^{2m} b^m, m \geq 0\}$

PDA: Example 13

- Give PDA M such that $L(M) = L!$
 - $L = \{w \in b^m a^m \cup a^{2m} b^m, m \geq 0\}$
 - $L = L_1 \cup L_2 = \{w \in b^m a^m, m \geq 0\} \cup \{a^{2m} b^m, m \geq 0\}$
 - using the construction theorem



- Solution:

q_1 — initial state

q_2 — 'b' reader state for L_1

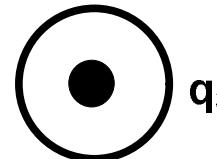
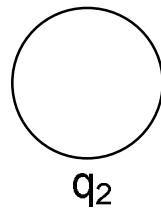
q_3 — 'a' reader state for L_1

q_4 — 'a' reader state for L_2

q_5 — 'b' reader state for L_2

PDA: Example 13

- Give PDA M such that $L(M) = L!$
 - $L = \{w \in b^m a^m \cup a^{2m} b^m, m \geq 0\}$
- Solution:



q_3

q_1 — initial state

q_2 — 'b' reader state for L_1

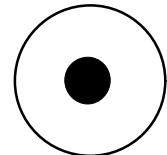
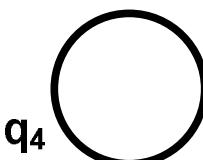
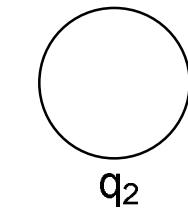
q_3 — 'a' reader state for L_1

q_4 — 'a' reader state for L_2

q_5 — 'b' reader state for L_2

PDA: Example 13

- Give PDA M such that $L(M) = L!$
 - $L = \{w \in b^m a^m \cup a^{2m} b^m, m \geq 0\}$
- Solution:



q_3

q_1 — initial state

q_2 — 'b' reader state for L_1

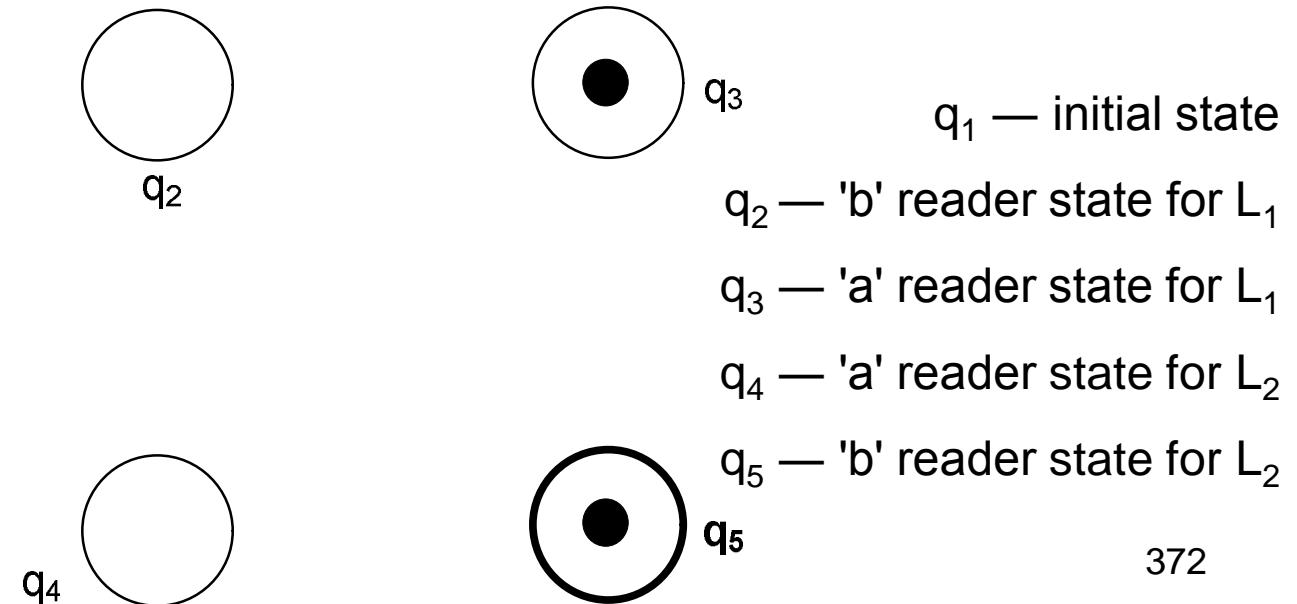
q_3 — 'a' reader state for L_1

q_4 — 'a' reader state for L_2

q_5 — 'b' reader state for L_2

PDA: Example 13

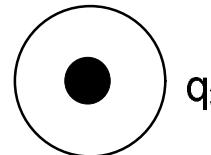
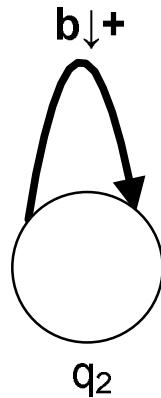
- Give PDA M such that $L(M) = L!$
 - $L = \{w \in b^m a^m \cup a^{2m} b^m, m \geq 0\}$
- Solution:



PDA: Example 13

- Give PDA M such that $L(M) = L!$
 - $L = \{w \in b^m a^m \cup a^{2m} b^m, m \geq 0\}$

- Solution:



q_3

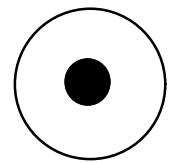
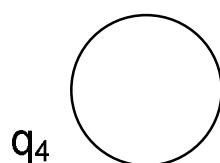
q_1 — initial state

q_2 — 'b' reader state for L_1

q_3 — 'a' reader state for L_1

q_4 — 'a' reader state for L_2

q_5 — 'b' reader state for L_2

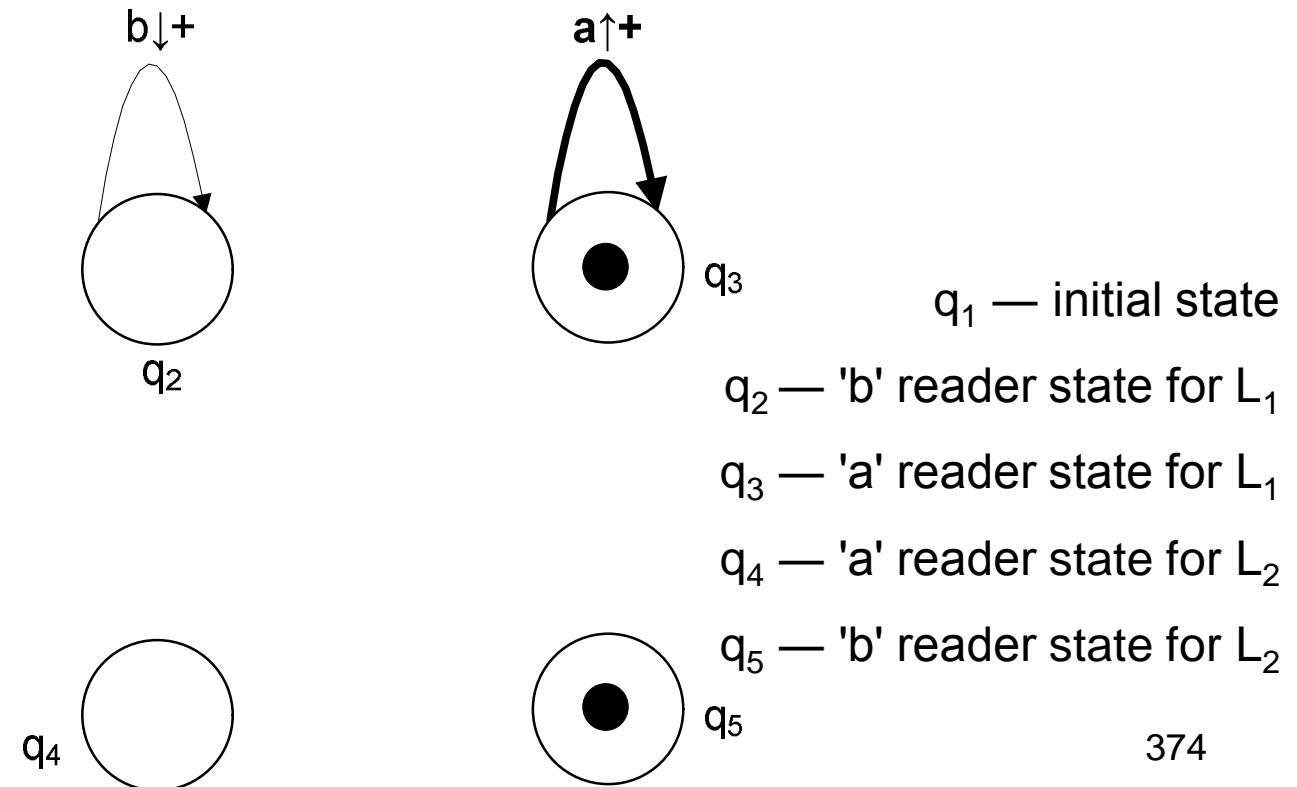


q_5

PDA: Example 13

- Give PDA M such that $L(M) = L!$
 - $L = \{w \in b^m a^m \cup a^{2m} b^m, m \geq 0\}$

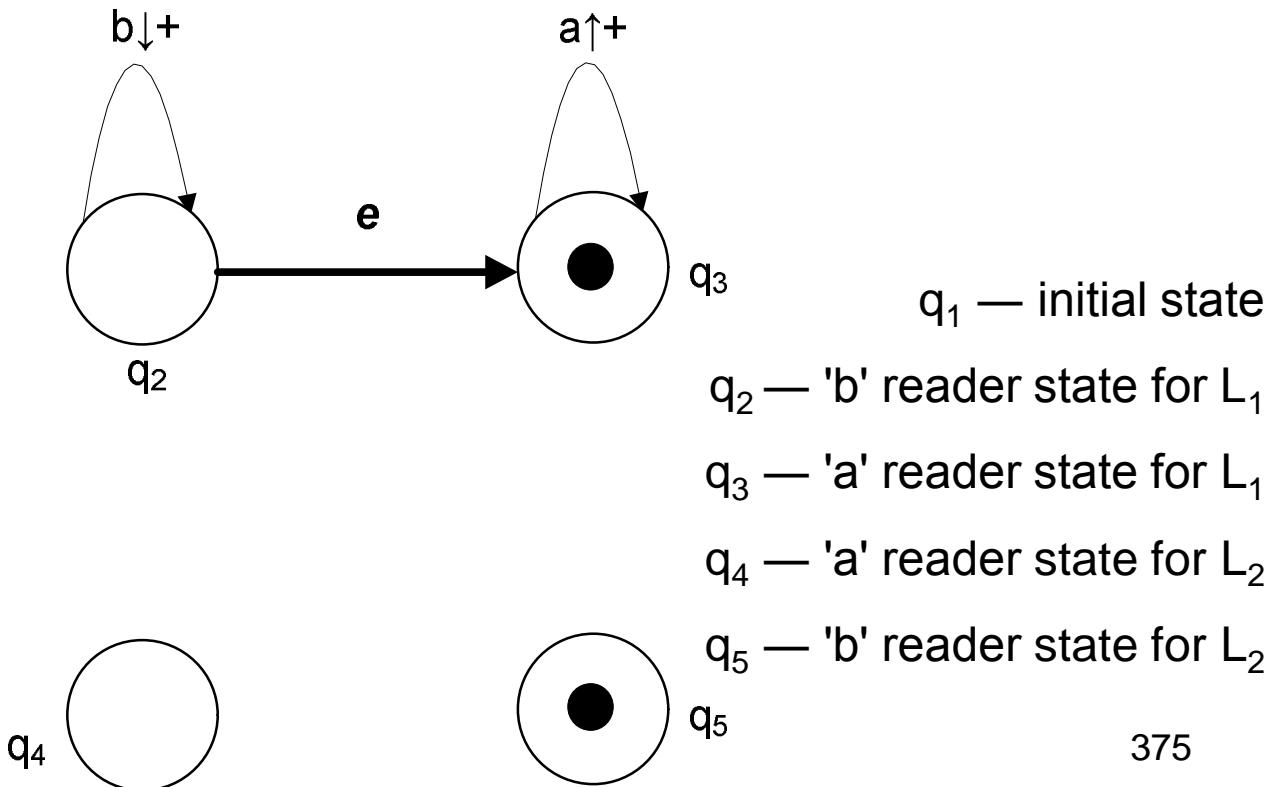
- Solution:



PDA: Example 13

- Give PDA M such that $L(M) = L!$
 - $L = \{w \in b^m a^m \cup a^{2m} b^m, m \geq 0\}$

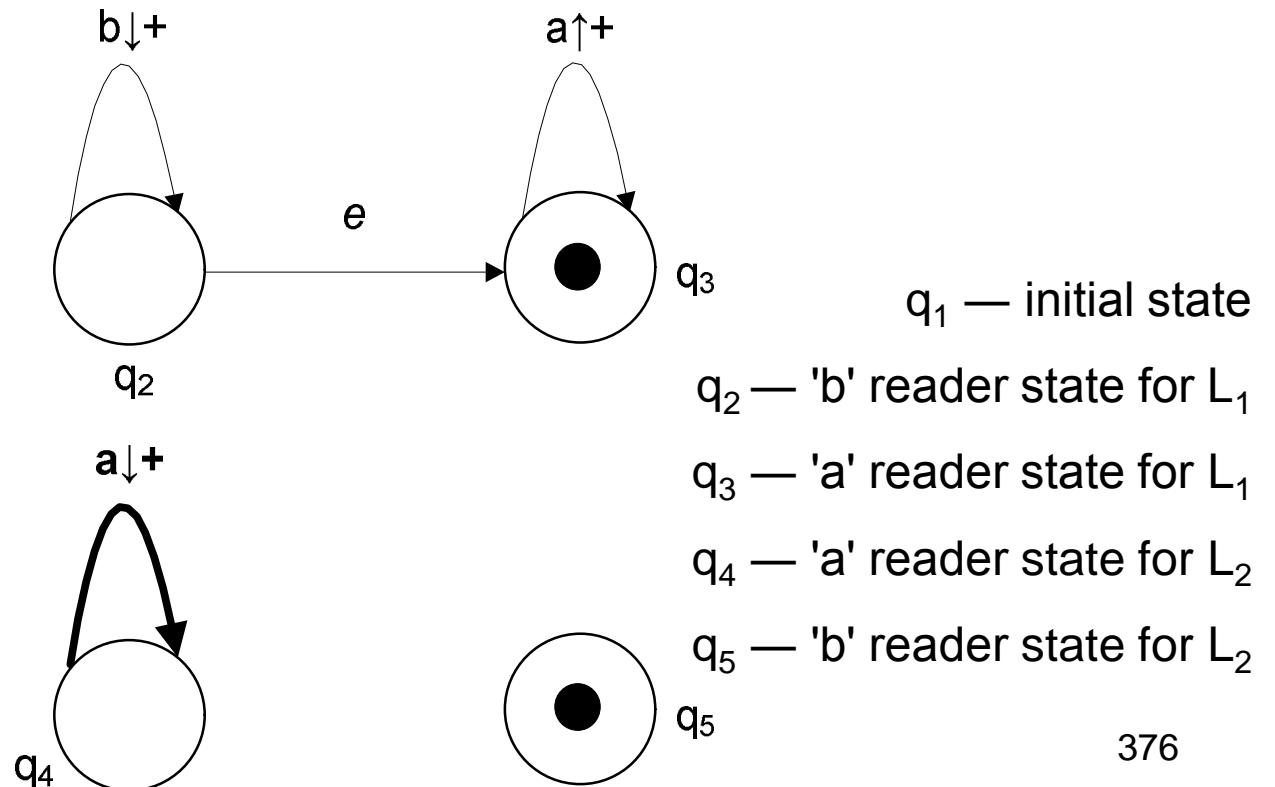
- Solution:



PDA: Example 13

- Give PDA M such that $L(M) = L!$
 - $L = \{w \in b^m a^m \cup a^{2m} b^m, m \geq 0\}$

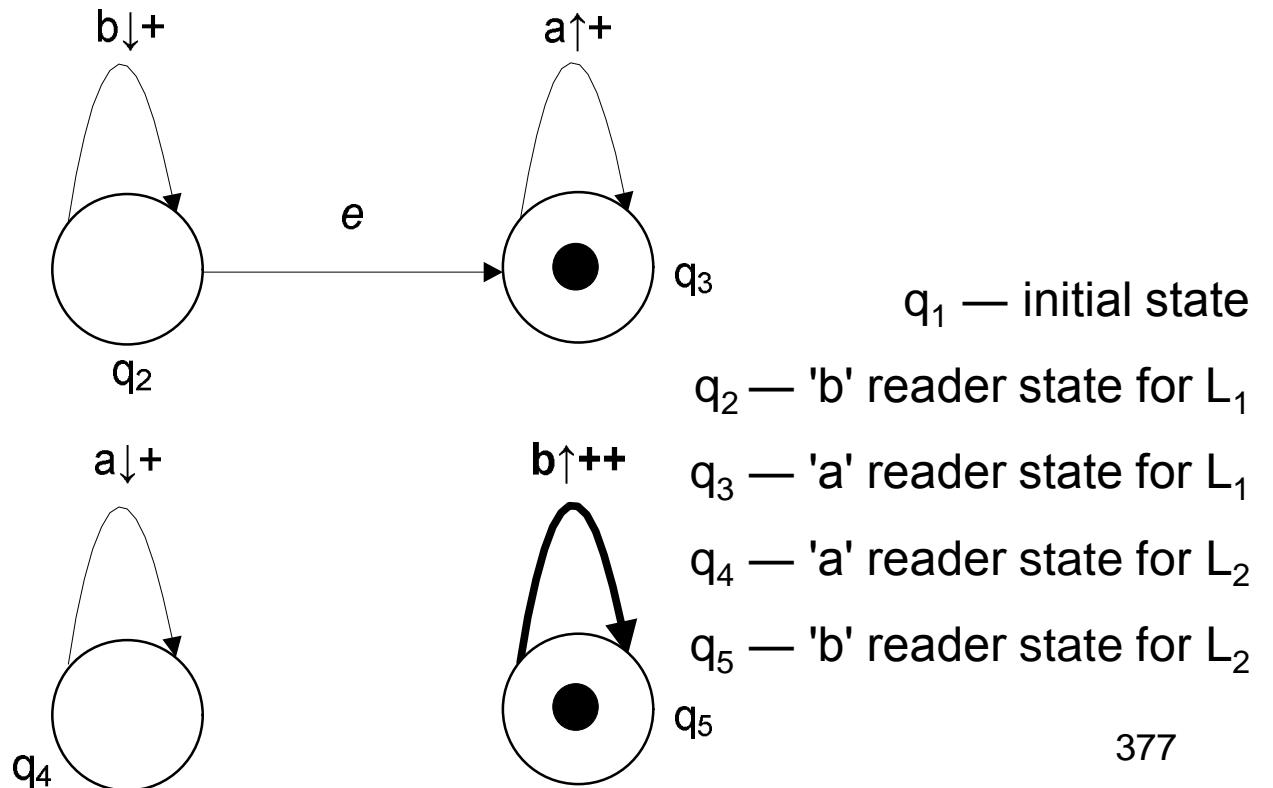
- Solution:



PDA: Example 13

- Give PDA M such that $L(M) = L!$
 - $L = \{w \in b^m a^m \cup a^{2m} b^m, m \geq 0\}$

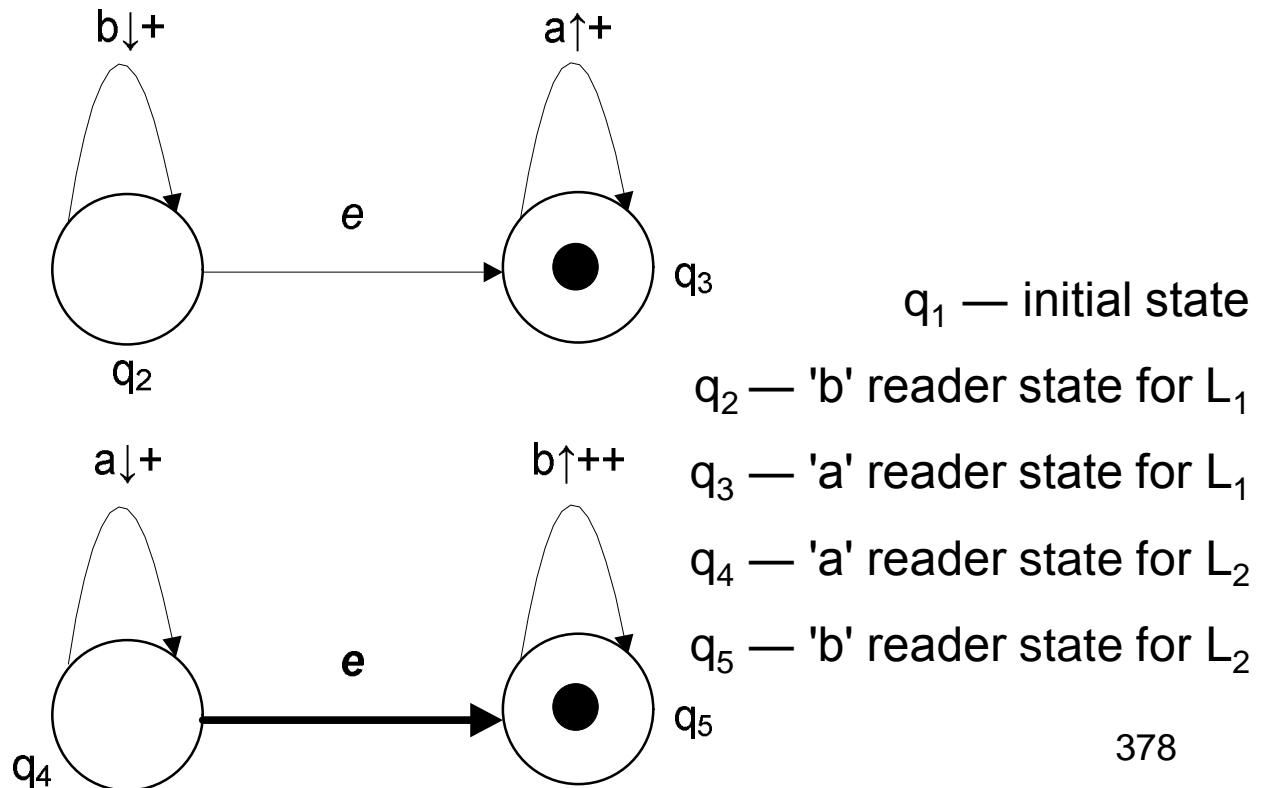
- Solution:



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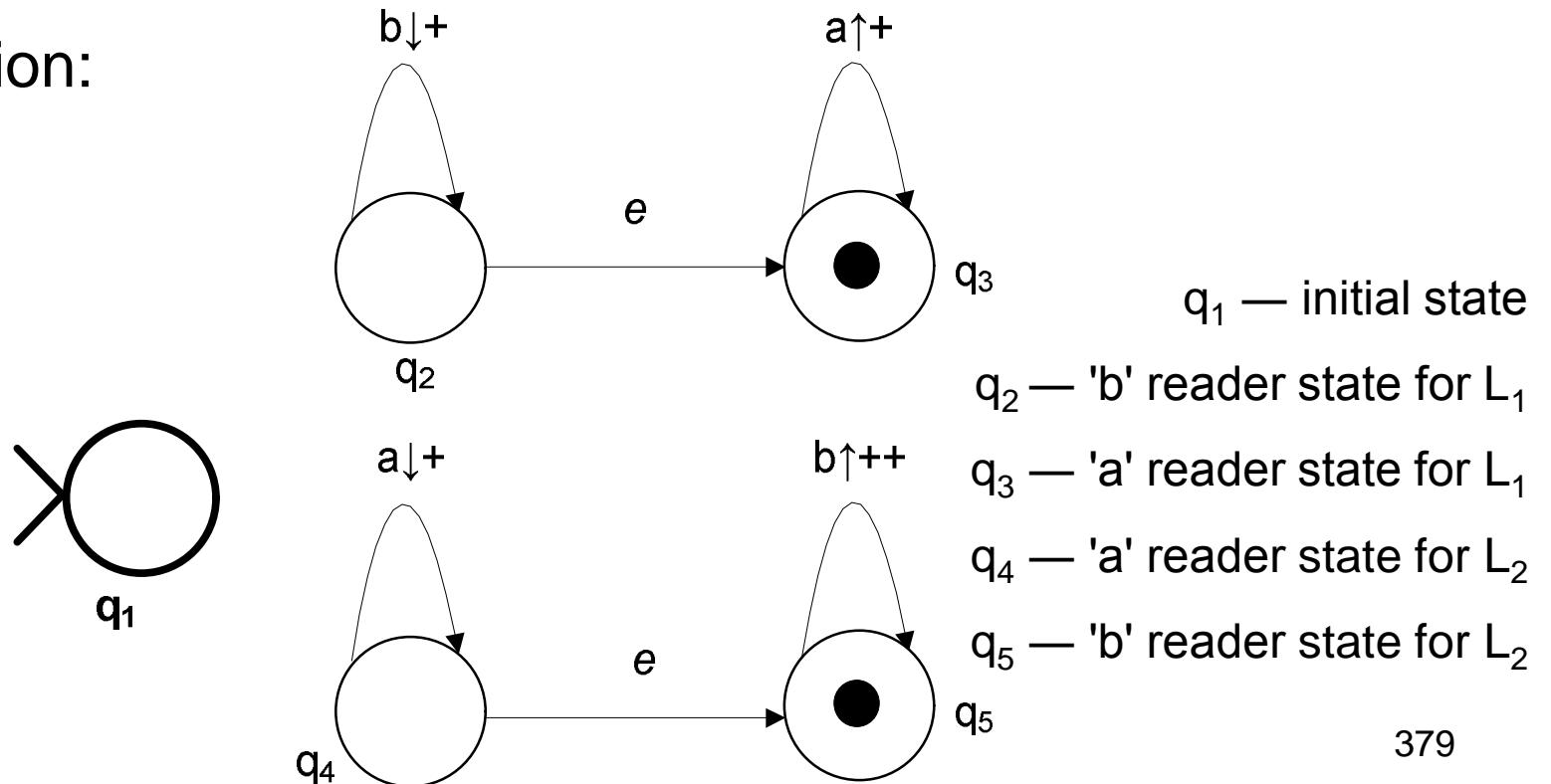
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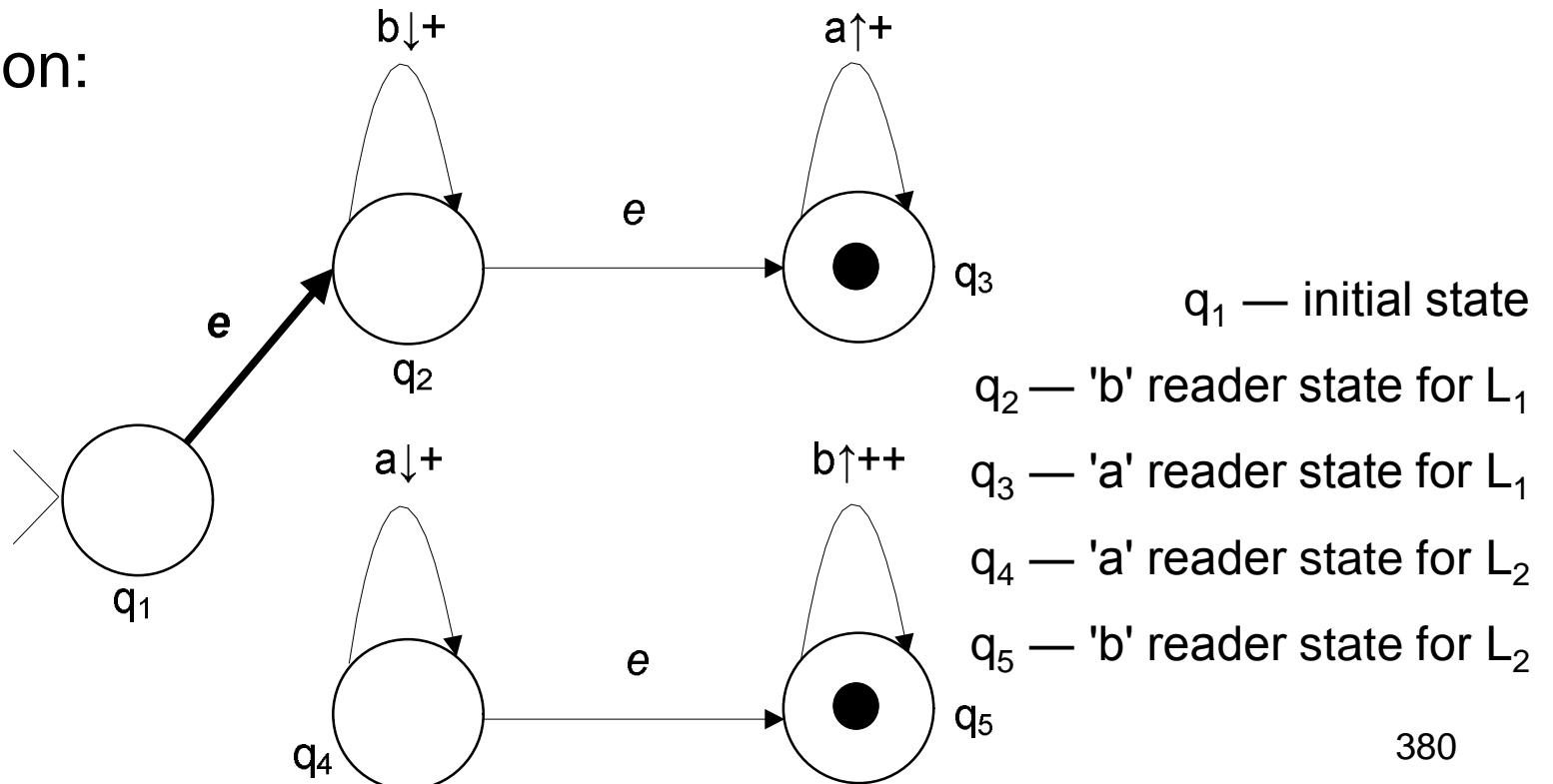
- Solution:



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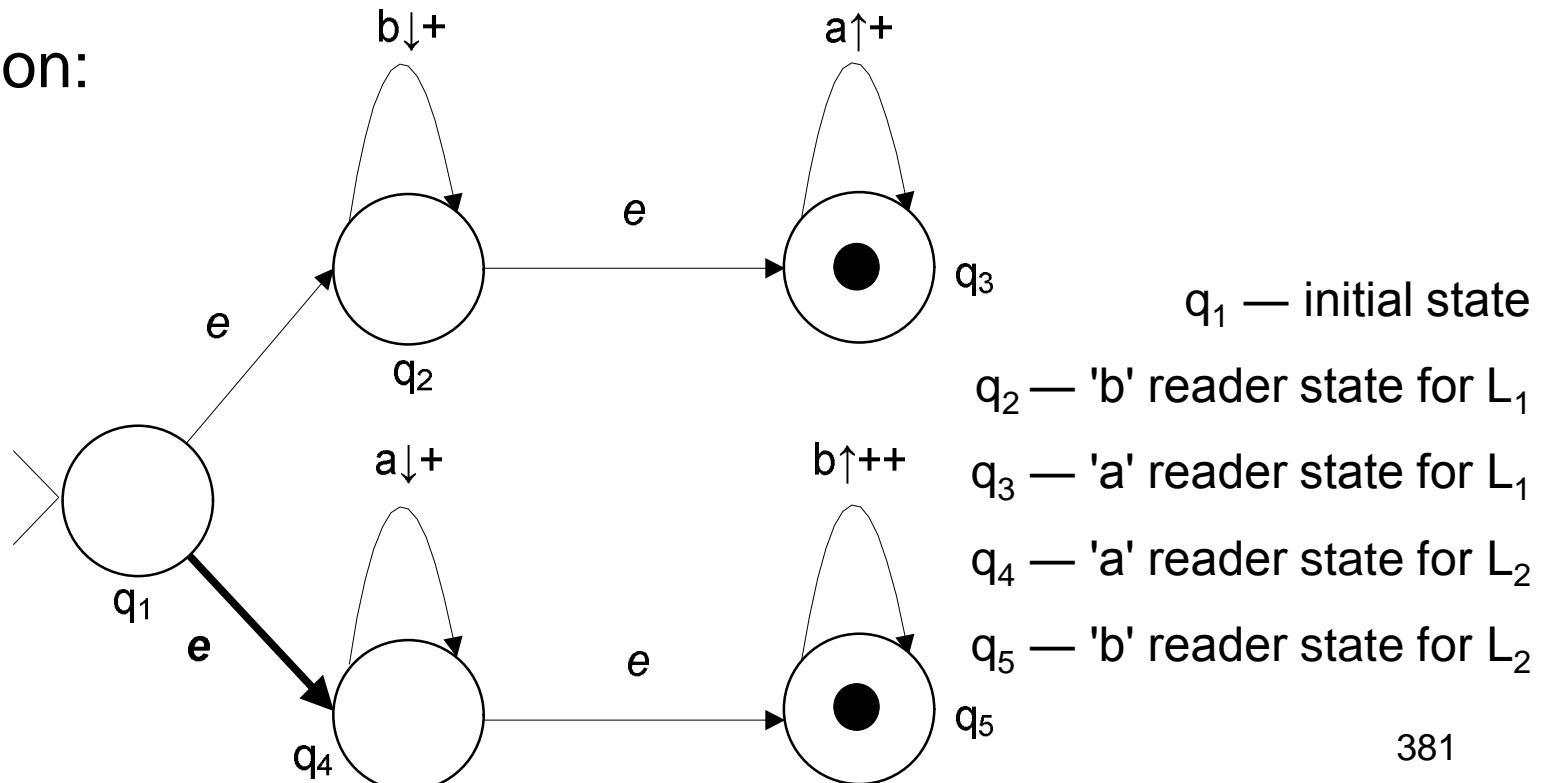
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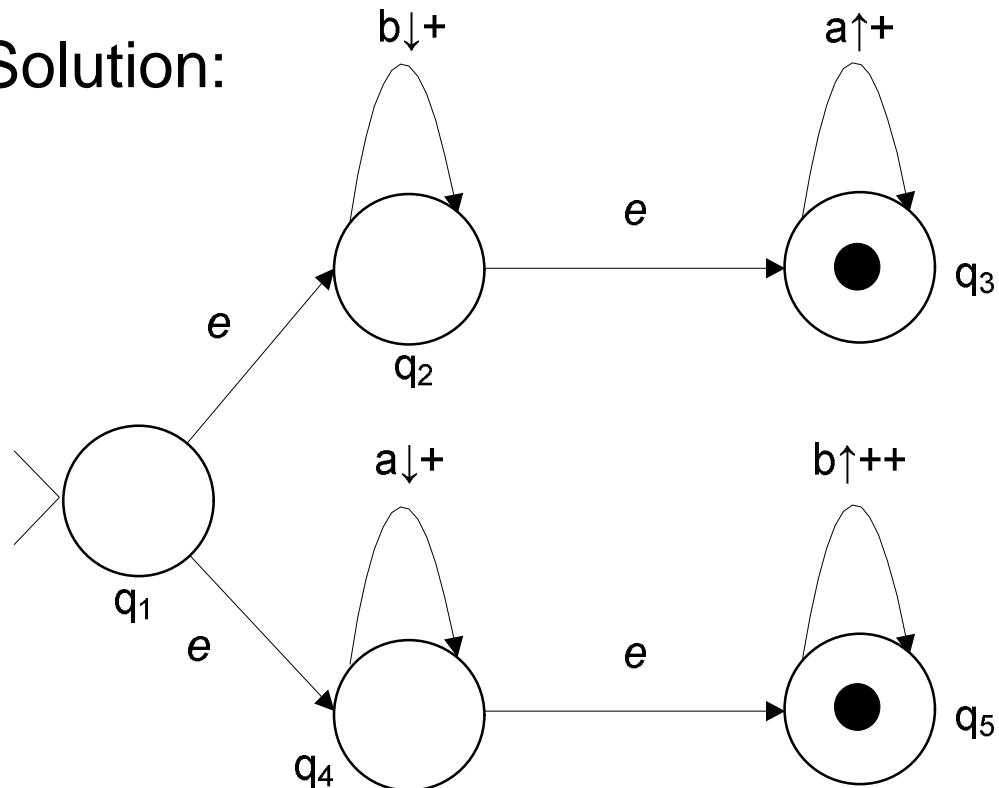
- Solution:



PDA: Example 13

- Give PDA M such that $L(M) = L!$
 - $L = \{w \in b^m a^m \cup a^{2m} b^m, m \geq 0\}$

- Solution:



q_1 — initial state

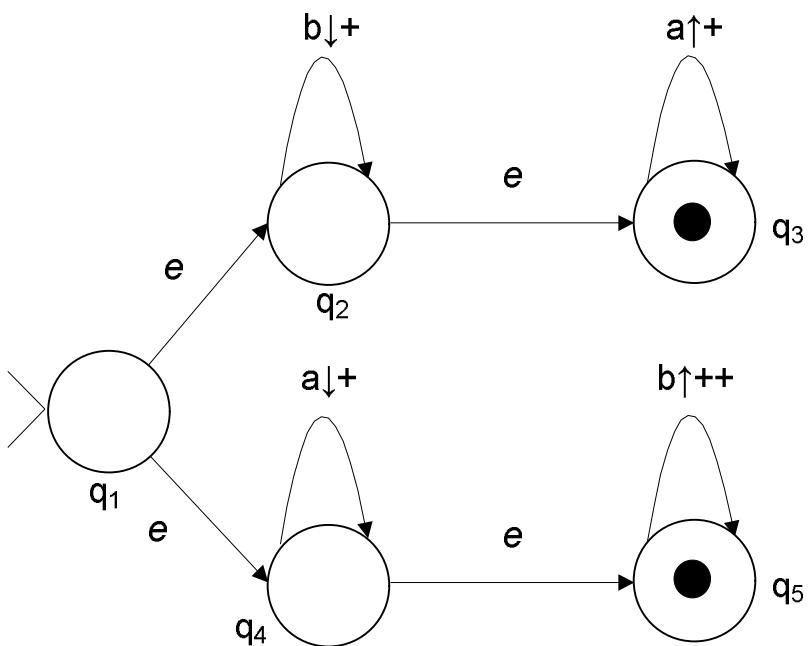
q_2 — 'b' reader state for L_1

q_3 — 'a' reader state for L_1

q_4 — 'a' reader state for L_2

q_5 — 'b' reader state for L_2

State	Unread input	Stack
q_1	bbaa	e
q_2	bbaa	e
q_2	baa	+
q_2	aa	++
q_3	aa	++
q_3	a	+
q_3	e	e

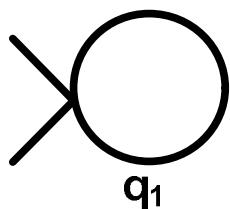


PDA: Example 14

- Give PDA M such that $L(M) = L!$
 - $L = \{w \in \{a, b\}^* \mid w = a^{2n}uu^Rb^{3n}\}$
 - nested problem

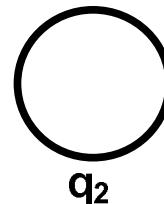
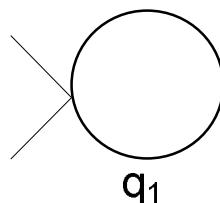
PDA: Example 14

- Give PDA M such that $L(M) = L!$
 - $L = \{w \in \{a, b\}^* \mid w = a^{2n}uu^Rb^{3n}\}$
- Solution:
 - q_1 — 'a' reader state
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 - q_4 — 'b' reader state
 - q_3 — reading u^R
 - '+' — 1/3 'a' excess, 1/2 'b' lack
 - 'a' — 'a' is read in u, 'b' — 'b' is read in u^R



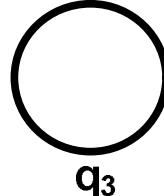
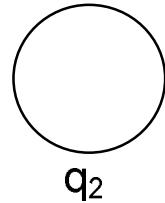
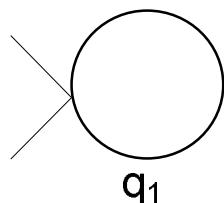
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 - $L = \{w \in \{a, b\}^* \mid w = a^{2n}uu^Rb^{3n}\}$
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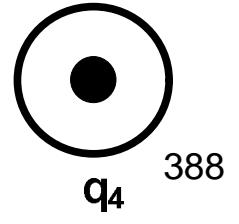
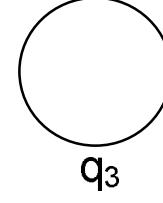
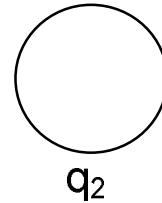
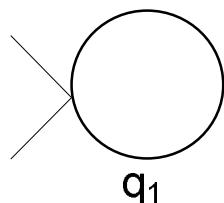
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PDA: Example 14

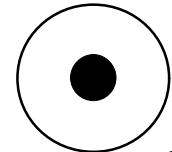
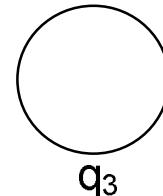
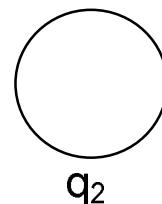
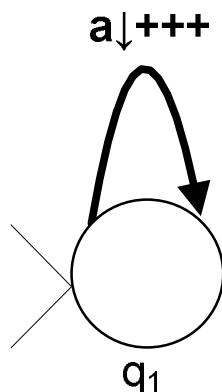
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PDA: Example 14

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 - q_2 — reading u
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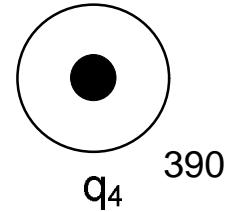
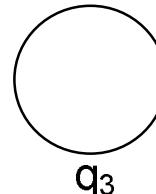
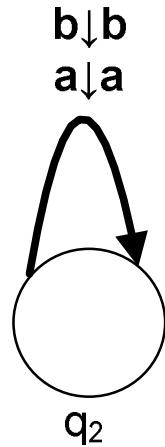
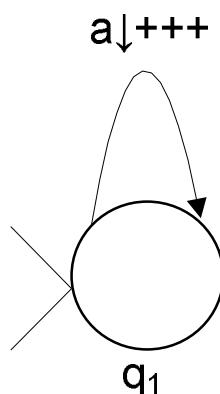
389

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390

PDA: Example 14

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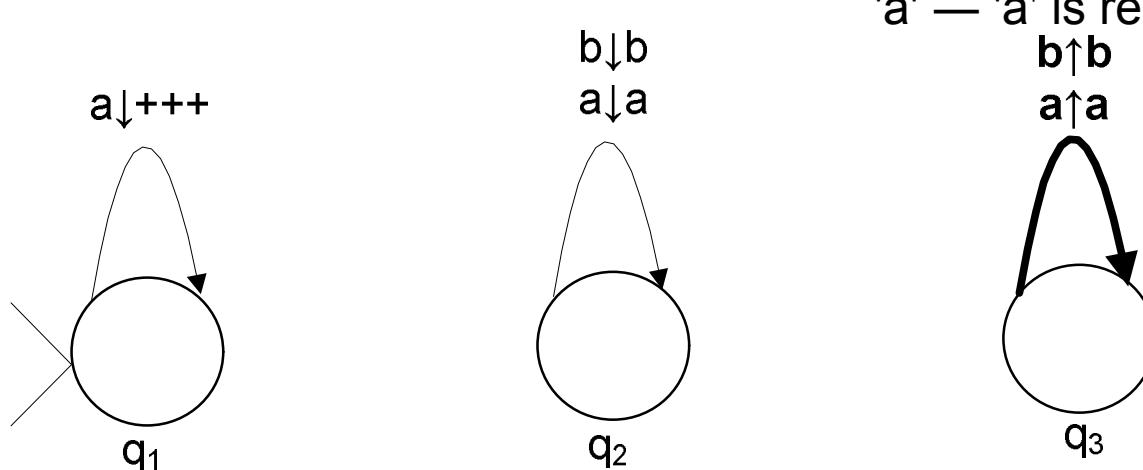
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'a' — 'a' is read in u, 'b' — 'b' is read in u

$b \uparrow b$

$a \uparrow a$



q₄

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PDA: Example 14

- Give PDA M such that $L(M) = L!$
 - $L = \{w \in \{a, b\}^* \mid w = a^{2n}uu^Rb^{3n}\}$

- Solution:

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q_2 — reading u

q_4 — 'b' reader state

q_3 — reading u^R

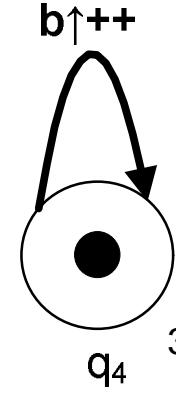
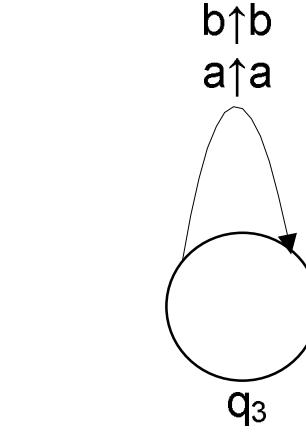
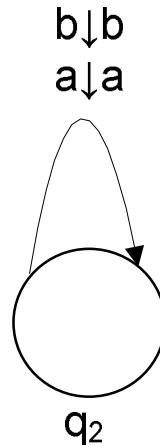
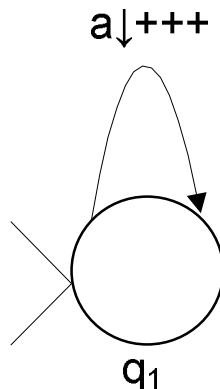
'+' — 1/3 'a' excess, 1/2 'b' lack

'a' — 'a' is read in u, 'b' — 'b' is read in u^R

$b \uparrow b$

$a \uparrow a$

$b \uparrow ++$



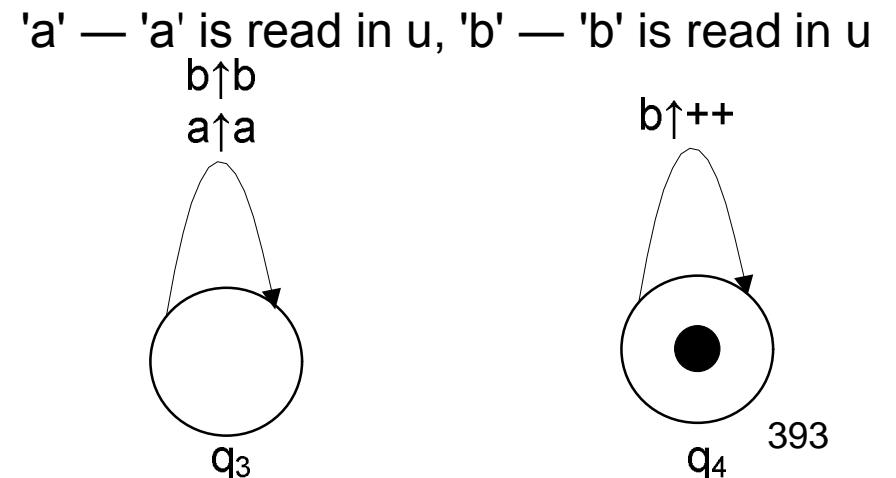
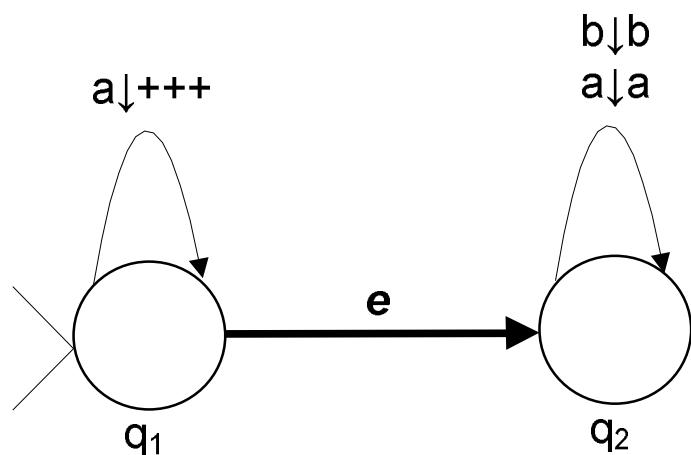
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PDA: Example 14

- Give PDA M such that $L(M) = L!$
 - $L = \{w \in \{a, b\}^* \mid w = a^{2n}uu^Rb^{3n}\}$

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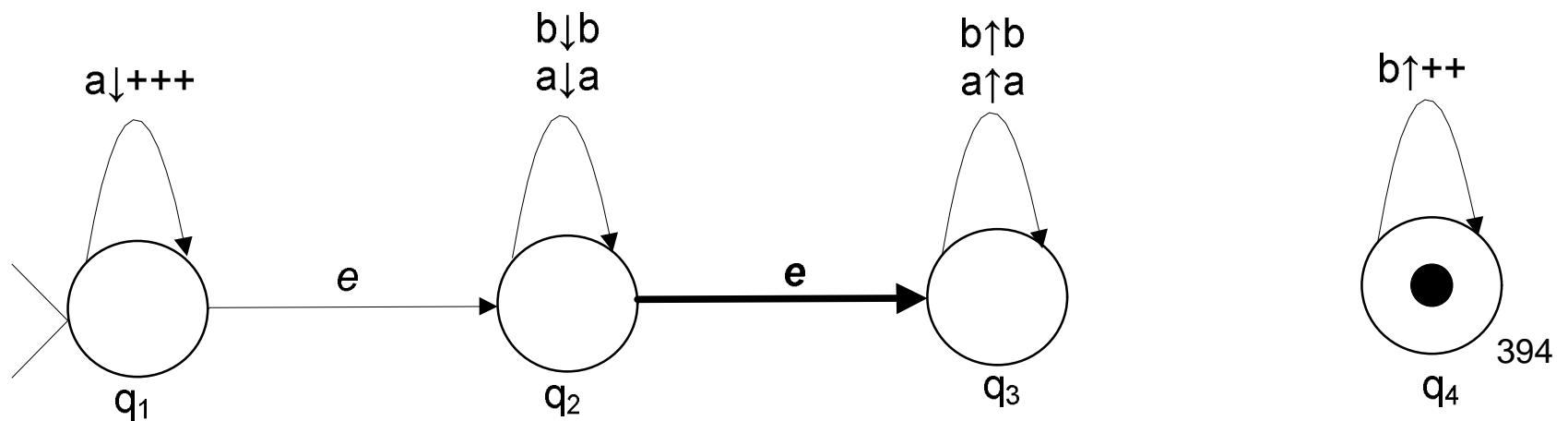


PDA: Example 14

- Give PDA M such that $L(M) = L!$
 $L = \{w \in \{a, b\}^* \mid w = a^{2n}uu^Rb^{3n}\}$

- Solution:

q_1 — 'a' reader state q_2 — reading u
 q_4 — 'b' reader state q_3 — reading u^R
 '+' — 1/3 'a' excess, 1/2 'b' lack
 'a' — 'a' is read in u, 'b' — 'b' is read in u^R

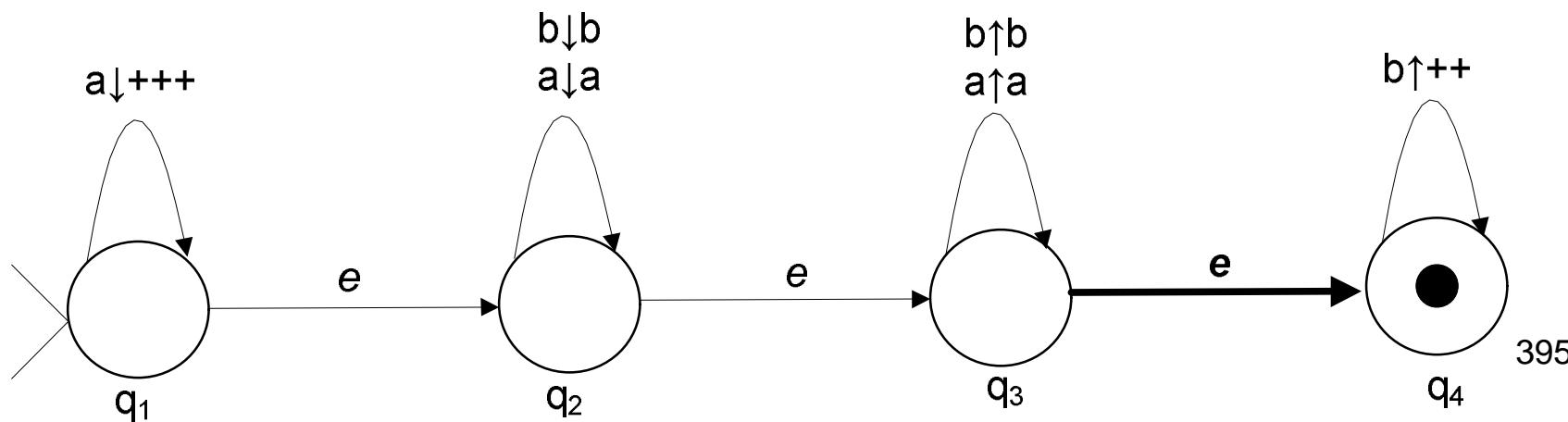


PDA: Example 14

- Give PDA M such that $L(M) = L!$
 - $L = \{w \in \{a, b\}^* \mid w = a^{2n}uu^Rb^{3n}\}$

- Solution:

q_1 — 'a' reader state q_2 — reading u
 q_4 — 'b' reader state q_3 — reading u^R
 '+' — 1/3 'a' excess, 1/2 'b' lack
 'a' — 'a' is read in u, 'b' — 'b' is read in u^R

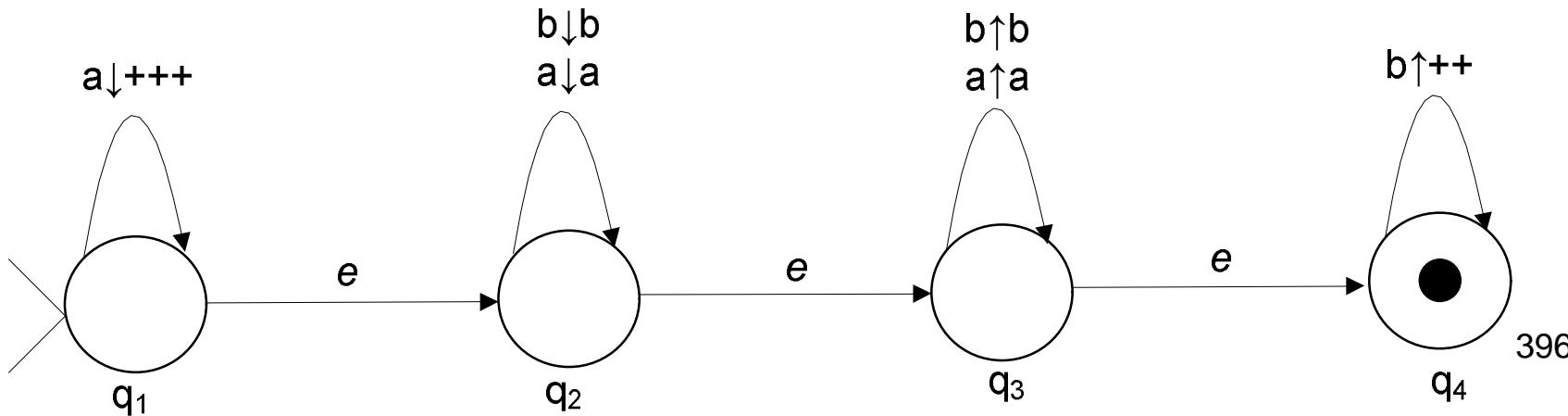


PDA: Example 14

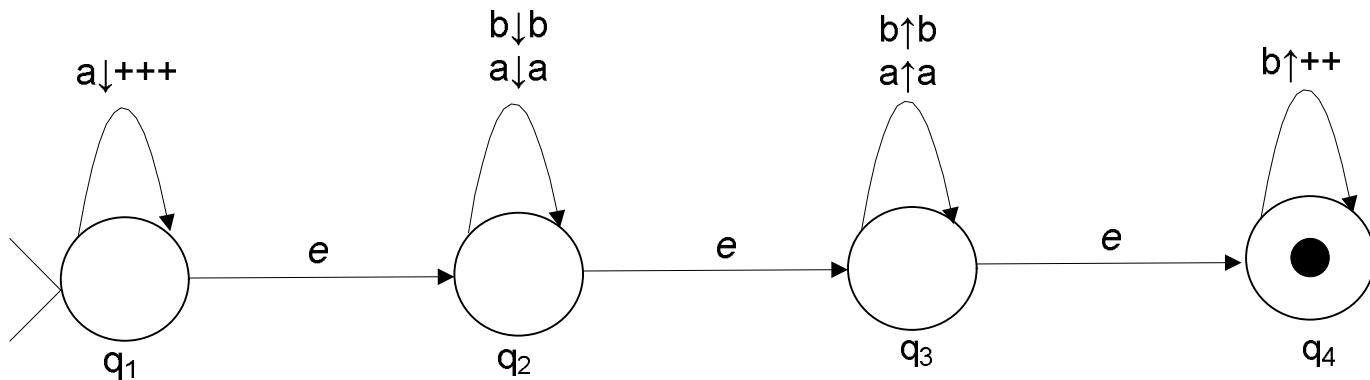
- Give PDA M such that $L(M) = L!$
 – $L = \{w \in \{a, b\}^* \mid w = a^{2n}uu^Rb^{3n}\}$

- Solution:

q_1 — 'a' reader state q_2 — reading u
 q_4 — 'b' reader state q_3 — reading u^R
 '+' — 1/3 'a' excess, 1/2 'b' lack
 'a' — 'a' is read in u, 'b' — 'b' is read in u^R



State	Unread input	Stack
q_1	aaabbabbbb	e
q_1	aabbabbbb	$+^3$
q_1	abbabbbb	$+^6$
q_2	abbabbbb	$+^6$
q_2	bbabbbb	$a+^6$
q_2	babbbb	$ba+^6$
q_3	babbbb	$ba+^6$
q_3	abbb	$a+^6$
q_3	bbb	$+^6$
q_4	bbb	$+^6$
q_4	bb	$+^4$
q_4	b	$+^2$
q_4	e	e



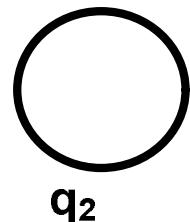
PDA: Example 15

- Give PDA M such that $L(M) = L!$
 - $L = \{w \in \{a, b\}^* \mid w = (a^{2n}b^{n+2})^*\}$

PDA: Example 15

- Give PDA M such that $L(M) = L!$
 - $L = \{w \in \{a, b\}^* \mid w = (a^{2n}b^{n+2})^*\}$
 - $L = \{w \in a^{2n}b^{n+2}\}^*$
 - using the construction theorem
 - '+' has new interpretation now
- Solution:

q_1 — start state
 q_2 — 'a' reader state
 q_3 — 'b' reader state
'+' — 2 'a' excess
1 'b' lack



PDA: Example 15

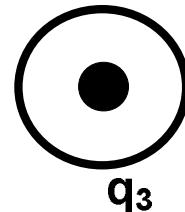
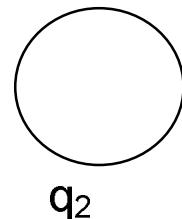
- Give PDA M such that $L(M) = L!$
 - $L = \{w \in \{a, b\}^* \mid w = (a^{2n}b^{n+2})^*\}$

q_1 — initial state

q_2 — 'a' reader state

q_3 — 'b' reader state

'+' — 2 'a' excess
1 'b' lack



PDA: Example 15

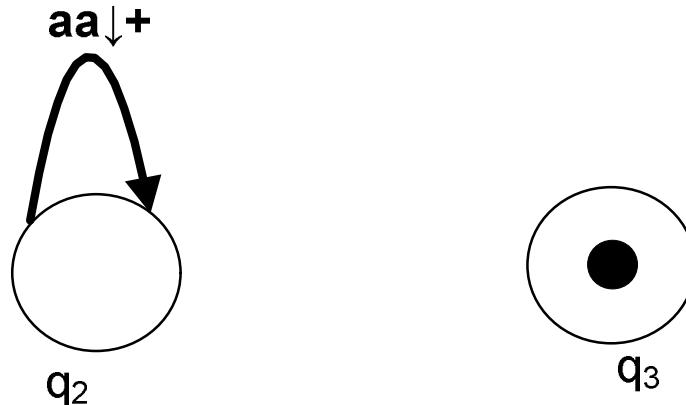
- Give PDA M such that $L(M) = L!$
 - $L = \{w \in \{a, b\}^* \mid w = (a^{2n}b^{n+2})^*\}$

q_1 — initial state

q_2 — 'a' reader state

q_3 — 'b' reader state

'+' — 2 'a' excess
1 'b' lack



PDA: Example 15

- Give PDA M such that $L(M) = L!$
 - $L = \{w \in \{a, b\}^* \mid w = (a^{2n}b^{n+2})^*\}$

- Solution:

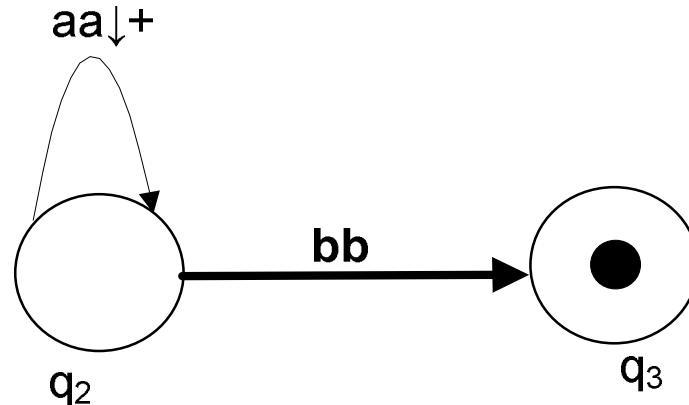
q_1 — initial state

q_2 — 'a' reader state

q_3 — 'b' reader state

'+' — 2 'a' excess

1 'b' lack



PDA: Example 15

- Give PDA M such that $L(M) = L!$
 - $L = \{w \in \{a, b\}^* \mid w = (a^{2n}b^{n+2})^*\}$

- Solution:

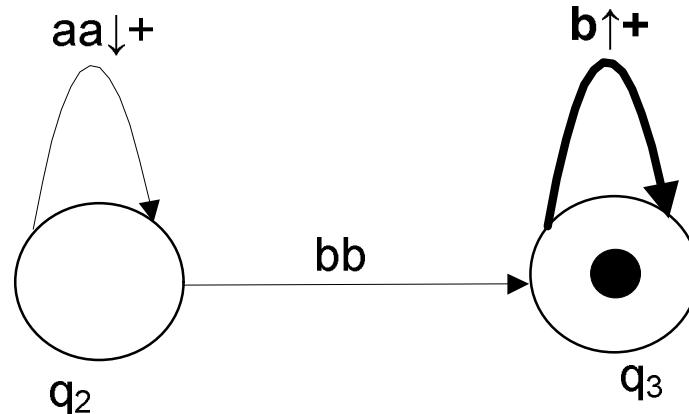
q_1 — initial state

q_2 — 'a' reader state

q_3 — 'b' reader state

'+' — 2 'a' excess

1 'b' lack



PDA: Example 15

- Give PDA M such that $L(M) = L!$
 - $L = \{w \in \{a, b\}^* \mid w = (a^{2n}b^{n+2})^*\}$

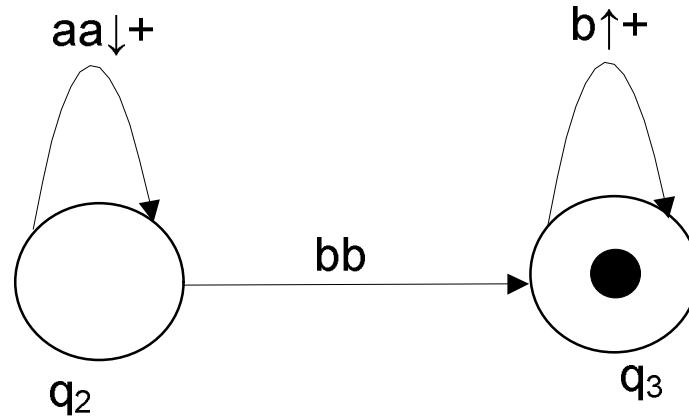
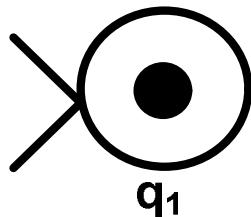
- Solution:

q_1 — initial state

q_2 — 'a' reader state

q_3 — 'b' reader state

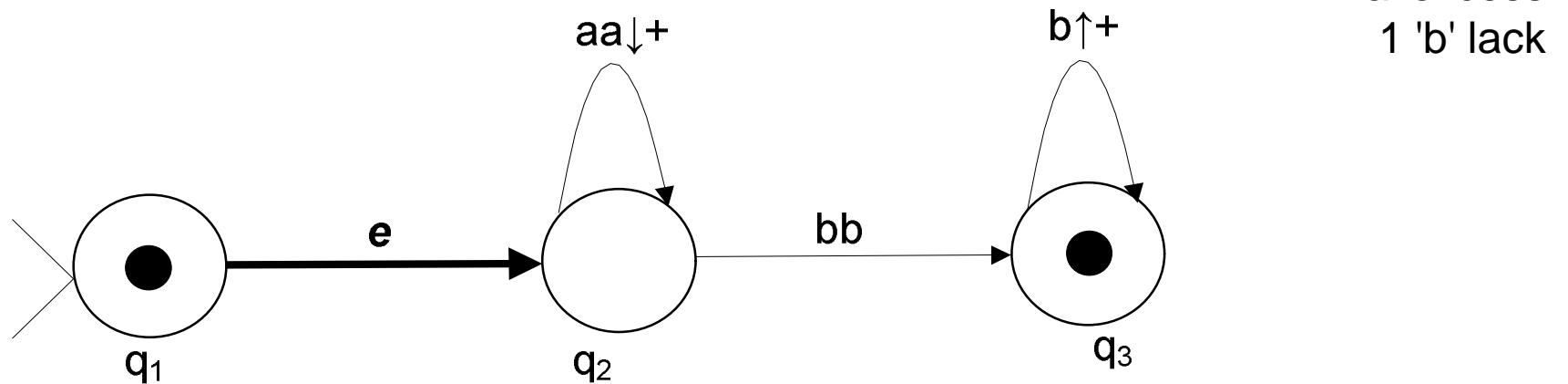
'+' — 2 'a' excess
1 'b' lack



PDA: Example 15

- Give PDA M such that $L(M) = L!$
 - $L = \{w \in \{a, b\}^* \mid w = (a^{2n}b^{n+2})^*\}$

- Solution:



PDA: Example 15

- Give PDA M such that $L(M) = L!$
 - $L = \{w \in \{a, b\}^* \mid w = (a^{2n}b^{n+2})^*\}$

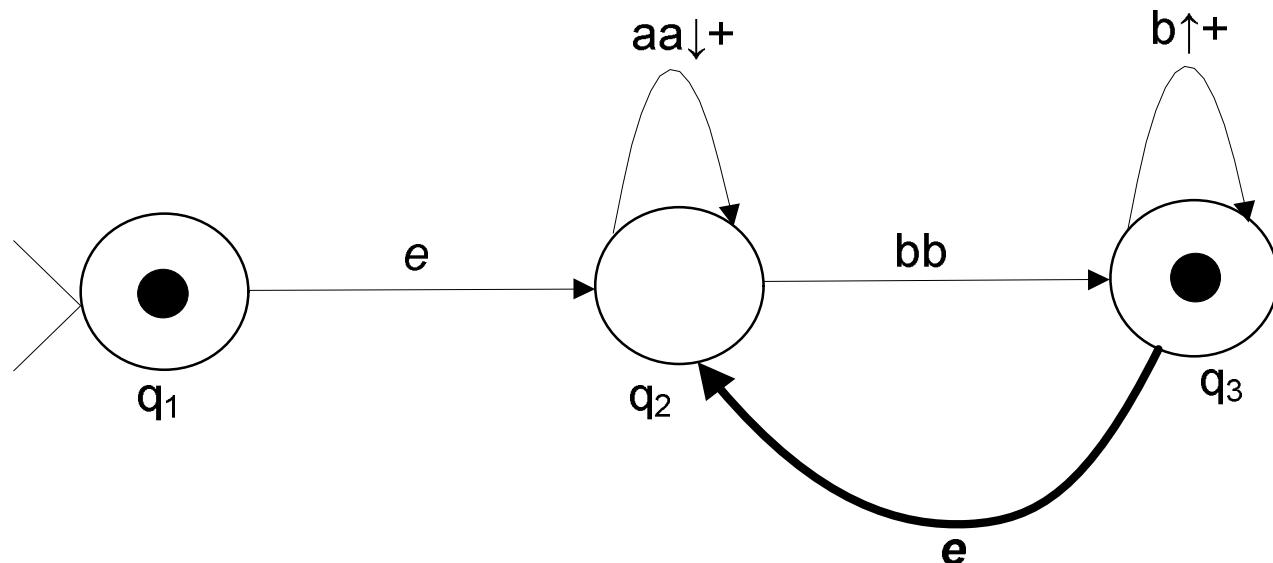
- Solution:

q_1 — initial state

q_2 — 'a' reader state

q_3 — 'b' reader state

'+' — 2 'a' excess
1 'b' lack



PDA: Example 15

- Give PDA M such that $L(M) = L!$
 - $L = \{w \in \{a, b\}^* \mid w = (a^{2n}b^{n+2})^*\}$

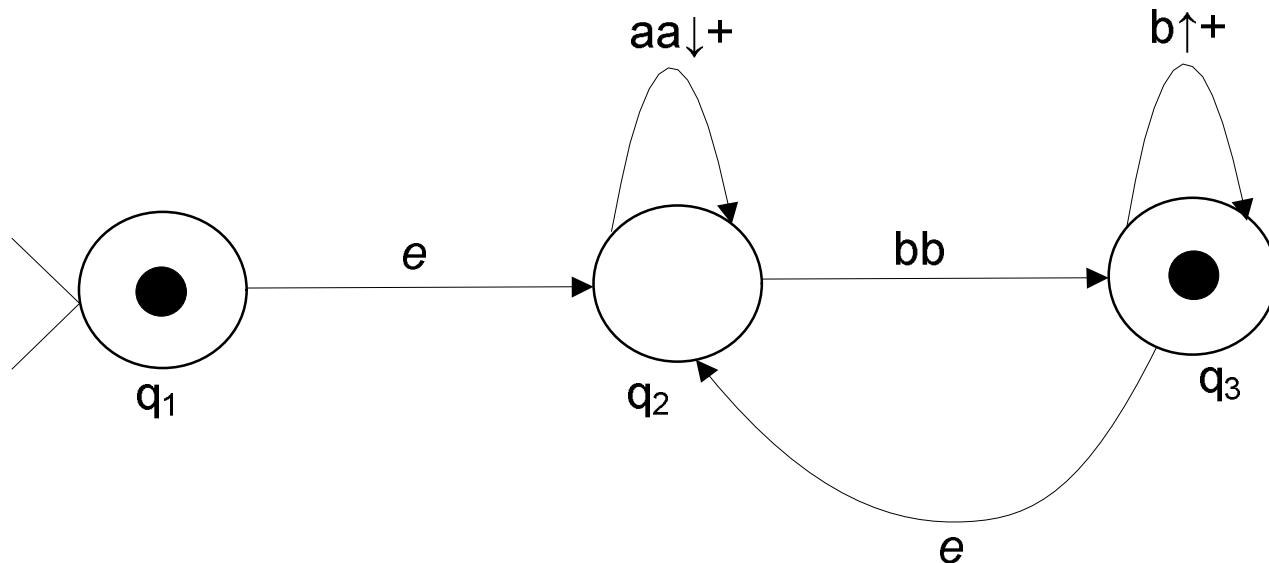
- Solution:

q_1 — initial state

q_2 — 'a' reader state

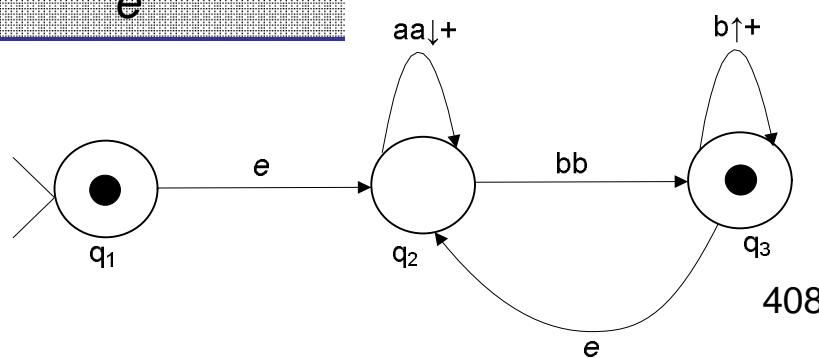
q_3 — 'b' reader state

'+' — 2 'a' excess
1 'b' lack



PDA: Example 15

State	Unread input	Stack
q_1	aabbbaabbb	e
q_2	aabbbaabbb	e
q_2	bbbbaabbb	$+$
q_3	baabbb	$+$
q_3	aabbb	e
q_2	aabbb	e
q_2	bbb	$+$
q_3	b	$+$
q_3	e	e

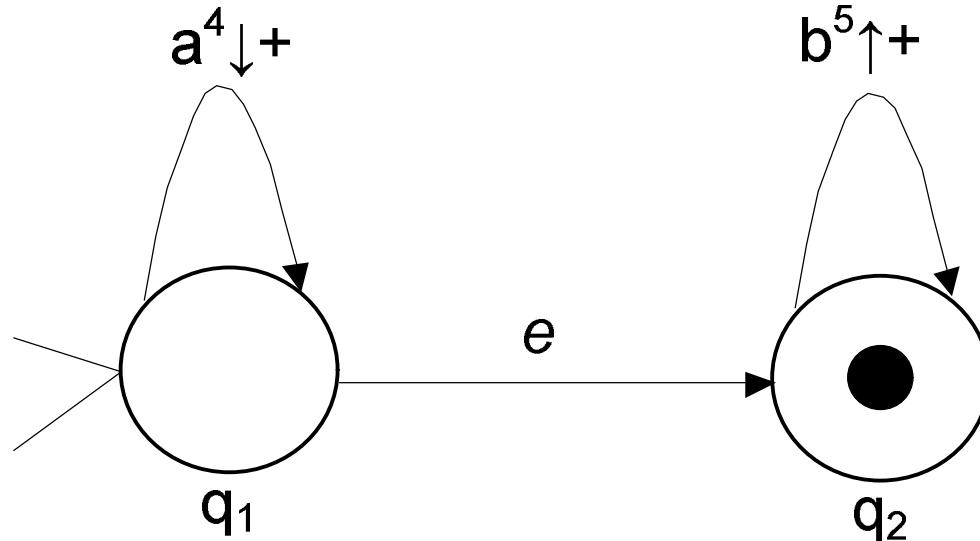


PDA: Example 16

- Give PDA M such that $L(M) = L!$
 - $L = \{w \in \{a, b\}^* \mid w = a^{4n}b^{5n}\}$

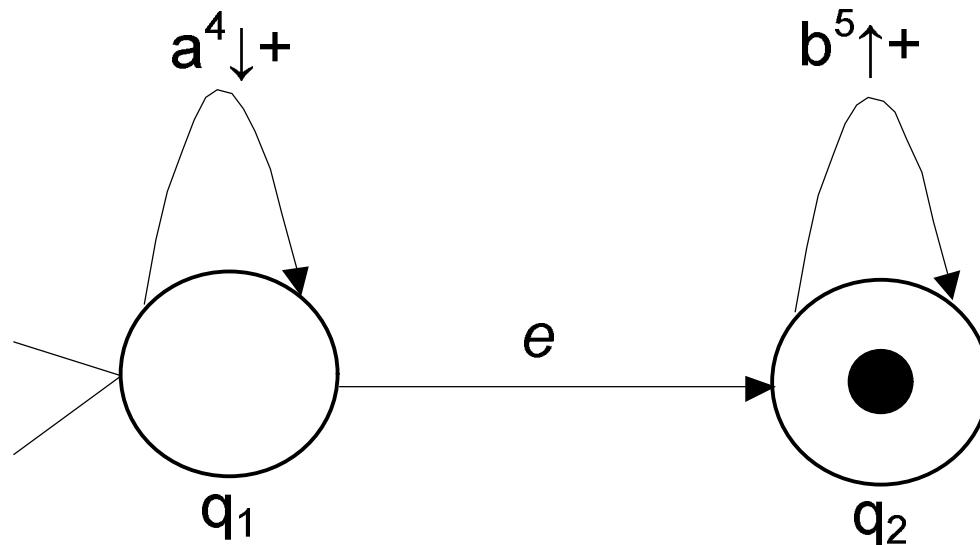
q_1 — 'a' reader state q_2 — 'b' reader state
'+' — 4 "a" excess, 5 "b" lack

- Solution:



PDA: Example 16

- Computation for "a⁸b¹⁰":

$$(q_1, a^8b^{10}, e) \xrightarrow{-} (q_1, a^4b^{10}, +) \xrightarrow{-} (q_1, b^{10},++) \xrightarrow{-} (q_2, b^{10},++) \xrightarrow{-} (q_2, b^5,+) \xrightarrow{-} (q_2, e, e)$$


PDA: Example 17

- Give PDA M such that $L(M) = L!$

- $L = \{w \in \{a, b\}^* \mid w = aba^{3n+1}bab^{4n+2}ab\}$

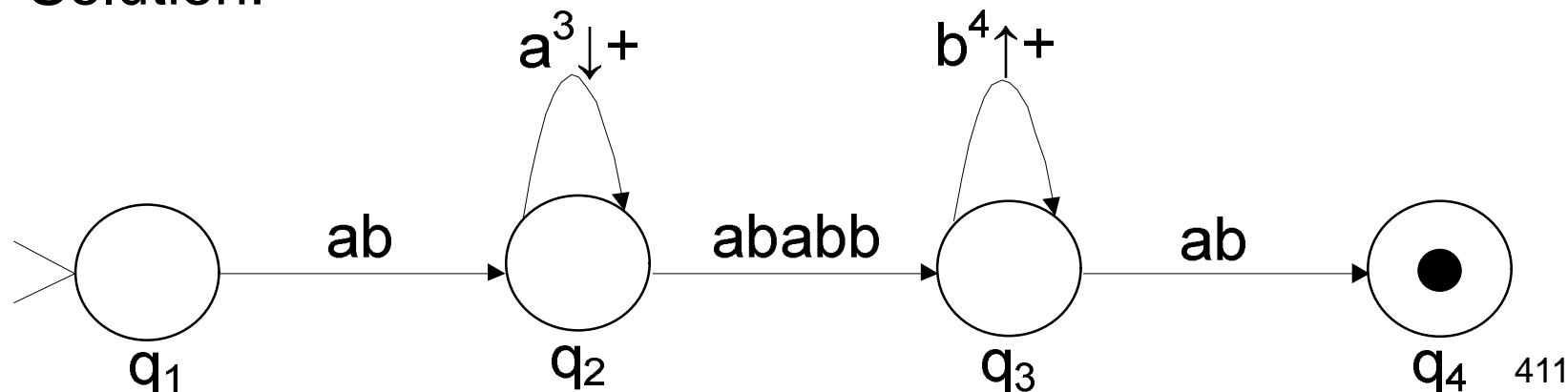
- $L = \{w \in \{a, b\}^* \mid w = aba^{3n}ababbb^{4n}ab\}$

q_1 — initial state q_2 — 'a' reader state

q_3 — 'b' reader state q_2 — final state

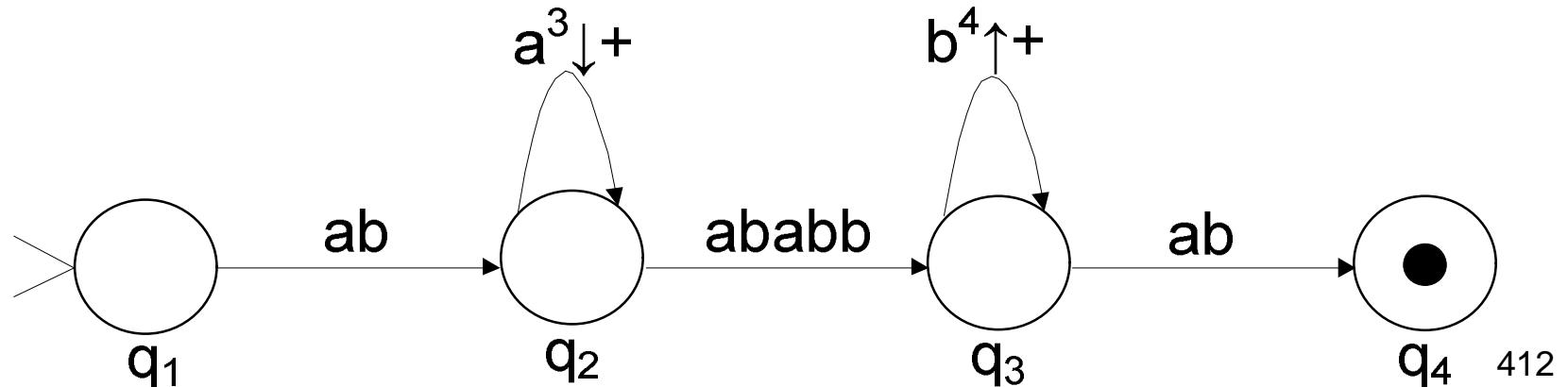
'+' — 3 "a" excess, 4 "b" lack

- Solution:



PDA: Example 17

- Computation for "abaaaababbbaaab":
 $(q_1, abaaaababbbaaab, e) \xrightarrow{-} (q_2, aaaababbbaaab, e)$
 $\xrightarrow{-} (q_2, ababbbaaab, +) \xrightarrow{-} (q_3, bbbbab, +)$
 $\xrightarrow{-} (q_3, ab, +) \xrightarrow{-} (q_4, e, e)$



Element of the Theory of the Computation

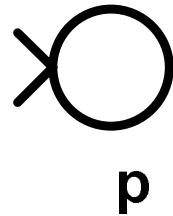
Lecture 9

CFG \rightarrow PDA: Example 1

- Construct PDA M such that $L(M) = L(G)$!
 - $V = \{a, b, S\}$
 - $\Sigma = \{a, b\}$
 - $S = S$
 - $R = \{S \rightarrow aSb \mid bSa \mid e\}$

CFG → PDA: Example 1

- Step a1: introduce an initial state
 - $V = \{a, b, S\}$
 - $\Sigma = \{a, b\}$
 - $S = S$
 - $R = \{S \rightarrow aSb \mid bSa \mid e\}$



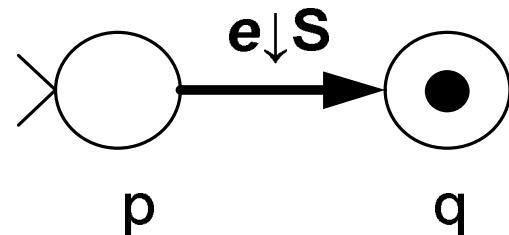
CFG → PDA: Example 1

- Step a2: introduce a final state
 - $V = \{a, b, S\}$
 - $\Sigma = \{a, b\}$
 - $S = S$
 - $R = \{S \rightarrow aSb \mid bSa \mid e\}$



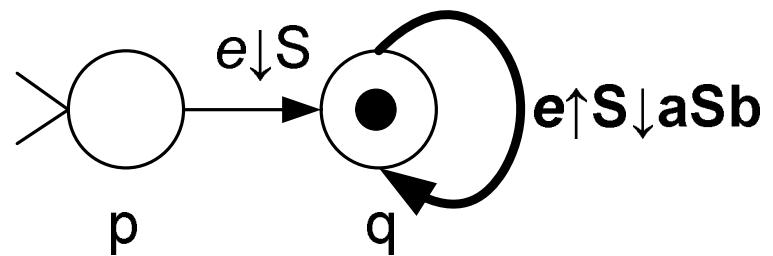
CFG → PDA: Example 1

- Step a3: initially push S into the stack
 - $V = \{a, b, S\}$
 - $\Sigma = \{a, b\}$
 - $S = S$
 - $R = \{S \rightarrow aSb \mid bSa \mid e\}$



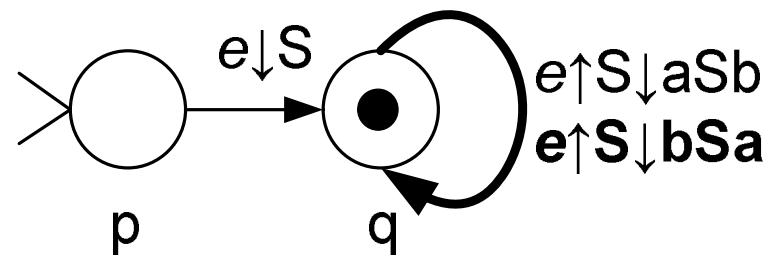
CFG → PDA: Example 1

- Step b1: introduce a new arc for $S \rightarrow aSb$
 - $V = \{a, b, S\}$
 - $\Sigma = \{a, b\}$
 - $S = S$
 - $R = \{S \rightarrow aSb \mid bSa \mid e\}$



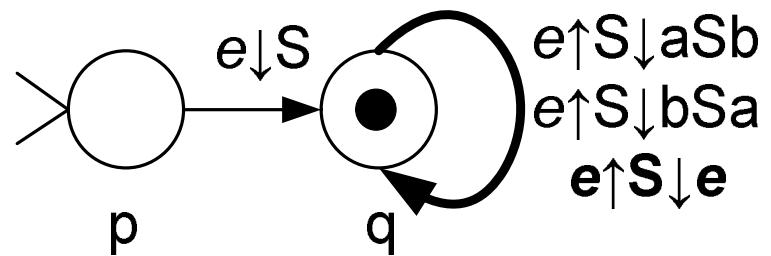
CFG → PDA: Example 1

- Step b2: introduce a new arc for $S \rightarrow bSa$
 - $V = \{a, b, S\}$
 - $\Sigma = \{a, b\}$
 - $S = S$
 - $R = \{S \rightarrow aSb \mid bSa \mid e\}$



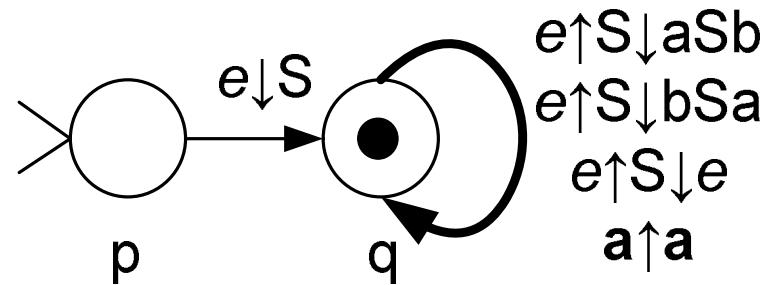
CFG → PDA: Example 1

- Step b3: introduce a new arc for $S \rightarrow e$
 - $V = \{a, b, S\}$
 - $\Sigma = \{a, b\}$
 - $S = S$
 - $R = \{S \rightarrow aSb \mid bSa \mid e\}$



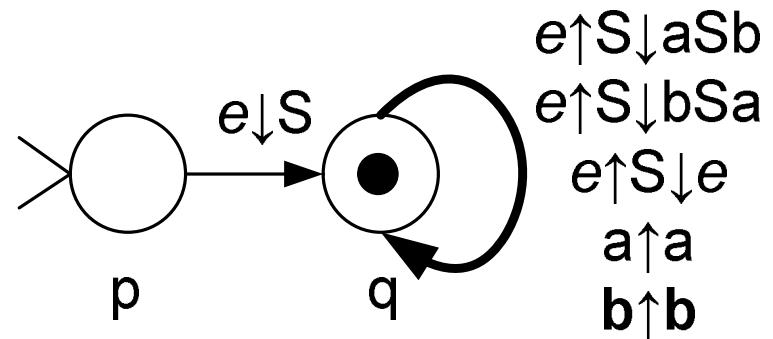
CFG → PDA: Example 1

- Step c1: introduce a new arc for $a \in \Sigma$
 - $V = \{a, b, S\}$
 - $\Sigma = \{a, b\}$
 - $S = S$
 - $R = \{S \rightarrow aSb \mid bSa \mid e\}$



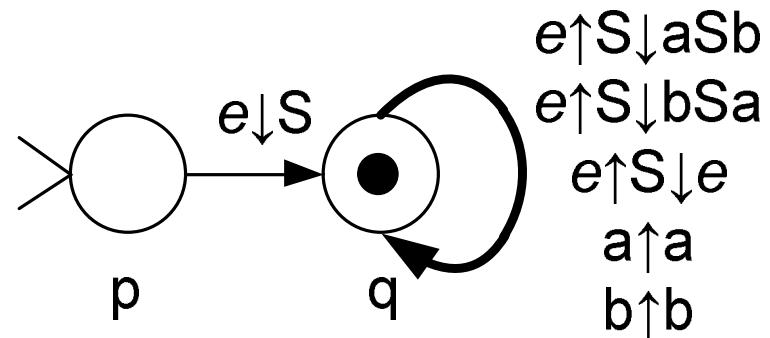
CFG → PDA: Example 1

- Step c2: introduce a new arc for $b \in \Sigma$
 - $V = \{a, b, S\}$
 - $\Sigma = \{a, \mathbf{b}\}$
 - $S = S$
 - $R = \{S \rightarrow aSb \mid bSa \mid e\}$



CFG → PDA: Example 1

- Example: abab
 - $S \Rightarrow aSb \Rightarrow abSab \Rightarrow abab$
 - $(p, abab, e) \vdash (q, abab, S) \vdash (q, abab, aSb) \vdash (q, bab, Sb) \vdash (q, bab, bSab) \vdash (q, ab, Sab) \vdash (q, ab, ab) \vdash (q, b, b) \vdash (q, e, e)$



CFG → PDA: Example 2

- Construct PDA M such that $L(M) = L(G)$! Use the set notations!
 - $V = \{a, b, S\}$
 - $\Sigma = \{a, b\}$
 - $S = S$
 - $R = \{S \rightarrow aSa \mid bSb \mid e\}$

CFG → PDA: Example 2

- Step a1: introduce an initial state
 - $V = \{a, b, S\}$
 - $\Sigma = \{a, b\}$
 - $S = S$
 - $R = \{S \rightarrow aSa \mid bSb \mid e\}$
 - $M = (\{p, q\}, \Sigma, V, \Delta, p, \{q\})$
 - $\Sigma = \{a, b\}$
 - $V = \{S, a, b\}$
 - $\Delta = \{\}$

CFG → PDA: Example 2

- Step a2: introduce a final state
 - $V = \{a, b, S\}$
 - $\Sigma = \{a, b\}$
 - $S = S$
 - $R = \{S \rightarrow aSa \mid bSb \mid e\}$
 - $M = (\{p, q\}, \Sigma, V, \Delta, p, \{q\})$
 - $\Sigma = \{a, b\}$
 - $V = \{S, a, b\}$
 - $\Delta = \{\}$

CFG → PDA: Example 2

- Step a3: initially push S into the stack
 - $V = \{a, b, S\}$
 - $\Sigma = \{a, b\}$
 - $S = S$
 - $R = \{S \rightarrow aSa \mid bSb \mid e\}$
 - $M = (\{p, q\}, \Sigma, V, \Delta, p, \{q\})$
 - $\Sigma = \{a, b\}$
 - $V = \{S, a, b\}$
 - $\Delta = \{\langle (p, e, e), (q, S) \rangle\}$

CFG → PDA: Example 2

- Step b1: introduce a new arc for $S \rightarrow aSa$
 - $V = \{a, b, S\}$
 - $\Sigma = \{a, b\}$
 - $S = S$
 - $R = \{S \rightarrow aSa \mid bSb \mid e\}$
 - $M = (\{p, q\}, \Sigma, V, \Delta, p, \{q\})$
 - $\Sigma = \{a, b\}$
 - $V = \{S, a, b\}$
 - $\Delta = \{((p, e, e), (q, S)), ((q, e, S), (q, aSa))\}$

CFG → PDA: Example 2

- Step b2: introduce a new arc for $S \rightarrow bSb$
 - $V = \{a, b, S\}$
 - $\Sigma = \{a, b\}$
 - $S = S$
 - $R = \{S \rightarrow aSa \mid bSb \mid e\}$
 - $M = (\{p, q\}, \Sigma, V, \Delta, p, \{q\})$
 - $\Sigma = \{a, b\}$
 - $V = \{S, a, b\}$
 - $\Delta = \{((p, e, e), (q, S)), ((q, e, S), (q, aSa)), ((q, e, S), (q, bSb))\}$

CFG → PDA: Example 2

- Step b3: introduce a new arc for $S \rightarrow e$
 - $V = \{a, b, S\}$
 - $\Sigma = \{a, b\}$
 - $S = S$
 - $R = \{S \rightarrow aSa \mid bSb \mid e\}$
 - $M = (\{p, q\}, \Sigma, V, \Delta, p, \{q\})$
 - $\Sigma = \{a, b\}$
 - $V = \{S, a, b\}$
 - $\Delta = \{((p, e, e), (q, S)), ((q, e, S), (q, aSa)), ((q, e, S), (q, bSb)), ((q, e, S), (q, e))\}$

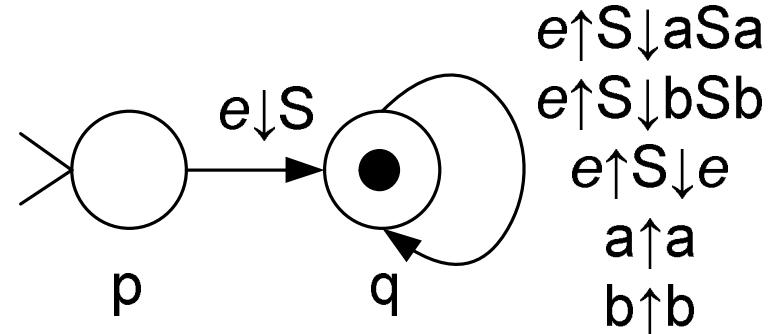
CFG → PDA: Example 2

- Step c1: introduce a new arc for $a \in \Sigma$
 - $V = \{a, b, S\}$
 - $\Sigma = \{a, b\}$
 - $S = S$
 - $R = \{S \rightarrow aSa \mid bSb \mid e\}$
 - $M = (\{p, q\}, \Sigma, V, \Delta, p, \{q\})$
 - $\Sigma = \{a, b\}$
 - $V = \{S, a, b\}$
 - $\Delta = \{((p, e, e), (q, S)), ((q, e, S), (q, aSa)), ((q, e, S), (q, bSb)), ((q, e, S), (q, e)), ((q, a, a), (q, e))\}$

CFG → PDA: Example 2

- Step c2: introduce a new arc for $b \in \Sigma$

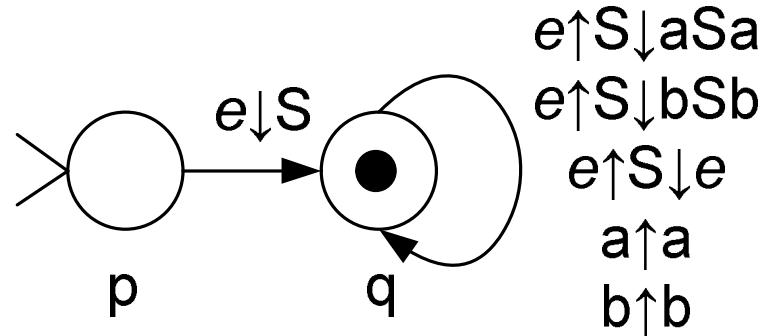
- $V = \{a, b, S\}$
- $\Sigma = \{a, \mathbf{b}\}$
- $S = S$
- $R = \{S \rightarrow aSa \mid bSb \mid e\}$



- $M = (\{p, q\}, \Sigma, V, \Delta, p, \{q\})$
 - $\Sigma = \{a, b\}$
 - $V = \{S, a, b\}$
 - $\Delta = \{((p, e, e), (q, S)), ((q, e, S), (q, aSa)), ((q, e, S), (q, bSb)), ((q, e, S), (q, e)), ((q, a, a), (q, e)), ((q, \mathbf{b}, \mathbf{b}), (q, e))\}$

CFG → PDA: Example 2

- Example: abba
 - $S \Rightarrow aSa \Rightarrow abSba \Rightarrow abba$
 - $(p, abba, e) \vdash (q, abba, S) \vdash (q, abba, aSa) \vdash (q, bba, Sa) \vdash (q, bba, bSba) \vdash (q, ba, Sba) \vdash (q, ba, ba) \vdash (q, a, a) \vdash (q, e, e)$

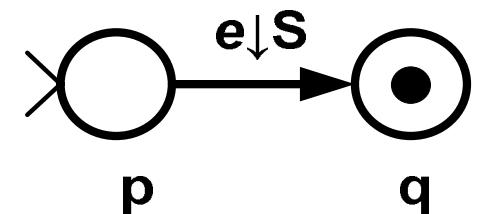


CFG → PDA: Example 3

- Construct PDA M such that $L(M) = L(G)$!
 - $V = \{a, b, c, d, S\}$
 - $\Sigma = \{a, b, c, d\}$
 - $S = S$
 - $R = \{S \rightarrow aSa \mid bSb \mid cSc \mid d\}$

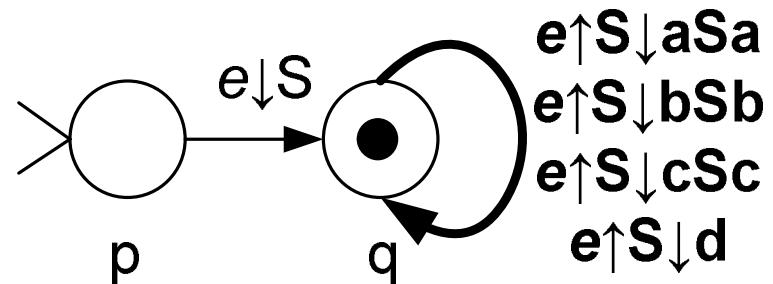
CFG → PDA: Example 3

- Step a: perform the initial steps
 - $V = \{a, b, c, d, S\}$
 - $\Sigma = \{a, b, c, d\}$
 - $S = S$
 - $R = \{S \rightarrow aSa \mid bSb \mid cSc \mid d\}$



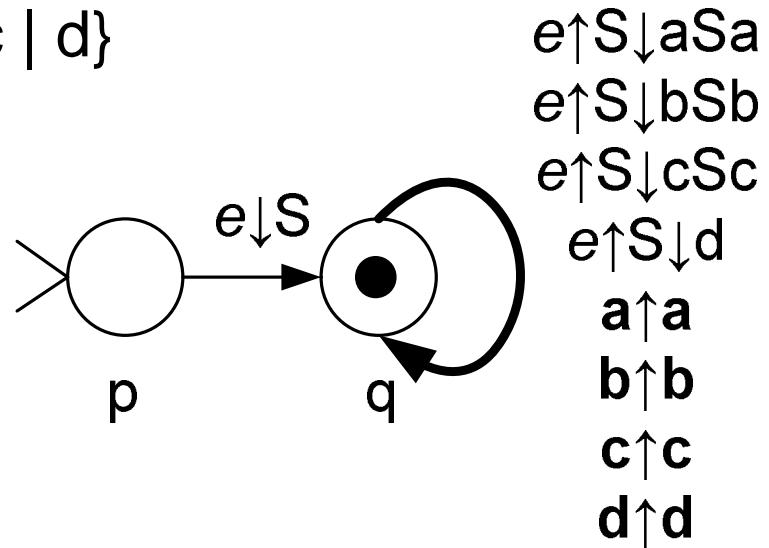
CFG → PDA: Example 3

- Step b: introduce new arcs for $S \rightarrow aSa | bSb | cSc | d$
 - $V = \{a, b, c, d, S\}$
 - $\Sigma = \{a, b, c, d\}$
 - $S = S$
 - $R = \{S \rightarrow aSa | bSb | cSc | d\}$



CFG → PDA: Example 3

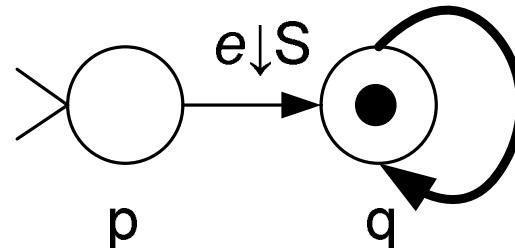
- Step c: introduce a new arcs for elements of Σ
 - $V = \{a, b, c, d, S\}$
 - $\Sigma = \{a, b, c, d\}$
 - $S = S$
 - $R = \{S \rightarrow aSa \mid bSb \mid cSc \mid d\}$



CFG → PDA: Example 3

- Example: abcdcba

- $S \Rightarrow aSa \Rightarrow abSba \Rightarrow abcScba \Rightarrow abcdcba$
 - $(p, abcdcba, e) \vdash (q, abcdcba, S) \vdash (q, abcdcba, aSa) \vdash (q, bcdcba, Sa) \vdash (q, bcdcba, bSba) \vdash (q, cdcba, Sba) \vdash (q, cdcba, cScba) \vdash (q, dcba, Scba) \vdash (q, dcba, dcba) \vdash (q, cba, cba) \vdash (q, ba, ba) \vdash (q, a, a) \vdash (q, e, e)$



$e \uparrow S \downarrow aSa$
 $e \uparrow S \downarrow bSb$
 $e \uparrow S \downarrow cSc$
 $e \uparrow S \downarrow d$
 $a \uparrow a$
 $b \uparrow b$
 $c \uparrow c$
 $d \uparrow d$

CFG → PDA: Example 4

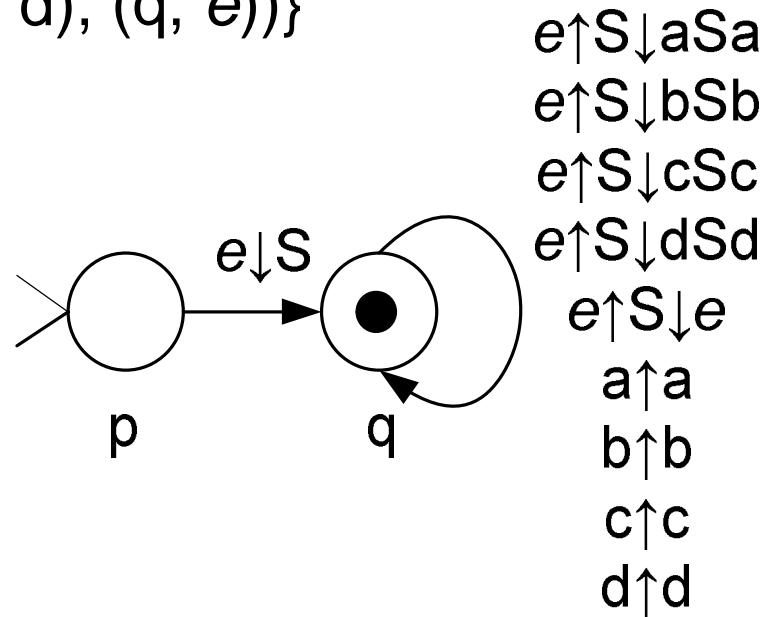
- Construct PDA M such that $L(M) = L(G)$! Use the set notations!
 - $V = \{a, b, c, d, S\}$
 - $\Sigma = \{a, b, c, d\}$
 - $S = S$
 - $R = \{S \rightarrow aSa \mid bSb \mid cSc \mid dSd \mid e\}$

CFG → PDA: Example 4

- $R = \{S \rightarrow aSa \mid bSb \mid cSc \mid dSd \mid e\}$
- $M = (\{p, q\}, \Sigma, V, \Delta, p, \{q\})$
 - $\Sigma = \{a, b, c, d, S\}$
 - $V = \{ S, a, b, c, d\}$
 - $\Delta = \{((p, e, e), (q, S)), ((q, e, S), (q, aSa)), ((q, e, S), (q, bSb)), ((q, e, S), (q, cSc)), ((q, e, S), (q, dSd)), ((q, e, S), (q, e)), ((q, a, a), (q, e)), ((q, b, b), (q, e)), ((q, c, c), (q, e)), ((q, d, d), (q, e))\}$

CFG → PDA: Example 4

- Give the state diagram for M!
 - $\Delta = \{((p, e, e), (q, S)), ((q, e, S), (q, aSa)), ((q, e, S), (q, bSb)), ((q, e, S), (q, cSc)), ((q, e, S), (q, dSd)), ((q, e, S), (q, e)), ((q, a, a), (q, e)), ((q, b, b), (q, e)), ((q, c, c), (q, e)), ((q, d, d), (q, e))\}$



CFG → PDA: Example 4

- Example: abcddcba
 - $S \Rightarrow aSa \Rightarrow abSba \Rightarrow abcScba \Rightarrow abcdSdcba \Rightarrow abcddcba$
 - $(p, abcddcba, e) \vdash (q, abcddcba, S) \vdash (q, abcddcba, aSa) \vdash (q, bcddcba, Sa) \vdash (q, bcddcba, bSba) \vdash (q, cddcba, Sba) \vdash (q, cddcba, cScba) \vdash (q, ddcba, Scba) \vdash (q, ddcba, dSdcba) \vdash (q, dcba, Sdcba) \vdash (q, dcba, dcba) \vdash (q, cba, cba) \vdash (q, ba, ba) \vdash (q, a, a) \vdash (q, e, e)$

Element of the Theory of the Computation

Lecture 10

Example 1: The T_{NEG} machine

- Create TM M which negates a binary number!
 - $\Sigma = \{\triangleright, \sqcup, 0, 1\}$
 - e.g.: $\triangleright \sqcup 1001 \rightarrow \triangleright \sqcup 0110 \sqcup$

Example 1: The T_{NEG} machine

- States:
 - q_0 : start
 - q_1 : negate
 - q_2 : move forward
 - h_1 : halt

Example 1: The T_{NEG} machine

- Transitions:
 - unused transitions are not displayed

q	σ	$\delta(q, \sigma)$
q_0	\sqcup	(q_1, \rightarrow)
q_1	0	$(q_2, 1)$
q_1	1	$(q_2, 0)$
q_2	0 or 1	(q_1, \rightarrow)
q_1	\sqcup	(h_1, \sqcup)

States:

- q_0 : start
- q_1 : negate
- q_2 : move forward
- h_1 : halt

Example 1: The T_{NEG} machine

- Computation for $\triangleright \underline{\sqcup} 0101 \sqcup$ without the yield in one step signs:
 - $(q_0, \triangleright \underline{\sqcup} 0101 \sqcup)$
 - $(q_1, \triangleright \sqcup \underline{0101} \sqcup)$
 - $(q_2, \triangleright \sqcup \underline{1101} \sqcup)$
 - $(q_1, \triangleright \sqcup \underline{1101} \sqcup)$
 - $(q_2, \triangleright \sqcup \underline{1001} \sqcup)$
 - $(q_1, \triangleright \sqcup \underline{1001} \sqcup)$
 - $(q_2, \triangleright \sqcup \underline{1011} \sqcup)$
 - $(q_1, \triangleright \sqcup \underline{1011} \sqcup)$
 - $(q_2, \triangleright \sqcup \underline{1010} \sqcup)$
 - $(q_1, \triangleright \sqcup \underline{1010} \sqcup)$
 - $(h_1, \triangleright \sqcup \underline{1010} \sqcup)$

q	σ	$\delta(q, \sigma)$
q_0	\sqcup	(q_1, \rightarrow)
q_1	0	$(q_2, 1)$
q_1	1	$(q_2, 0)$
q_2	0 or 1	(q_1, \rightarrow)
q_1	\sqcup	(h_1, \sqcup)

Example 2: The L_{\sqcup} machine

- Create TM M, which scans for the first \sqcup to the left!
 - $\Sigma = \{\triangleright, \sqcup, a, b, c\}$
 - e.g.: $\triangleright \sqcup a a c b b \underline{\sqcup} \rightarrow \triangleright \underline{\sqcup} a a c b b$
 - this TM never stops if there is no \sqcup to the left

Example 2: The L_U machine

- States:
 - q_0 : start
 - q_1 : read
 - h_1 : halt

Example 2: The L_{\sqcup} machine

- Transitions:

q	σ	$\delta(q, \sigma)$
q_0	\sqcup	(q_1, \leftarrow)
q_1	a	(q_1, \leftarrow)
q_1	b	(q_1, \leftarrow)
q_1	c	(q_1, \leftarrow)
q_1	\sqcup	(h_1, \sqcup)

States:

- q_0 : start
- q_1 : read
- h_1 : halt

Example 2: The L_{\sqcup} machine

- Computation for $\triangleright \sqcup abc \sqcup$:

- $(q_0, \triangleright \sqcup abc \sqcup)$
- $(q_1, \triangleright \sqcup ab \underline{c} \sqcup)$
- $(q_1, \triangleright \sqcup ab \underline{c} \sqcup)$
- $(q_1, \triangleright \sqcup \underline{a} bc \sqcup)$
- $(q_1, \triangleright \sqcup abc \sqcup)$
- $(h_1, \triangleright \sqcup abc \sqcup)$

q	σ	$\delta(q, \sigma)$
q_0	\sqcup	(q_1, \rightarrow)
q_1	x	(q_1, \rightarrow)
q_1	y	(q_1, \rightarrow)
q_1	\sqcup	(q_1, \rightarrow)
q_1	z	(h_1, z)

Example 3: The R_z machine

- Create TM M, which scans for the first z to the right!
 - $\Sigma = \{\triangleright, \sqcup, x, y, z\}$
 - e.g.: $\triangleright \sqcup xy \sqcup x \sqcup yzxx \rightarrow \triangleright \sqcup xy \sqcup x \sqcup y \underline{z} xx$
 - this TM never stops if there is no z to the right

Example 3: The R_z machine

- States:
 - q_0 : start
 - q_1 : read
 - h_1 : halt

Example 3: The R_z machine

- Transitions:

q	σ	$\delta(q, \sigma)$
q_0	\sqcup	(q_1, \rightarrow)
q_1	x	(q_1, \rightarrow)
q_1	y	(q_1, \rightarrow)
q_1	\sqcup	(q_1, \rightarrow)
q_1	z	(h_1, z)

States:

- q_0 : start
- q_1 : read
- h_1 : halt

Example 3: The R_z machine

- Computation for $\triangleright \underline{L}xyzx$:

- $(q_0, \triangleright \underline{L}xyzx)$
- $(q_1, \triangleright \underline{L}xyzx)$
- $(q_1, \triangleright \underline{L}xyzx)$
- $(q_1, \triangleright \underline{L}xy\underline{z}x)$
- $(h_1, \triangleright \underline{L}xy\underline{z}x)$

q	σ	$\delta(q, \sigma)$
q_0	\sqcup	(q_1, \rightarrow)
q_1	x	(q_1, \rightarrow)
q_1	y	(q_1, \rightarrow)
q_1	\sqcup	(q_1, \rightarrow)
q_1	z	(h_1, z)

Example 4: The c machine

- Create TM M, which writes c to the actual position!
 - $\Sigma = \{\triangleright, \sqcup, a, b, c\}$
 - e.g.: $\triangleright \sqcup aa\underline{ab}b \rightarrow \triangleright \sqcup aac\underline{cb}b$

Example 4: The c machine

- States:
 - q_0 : start
 - h_1 : halt

Example 4: The c machine

- Transitions:

q	σ	$\delta(q, \sigma)$
q_0	a	(h_1 , c)
q_0	b	(h_1 , c)
q_0	c	(h_1 , c)

States:

- q_0 : start
- h_1 : halt

Example 4: The c machine

- Computation for $\triangleright aab\underline{b}cc$:

- $(q_0, \triangleright aab\underline{b}cc)$
- $(h_1, \triangleright aac\underline{b}cc)$

q	σ	$\delta(q, \sigma)$
q_0	a	(h_1, c)
q_0	b	(h_1, c)
q_0	c	(h_1, c)

Example 5: Create the RR machine

- Create TM M, which steps two positions to the right!
 - $\Sigma = \{\triangleright, \sqcup, a, b\}$
 - e.g.: $\triangleright \sqcup aa\underline{ab}b \rightarrow \triangleright \sqcup aaabb\underline{b}$

Example 5: Create the RR machine

- States:
 - q_0 : two more right
 - q_1 : one more right
 - h_1 : halt

Example 5: Create the RR machine

- Transitions:

q	σ	$\delta(q, \sigma)$
q_0	\triangleright	(q_1, \rightarrow)
q_0	\sqcup	(q_1, \rightarrow)
q_0	a	(q_1, \rightarrow)
q_0	b	(q_1, \rightarrow)
q_1	a	(h_1, \rightarrow)
q_1	b	(h_1, \rightarrow)
q_1	\sqcup	(h_1, \rightarrow)

States:

- q_0 : two more right
- q_1 : one more right
- h_1 : halt

Example 5: Create the RR machine

- Computation for $\triangleright \sqcup aba$:

- $(q_0, \triangleright \sqcup aba)$
- $(q_1, \triangleright \sqcup aba)$
- $(h_1, \triangleright \sqcup aba)$

q	σ	$\delta(q, \sigma)$
q_0	\triangleright	(q_1, \rightarrow)
q_0	\sqcup	(q_1, \rightarrow)
q_0	a	(q_1, \rightarrow)
q_0	b	(q_1, \rightarrow)
q_1	a	(h_1, \rightarrow)
q_1	b	(h_1, \rightarrow)
q_1	\sqcup	(h_1, \rightarrow)

- Computation $\triangleright \sqcup aba_\underline{a}$:

- $(q_0, \triangleright \sqcup aba_\underline{a})$
- $(q_1, \triangleright \sqcup aba_\underline{a})$
- $(h_1, \triangleright \sqcup aba_\underline{a})$

Example 6: Create the La machine

- Create TM M, which steps to the left and writes a!
 - $\Sigma = \{\triangleright, \sqcup, a, b\}$
 - e.g.: $\triangleright \sqcup ab\underline{abb} \rightarrow \triangleright \sqcup ab\underline{abb} \rightarrow \triangleright \sqcup aa\underline{abb}$
- What should we do on \triangleright ?
 - step to right and halt

Example 6: Create the La machine

- States:
 - q_0 : go left
 - q_1 : write 'a'
 - h_1 : halt

Example 6: Create the La machine

- Transitions:

q	σ	$\delta(q, \sigma)$
q_0	\triangleright	(h_1, \rightarrow)
q_0	\sqcup	(q_1, \leftarrow)
q_0	a	(q_1, \leftarrow)
q_0	b	(q_1, \leftarrow)
q_1	\triangleright	(h_1, \rightarrow)
q_1	a	(h_1, a)
q_1	b	(h_1, a)
q_1	\sqcup	(h_1, a)

States:

- q_0 : go left
- q_1 : write 'a'
- h_1 : halt

Example 6: Create the La machine

- Computation for $\triangleright \sqcup \underline{abb}$:
 - $(q_0, \triangleright \sqcup \underline{abb})$
 - $(q_1, \triangleright \sqcup \underline{abb})$
 - $(h_1, \triangleright \sqcup aab)$
- Computation for $\triangleright \sqcup \underline{abb}$:
 - $(q_0, \triangleright \sqcup \underline{abb})$
 - $(q_1, \triangleleft \sqcup \underline{abb})$
 - $(h_1, \triangleright \sqcup \underline{abb})$

q	σ	$\delta(q, \sigma)$
q_0	\triangleright	(h_1, \rightarrow)
q_0	\sqcup	(q_1, \leftarrow)
q_0	a	(q_1, \leftarrow)
q_0	b	(q_1, \leftarrow)
q_1	\triangleright	(h_1, \rightarrow)
q_1	a	(h_1, a)
q_1	b	(h_1, a)
q_1	\sqcup	(h_1, a)

Example 7: Create the RbR machine

- Create TM M, which steps to the right, writes b, and steps to right again!
 - $\Sigma = \{\triangleright, \sqcup, a, b\}$
 - e.g.: $\triangleright \sqcup a \underline{b} a a b \rightarrow \triangleright \sqcup a b \underline{a} a b \rightarrow \triangleright \sqcup a b b \underline{a} b$

Example 7: Create the RbR machine

- States:
 - q_0 : go right
 - q_1 : write 'b'
 - q_2 : go right
 - h_1 : halt

Example 7: Create the RbR machine

- Transitions:

q	σ	$\delta(q, \sigma)$
q_0	x	(q_1, \rightarrow)
q_1	y	(q_2, b)
q_2	y	(h_1, \rightarrow)

$$\begin{aligned}x &\in \{a, b, \triangleright, \sqcup\} \\y &\in \{a, b, \sqcup\}\end{aligned}$$

- States:
 - q_0 : go right
 - q_1 : write 'b'
 - q_2 : go right
 - h_1 : halt

Example 7: Create the RbR machine

- Computation for $\triangleright \sqcup \underline{aba}$:

- $(q_0, \triangleright \sqcup \underline{aba})$
- $(q_1, \triangleright \sqcup \underline{aba})$
- $(q_2, \triangleright \sqcup \underline{abb})$
- $(h_1, \triangleright \sqcup \underline{aab} \sqcup)$

q	σ	$\delta(q, \sigma)$
q_0	x	(q_1, \rightarrow)
q_1	y	(q_2, b)
q_2	y	(h_1, \rightarrow)

Example 8: Sort machine

- Create a TM M, which arranges all 'b' in a word to the beginning of the word!
 - $\Sigma = \{\triangleright, \sqcup, a, b\}$
 - e.g.: $\triangleright \sqcup abaab \rightarrow \triangleright \sqcup bbaaa \sqcup$

Example 8: Sort machine

- Logic of the operation:
 - find b to the right
 - move this b to the beginning part of the string by consecutively swapping b and the preceding a
 - find the next b
 - e.g.:
 - bbaaabaab, we are in the middle of the algorithm
 - bbaaabaab, b is found
 - bbaabaab, first swap
 - ...
 - bbaaaaab, last swap, b is at the final position
 - bbbaaaaab, start of the next cycle

Example 8: Sort machine

```
move to the right
while not blank
    if b is found
        move to the left
        while a is found
            swap a and b, head on b
            move to the left
        move to the right
        move to the right
    move to the right
```

- The transition function is not implemented exactly according to the algorithm

Example 8: Sort machine

- States:
 - q_i : initial state, we has to move to the right
 - q_0 : go to the right until b or \sqcup
 - q_1 : b is found
 - q_2 : $a \rightarrow b$ change is performed
 - q_3 : right move is done
 - q_4 : $b \rightarrow a$ change is performed
 - q_5 : the character before b was not 'a'
 - h_1 : halt

Example 8: Sort machine

- Transitions:

q	σ	$\delta(q, \sigma)$
q_i	\sqcup	(q_0, \rightarrow)
q_0	a	(q_0, \rightarrow)
q_0	\sqcup	(h_1, \sqcup)
q_0	b	(q_1, \leftarrow)
q_1	a	(q_2, b)
q_2	b	(q_3, \rightarrow)
q_3	b	(q_4, a)
q_4	a	(q_0, \leftarrow)
q_1	b	(q_5, \rightarrow)
q_1	\sqcup	(q_5, \rightarrow)
q_5	b	(q_0, \rightarrow)

States:

- q_i : initial state, we has to move to the right
- q_0 : go to the right until b or \sqcup
- q_1 : b is found
- q_2 : a \rightarrow b change is performed
- q_3 : right move is done
- q_4 : b \rightarrow a change is performed
- q_5 : the character before b was not 'a'
- h_1 : halt

It is not easy to represent and understand δ

- if the algorithm was followed then more states would be required

Example 8: Sort machine

- Computation:
 - $(q_i, \triangleright \underline{ababaa} \sqcup)$, search for b
 - $(q_0, \triangleright \underline{ababaa} \sqcup)$
 - $(q_0, \triangleright \underline{ababaa} \sqcup)$, b is found
 - $(q_1, \triangleright \underline{ababaa} \sqcup)$, character before b is checked
 - $(q_2, \triangleright \underline{bbabaa} \sqcup)$, a \rightarrow b
 - $(q_3, \triangleright \underline{bbabaa} \sqcup)$
 - $(q_4, \triangleright \underline{baabaa} \sqcup)$, b \rightarrow a
 - $(q_0, \triangleright \underline{baabaa} \sqcup)$, b is found
 - $(q_1, \triangleright \underline{baabaa} \sqcup)$, character before b is checked

Example 8: Sort machine

- $(q_5, \triangleright \sqcup \underline{baabaa} \sqcup)$, swap was not needed
- $(q_0, \triangleright \sqcup \underline{baabaa} \sqcup)$, first b is in position
...
- $(q_0, \triangleright \sqcup baab\underline{baa} \sqcup)$
- $(q_1, \triangleright \sqcup ba\underline{aab}baa \sqcup)$
- $(q_2, \triangleright \sqcup bab\underline{baa} \sqcup)$
- $(q_3, \triangleright \sqcup bab\underline{baa} \sqcup)$

Example 8: Sort machine

- $(q_4, \triangleright \sqcup baba\underline{aa}a \sqcup)$
- $(q_0, \triangleright \sqcup bab\underline{aaa}a \sqcup)$
- $(q_1, \triangleright \sqcup b\underline{abaaa}a \sqcup)$
- ...
- $(q_0, \triangleright \sqcup b\underline{b}aaaaa \sqcup)$
- $(q_1, \triangleright \sqcup \underline{b}baaaa \sqcup)$
- $(q_5, \triangleright \sqcup b\underline{b}aaaaa \sqcup)$
- $(q_0, \triangleright \sqcup bb\underline{aaa}aa \sqcup)$, second b is in position
 ...
- $(q_0, \triangleright \sqcup bbaaaa \sqcup)$, searching for b
- $(h_1, \triangleright \sqcup bbaaaa \sqcup)$

Turing Machines with 2 tape

- A Turing Machine can have 2 or more tapes
 - such a machine can be simulated with a regular TM
- A TM with two tapes has transition function like $\delta(q, a_1, a_2) = (p, b_1, b_2)$
 - if TM M is in state q and reads a_1 from the first tape, a_2 from the second tape, then it goes to state p and writes b_1 to the first tape, b_2 to the second tape

Example 9: Copy machine

- Create TM M with two tapes, which copies a word from the first tape to the second tape!
 - $\Sigma = \{\triangleright, \sqcup, a, b, c\}$
 - e.g.: $T_1: \triangleright \underline{\sqcup} abaab, T_2: \triangleright \underline{\sqcup} \rightarrow T_1: \triangleright \underline{\sqcup} abaab,$
 $T_2: \triangleright \sqcup abaab \underline{\sqcup}$

Example 9: Copy machine

- States:
 - q_i : we have to move to the right
 - q_0 : we moved to the right
 - q_1 : character is copied
 - h_1 : halt

Example 9: Copy machine

- Transitions:

q	σ_1	σ_2	$\delta(q, \sigma_1, \sigma_2)$
q_i	□	□	$(q_0, \rightarrow, \rightarrow)$
q_0	▷	▷	$(q_0, \rightarrow, \rightarrow)$
q_0	a	□	(q_1, a, a)
q_0	b	□	(q_1, b, b)
q_0	c	□	(q_1, c, c)
q_1	a	a	$(q_0, \rightarrow, \rightarrow)$
q_1	b	b	$(q_0, \rightarrow, \rightarrow)$
q_1	c	c	$(q_0, \rightarrow, \rightarrow)$
q_0	□	□	(h_1, \square, \square)

- States:

- q_i : we have to move to the right
- q_0 : we moved to the right
- q_1 : character is copied
- h_1 : halt

Example 9: Copy machine

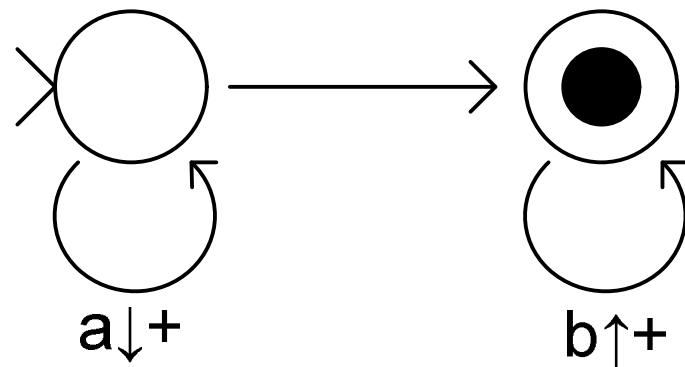
- Computation:

- $(q_i, \triangleright \underline{abc}\sqcup, \triangleright \underline{\underline{a}}\underline{\underline{b}}\underline{\underline{c}}\sqcup)$
- $(q_0, \triangleright \underline{a}\underline{bc}\sqcup, \triangleright \underline{\underline{a}}\underline{\underline{b}}\underline{\underline{c}}\sqcup)$
- $(q_1, \triangleright \underline{a}\underline{bc}\sqcup, \triangleright \underline{\underline{a}}\underline{\underline{a}}\underline{\underline{c}}\sqcup)$
- $(q_0, \triangleright \underline{a}\underline{ab}\underline{c}\sqcup, \triangleright \underline{\underline{a}}\underline{\underline{a}}\underline{\underline{c}}\sqcup)$
- ...
- $(q_1, \triangleright \underline{a}\underline{ab}\underline{c}\sqcup, \triangleright \underline{\underline{a}}\underline{\underline{ab}}\underline{\underline{c}}\sqcup)$
- $(q_0, \triangleright \underline{a}\underline{ab}\underline{c}\sqcup, \triangleright \underline{\underline{a}}\underline{\underline{ab}}\underline{\underline{c}}\sqcup)$
- $(h_1, \triangleright \underline{a}\underline{ab}\underline{c}\sqcup, \triangleright \underline{\underline{a}}\underline{\underline{ab}}\underline{\underline{c}}\sqcup)$

q	σ_1	σ_2	$\delta(q, \sigma_1, \sigma_2)$
q_i	\sqcup	\sqcup	$(q_0, \rightarrow, \rightarrow)$
q_0	\triangleright	\triangleright	$(q_0, \rightarrow, \rightarrow)$
q_0	a	\sqcup	(q_1, a, a)
q_0	b	\sqcup	(q_1, b, b)
q_0	c	\sqcup	(q_1, c, c)
q_1	a	a	$(q_0, \rightarrow, \rightarrow)$
q_1	b	b	$(q_0, \rightarrow, \rightarrow)$
q_1	c	c	$(q_0, \rightarrow, \rightarrow)$
q_0	\sqcup	\sqcup	(h_1, \sqcup, \sqcup)

Example 10: Simulate PDA with TM

- Create such a TM that simulates PDA M!
 - PDA M accepts $L(M) = \{w \in a^n b^n\}$
 - e.g.: $\triangleright \underline{aabb} \rightarrow h_1$ (accepting halting state)
- PDA:
 - '+' symbols indicates the number of 'a'



Example 10: Simulate PDA with TM

- The TM requires two tapes
 - tape 1: input tape, we move only to the right on this tape
 - tape 2: stack, '+' symbols indicates the number of 'a'

Example 10: Simulate PDA with TM

- States:
 - q_i : we have to move to the right
 - q_0 : input has to be read, '+' is pushed
 - q_1 : 'a' is read
 - q_2 : pop is finished
 - q_3 : 'b' is read, '+' is deleted
 - h_1 : string accepted
 - h_2 : $|b| > |a|$
 - h_3 : $|a| > |b|$
 - h_4 : 'a' is read after 'b'
- A state can be reached after different actions, different actions can be performed after a state, thus a state can have different names

Example 10: Simulate PDA with TM

- Transitions:

q	σ_1	σ_2	$\delta(\sigma_1, \sigma_2)$
q_i	\sqcup	\sqcup	$(q_0, \rightarrow, \rightarrow)$
q_0	a	\sqcup	$(q_1, \rightarrow, \sqcup)$
q_0	a	+	$(q_1, \rightarrow, \rightarrow)$
q_1	a	\sqcup	$(q_0, a, +)$
q_1	b	\sqcup	$(q_0, b, +)$
q_2	b	+	$(q_3, \rightarrow, \sqcup)$
q_3	b	\sqcup	(q_2, b, \leftarrow)
q_3	\sqcup	\sqcup	$(q_2, \sqcup, \leftarrow)$

States:

- q_i : we have to move to the right
- q_0 : input has to be read, '+' is pushed
- q_1 : 'a' is read
- q_2 : pop is finished
- q_3 : 'b' is read, '+' is deleted
- h_1 : string accepted
- h_2 : $|b| > |a|$
- h_3 : $|a| > |b|$
- h_4 : 'a' is read after 'b'

Example 10: Simulate PDA with TM

- Transitions:

q_2	\sqcup	\sqcup	(h_1, \sqcup, \sqcup)
q_0	\sqcup	\sqcup	(h_1, \sqcup, \sqcup)
q_0	b	\sqcup	(h_2, b, \sqcup)
q_2	b	\sqcup	(h_2, b, \sqcup)
q_2	\sqcup	$+$	$(h_3, \sqcup, +)$
q_2	a	$+$	$(h_4, a, +)$

States:

- q_i : we have to move to the right
- q_0 : input has to be read, ' $+$ ' is pushed
- q_1 : 'a' is read
- q_2 : pop is finished
- q_3 : 'b' is read, ' $+$ ' is deleted
- h_1 : string accepted
- h_2 : $|b| > |a|$
- h_3 : $|a| > |b|$
- h_4 : 'a' is read after 'b'

Example 10: Simulate PDA with TM

- Computation:
 - $(q_i, \triangleright \underline{\sqcup}aabb\sqcup, \triangleright \underline{\sqcup})$
 - $(q_0, \triangleright \sqcup\underline{aabb}\sqcup, \triangleright \sqcup\underline{\sqcup})$
 - $(q_1, \triangleright \sqcup\underline{aab}\underline{bb}\sqcup, \triangleright \sqcup\underline{\sqcup})$
 - $(q_0, \triangleright \sqcup\underline{aab}\underline{bb}\sqcup, \triangleright \sqcup\underline{+})$, push 0
 - $(q_1, \triangleright \sqcup\underline{aab}\underline{b}\sqcup, \triangleright \sqcup\underline{+}\underline{\sqcup})$
 - $(q_0, \triangleright \sqcup\underline{aab}\underline{b}\sqcup, \triangleright \sqcup\underline{+}\underline{+})$, push 2

q	σ_1	σ_2	$\delta(\sigma_1, \sigma_2)$
q_i	\sqcup	\sqcup	$(q_0, \rightarrow, \rightarrow)$
q_0	a	\sqcup	$(q_1, \rightarrow, \sqcup)$
q_0	a	+	$(q_1, \rightarrow, \rightarrow)$
q_1	a	\sqcup	$(q_0, a, +)$
q_1	b	\sqcup	$(q_0, b, +)$
q_2	b	+	$(q_3, \rightarrow, \sqcup)$
q_3	b	\sqcup	(q_2, b, \leftarrow)
q_3	\sqcup	\sqcup	$(q_2, \sqcup, \leftarrow)$
q_0	b	+	$(q_2, b, +)$

q_2	\sqcup	\sqcup	(h_1, \sqcup, \sqcup)
q_0	\sqcup	\sqcup	(h_1, \sqcup, \sqcup)
q_0	b	\sqcup	(h_2, b, \sqcup)
q_2	b	\sqcup	(h_2, b, \sqcup)
q_2	\sqcup	+	$(h_3, \sqcup, +)$
q_2	a	+	$(h_4, a, +)$
q_2	a	\sqcup	(h_4, a, \sqcup)

Example 10: Simulate PDA with TM

- Computation:
 - $(q_2, \triangleright \underline{aab} \underline{b} \sqcup, \triangleright \sqcup + \underline{\sqcup} \sqcup)$
 - $(q_3, \triangleright \underline{aab} \underline{b} \sqcup, \triangleright \sqcup + \underline{\sqcup} \sqcup)$
 - $(q_2, \triangleright \underline{aab} \underline{b} \sqcup, \triangleright \sqcup \underline{\sqcup} \sqcup \sqcup)$, pop 1
 - $(q_3, \triangleright \underline{aab} \underline{b} \sqcup, \triangleright \sqcup \underline{\sqcup} \sqcup \sqcup)$
 - $(q_2, \triangleright \underline{aab} \underline{b} \sqcup, \triangleright \underline{\sqcup} \sqcup \sqcup \sqcup)$, pop 2
 - $(h_1, \triangleright \underline{aab} \underline{b} \sqcup, \triangleright \underline{\sqcup} \sqcup \sqcup \sqcup)$

q	σ_1	σ_2	$\delta(\sigma_1, \sigma_2)$
q_i	\sqcup	\sqcup	$(q_0, \rightarrow, \rightarrow)$
q_0	a	\sqcup	$(q_1, \rightarrow, \sqcup)$
q_0	a	+	$(q_1, \rightarrow, \rightarrow)$
q_1	a	\sqcup	$(q_0, a, +)$
q_1	b	\sqcup	$(q_0, b, +)$
q_2	b	+	$(q_3, \rightarrow, \sqcup)$
q_3	b	\sqcup	(q_2, b, \leftarrow)
q_3	\sqcup	\sqcup	$(q_2, \sqcup, \leftarrow)$
q_0	b	+	$(q_2, b, +)$

q_2	\sqcup	\sqcup	(h_1, \sqcup, \sqcup)
q_0	\sqcup	\sqcup	(h_1, \sqcup, \sqcup)
q_0	b	\sqcup	(h_2, b, \sqcup)
q_2	b	\sqcup	(h_2, b, \sqcup)
q_2	\sqcup	+	$(h_3, \sqcup, +)$
q_2	a	+	$(h_4, a, +)$
q_2	a	\sqcup	(h_4, a, \sqcup)

Example 11: The T_{ADD} machine

- Create TM M with two tapes, which adds 2 binary numbers!
 - $\Sigma = \{\triangleright, \sqcup, 0, 1\}$
 - the binary numbers are written in reverse
 - the numbers should have the same length
 - use extra zeros if needed
 - the result should be on the first tape
 - e.g.: T1: 0111, T2: 1010 \rightarrow T1: 11001
 - $14_{10} + 5_{10} = 19_{10} \rightarrow 1110_2 + 0101_2 = 10011_2$

Example 11: The T_{ADD} machine

- States:
 - q_i : start
 - q_0 : add, no carry
 - q_1 : move right, no carry
 - q_2 : add, carry
 - q_3 : move right, carry
 - h_1 : halt

Example 11: The T_{ADD} machine

- Transitions:

q	σ_1	σ_2	$\delta(q, \sigma_1, \sigma_2)$
q_i	◻	◻	$(q_0, \rightarrow, \rightarrow)$
q_0	0	0	$(q_1, 0, 0)$
q_0	0	1	$(q_1, 1, 1)$
q_0	1	0	$(q_1, 1, 0)$
q_0	1	1	$(q_3, 0, 1)$
q_2	0	0	$(q_1, 1, 0)$
q_2	0	1	$(q_3, 0, 1)$
q_2	1	0	$(q_3, 0, 0)$
q_2	1	1	$(q_3, 1, 1)$

States:

- q_i : start
- q_0 : add, no carry
- q_1 : move right, no carry
- q_2 : add, carry
- q_3 : move right, carry
- h_1 : halt

Example 11: The T_{ADD} machine

- Transitions:

q_1	0 or 1	0 or 1	$(q_0, \rightarrow, \rightarrow)$
q_3	0 or 1	0 or 1	$(q_2, \rightarrow, \rightarrow)$
q_2	\sqcup	\sqcup	$(h_1, 1, \sqcup)$
q_0	\sqcup	\sqcup	(h_1, \sqcup, \sqcup)

States:

- q_i : start
- q_0 : add, no carry
- q_1 : move right, no carry
- q_2 : add, carry
- q_3 : move right, carry
- h_1 : halt

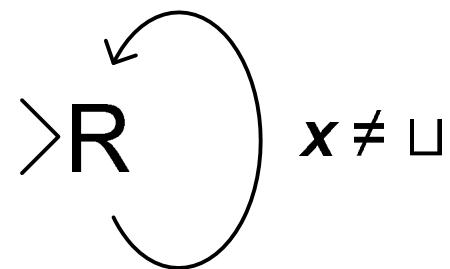
Example 11: The T_{ADD} machine

- $(q_i, \triangleright \underline{0}111, \triangleright \underline{1}010)$
- $(q_0, \triangleright \underline{0}111, \triangleright \underline{1}010) // 0 + 1$
- $(q_1, \triangleright \underline{1}111, \triangleright \underline{1}010)$
- $(q_0, \triangleright \underline{1}111, \triangleright \underline{1}010) // 1 + 0$
- $(q_1, \triangleright \underline{1}111, \triangleright \underline{1}010)$
- $(q_0, \triangleright \underline{1}111, \triangleright \underline{1}010) // 1 + 1$
- $(q_3, \triangleright \underline{1}1\underline{0}1, \triangleright \underline{1}010)$
- $(q_2, \triangleright \underline{1}10\underline{1}, \triangleright \underline{1}010) // 1 + 0 + \text{carry}$
- $(q_3, \triangleright \underline{1}10\underline{0}, \triangleright \underline{1}010)$
- $(q_2, \triangleright \underline{1}100\underline{\sqcup}, \triangleright \underline{1}010\underline{\sqcup}) // \text{carry}$
- $(h_1, \triangleright \underline{1}100\underline{1}, \triangleright \underline{1}010\underline{\sqcup}) // \text{halt}$

q	σ_1	σ_2	$\delta(q, \sigma_1, \sigma_2)$
q_i	\sqcup	\sqcup	$(q_0, \rightarrow, \rightarrow)$
q_0	0	0	$(q_1, 0, 0)$
q_0	0	1	$(q_1, 1, 1)$
q_0	1	0	$(q_1, 1, 0)$
q_0	1	1	$(q_3, 0, 1)$
q_2	0	0	$(q_1, 1, 0)$
q_2	0	1	$(q_3, 0, 1)$
q_2	1	0	$(q_3, 0, 0)$
q_2	1	1	$(q_3, 1, 1)$
q_1	0 or 1	0 or 1	$(q_0, \rightarrow, \rightarrow)$
q_3	0 or 1	0 or 1	$(q_2, \rightarrow, \rightarrow)$
q_2	\sqcup	\sqcup	$(h_1, 1, \sqcup)$
q_0	\sqcup	\sqcup	(h_1, \sqcup, \sqcup)

Example 12: The R_{\sqcup} machine

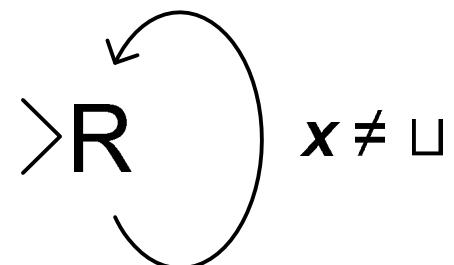
- Create a machine schema, which scans for the first \sqcup to the right!



Example 12: The R_{\sqcup} machine

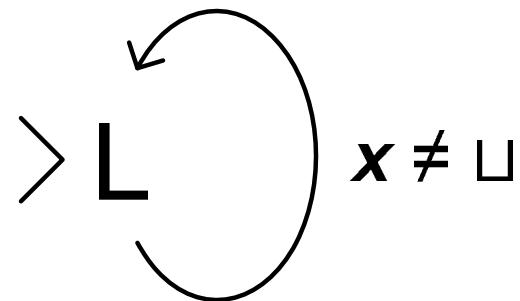
- Computation for $\sqcup abc$:

- $\sqcup abc$
- $\sqcup \underline{a}bc$
- $\sqcup a\underline{b}c$
- $\sqcup ab\underline{c}$
- $\sqcup abc\underline{\sqcup}$



Example 13: The L_{\sqcup} machine

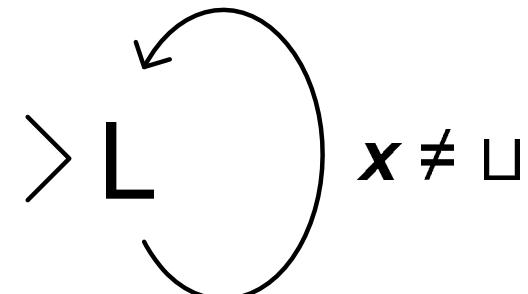
- Create a machine schema, which scans for the first \sqcup to the left!
 - this TM never stops if there is no \sqcup to the left



Example 13: The L_{\sqcup} machine

- Computation for $\sqcup abc \sqcup$:

- $\sqcup abc \sqcup$
 - $\sqcup abc \underline{c}$
 - $\sqcup ab\underline{c}c$
 - $\sqcup a\underline{bc}c$
 - $\sqcup abc \underline{c}$

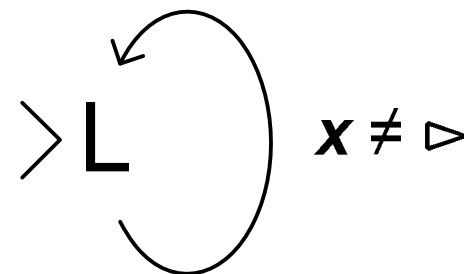


- Computation for $\triangleright abc$:

- $\triangleright a\underline{bc}$
 - $\triangleright \underline{abc}$
 - $\triangleright abc$
 - $\triangleright \underline{abc}$
 - $\triangleright abc$ // loop again

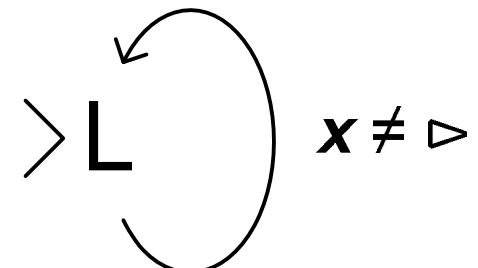
Example 14: The L_{\triangleright} machine

- Create a machine, which goes to the beginning of the tape!
 - this machine never stops



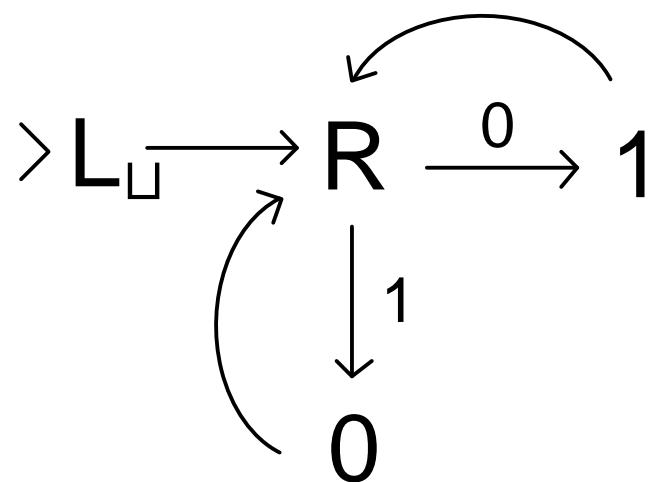
Example 14: The L_{\triangleright} machine

- Computation for $\triangleright \sqcup 012$:
 - $\triangleright \sqcup \underline{012}$
 - $\triangleright \sqcup \underline{012}$
 - $\triangleright \underline{\sqcup} 012$
 - $\triangleright \sqcup 012$
 - $\triangleright \underline{\sqcup} 012$
 - $\triangleright \underline{\sqcup} 012$ // loop again



Example 15: The T_{NEG} machine

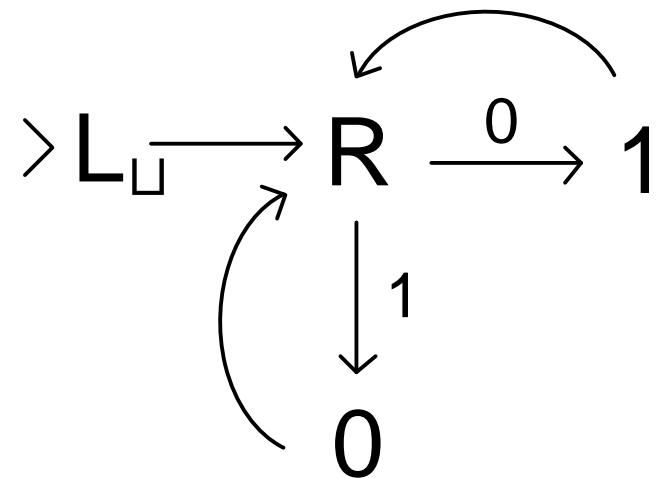
- Create a machine schema, which negates a binary number!
 - $\Sigma = \{\triangleright, \sqcup, 0, 1\}$
 - e.g.: $\sqcup 0111 \sqcup \rightarrow \sqcup 1000 \sqcup$



Example 15: The T_{NEG} machine

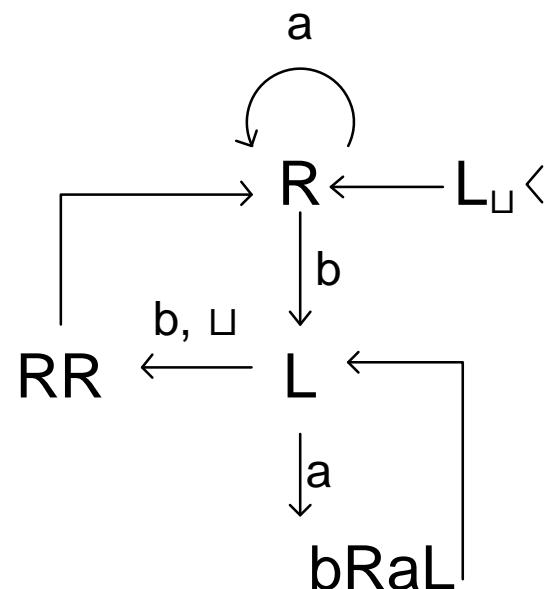
- Computation for $\sqcup 0101 \sqcup$:

- $\sqcup 0101 \sqcup$
- $\sqcup 0101$
- $\sqcup 0101$
- $\sqcup 1101$
- $\sqcup 1101$
- $\sqcup 1001$
- $\sqcup 1001$
- $\sqcup 1011$
- $\sqcup 1011$
- $\sqcup 1010$
- $\sqcup 1010 \sqcup$



Example 16: Sort machine

- Create a machine schema, which arranges all 'b' in a word to the beginning of the word!
 - $\Sigma = \{\triangleright, \sqcup, a, b\}$
 - e.g.: $\sqcup abaab\sqcup \rightarrow \sqcup bbaaa\sqcup$

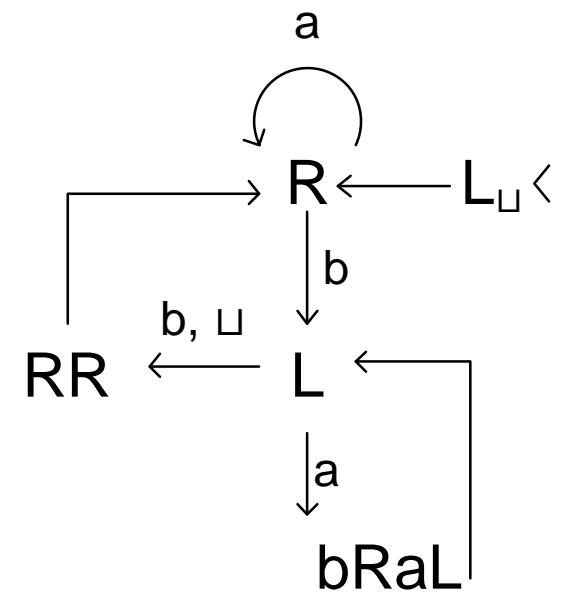


Example 16: Sort machine

- Logic of the operation:
 - find b to the right
 - move this b to the beginning part of the string by consecutively swapping b and the preceding a
 - find the next b to the right
 - e.g.:
 - bbaaabaab, we are in the middle of the algorithm
 - bbaaabaab, b is found
 - bbaabaab, first swap
 - ...
 - bbaaaaab, last swap, b is at the final position
 - bbbaaaaab, start again, find b to the right

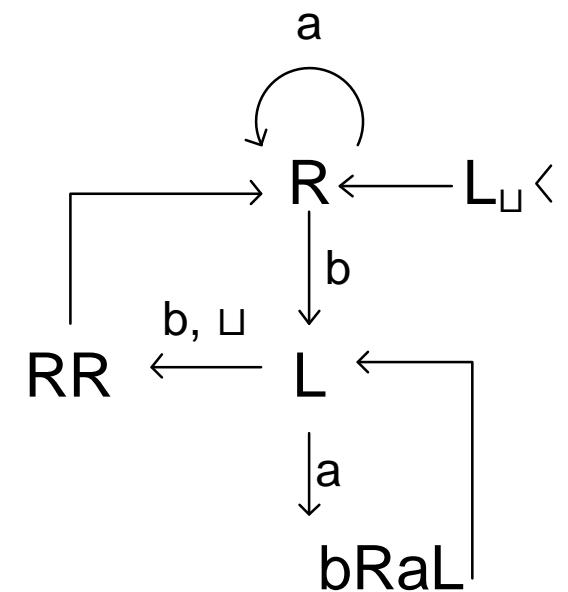
Example 16: Sort machine

move to the right
while not blank
 if b is found
 move to the left
 while a is found
 swap a and b, head on b
 move to the left
 move to the right
 move to the right
move to the right



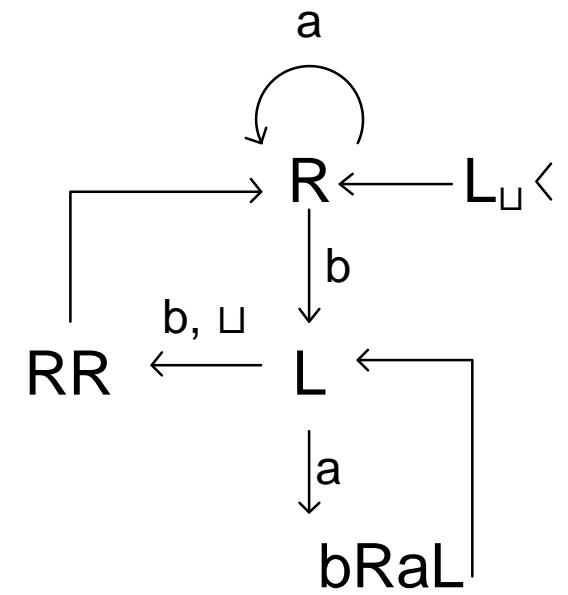
Example 16: Sort machine

- Computation for $\sqcup ababaa \sqcup$:
 - $\sqcup ababaa \sqcup$
 - $\sqcup ababaa$
 - $\sqcup \underline{ababaa}$
 - $\sqcup a \underline{babaa} // \downarrow b$
 - $\sqcup a \underline{babaa} // \downarrow a$
 - $\sqcup \underline{b}babaa$
 - $\sqcup b \underline{babaa}$
 - $\sqcup b \underline{aabaa}$
 - $\sqcup b \underline{aabaa}$
 - $\sqcup b \underline{aabaa} // \leftarrow b, \sqcup$; the schema goes to the left, the head goes to the right



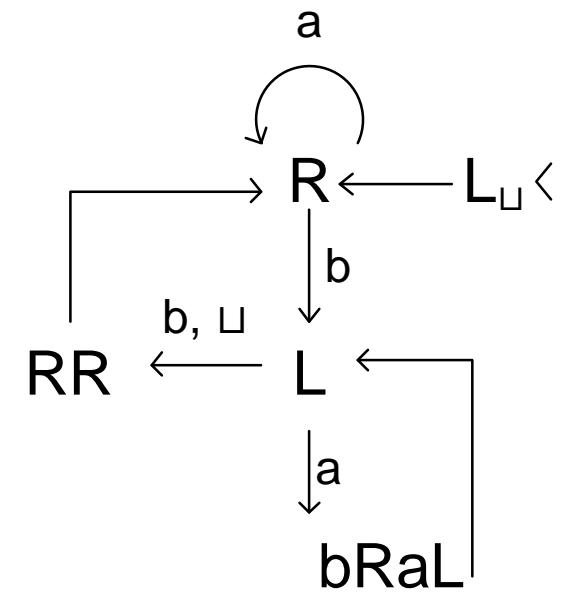
Example 16: Sort machine

- $\sqcup \underline{baabaa}$
- $\sqcup b \underline{aabaa}$
- $\sqcup ba \underline{abaa}$
- $\sqcup baab \underline{baa} // \downarrow b$
- $\sqcup baab \underline{baa} // \downarrow a$
- $\sqcup bab \underline{bbaa}$
- $\sqcup bab \underline{bbaa}$
- $\sqcup bab \underline{aaa}$
- $\sqcup b \underline{aba}aa // \downarrow a$
- $\sqcup b \underline{bb}aaa$



Example 16: Sort machine

- $\sqcup \underline{bbb}aaa$
- $\sqcup b\underline{b}aaaa$
- $\sqcup b\underline{ba}aaa$
- $\sqcup \underline{bba}aaa // \leftarrow b, \sqcup$
- $\sqcup b\underline{ba}aaa$
- $\sqcup b\underline{ba}aaa$
- $\sqcup bba\underline{aa}$
- $\sqcup bbaa\underline{aa}$
- $\sqcup bbaaa\underline{a}$
- $\sqcup bbaaaa\underline{\sqcup}$



Example 17: Copy machine ($C^{1 \rightarrow 2}$)

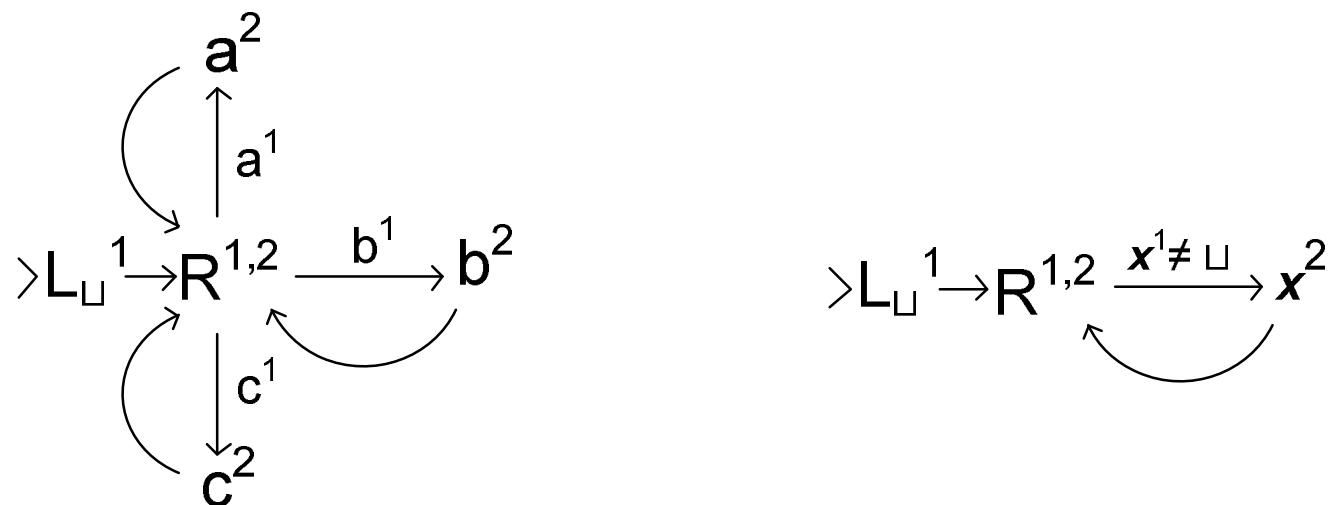
- Create a machine schema, which copies a word from the first tape to the second tape!
 - $\Sigma = \{\triangleright, \sqcup, a, b, c\}$
 - e.g.: $\sqcup bab \sqcup, \sqcup \rightarrow \sqcup bab \sqcup, \sqcup bab \sqcup$

Example 17: Copy machine ($C^{1 \rightarrow 2}$)

- Create a machine schema, which copies a word from the first tape to the second tape!
 - use superscript to denote that an elementary machine works on the first or the second tape
 - e.g.: R^1 – move to the right on tape 1
 - e.g.: $L^{1,2}$ – move to the left on both tapes
 - also use superscript on the labels of the arcs
 - e.g.: b^1c^2 – follow this arc if the head of the first tape reads b, and the head of the second tape reads c

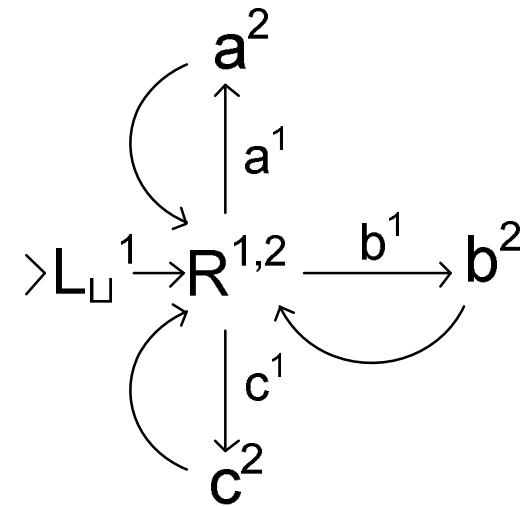
Example 17: Copy machine ($C^{1 \rightarrow 2}$)

- Create a machine schema, which copies a word from the first tape to the second tape!



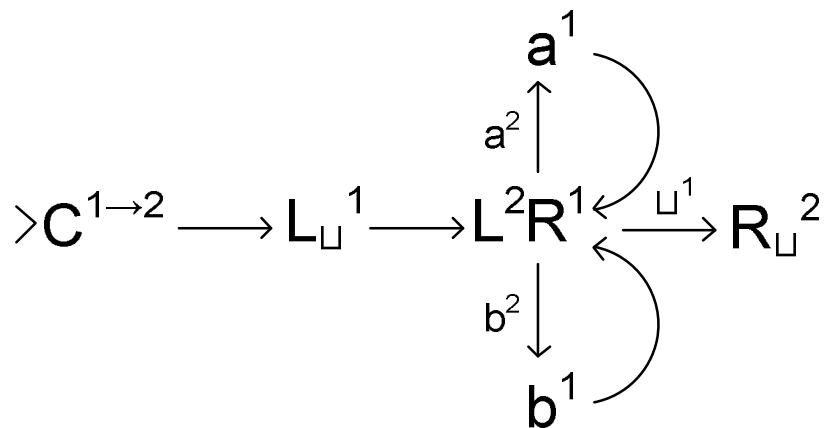
Example 17: Copy machine ($C^{1 \rightarrow 2}$)

- Computation for $\llbracket abc \rrbracket_U, \llbracket \cdot \rrbracket$:
 - $\llbracket abc \rrbracket_U, \llbracket \cdot \rrbracket$
 - $\llbracket abc \rrbracket_U, \llbracket \cdot \rrbracket$
 - $\llbracket abc \rrbracket_U, \llbracket \cdot \rrbracket$
 - $\llbracket abc \rrbracket_U, \llbracket a \rrbracket$
 - $\llbracket abc \rrbracket_U, \llbracket a \rrbracket$
 - $\llbracket abc \rrbracket_U, \llbracket a \rrbracket$
 - $\llbracket abc \rrbracket_U, \llbracket ab \rrbracket$
 - $\llbracket abc \rrbracket_U, \llbracket ab \rrbracket$
 - $\llbracket abc \rrbracket_U, \llbracket abc \rrbracket$
 - $\llbracket abc \rrbracket_U, \llbracket abc \rrbracket$



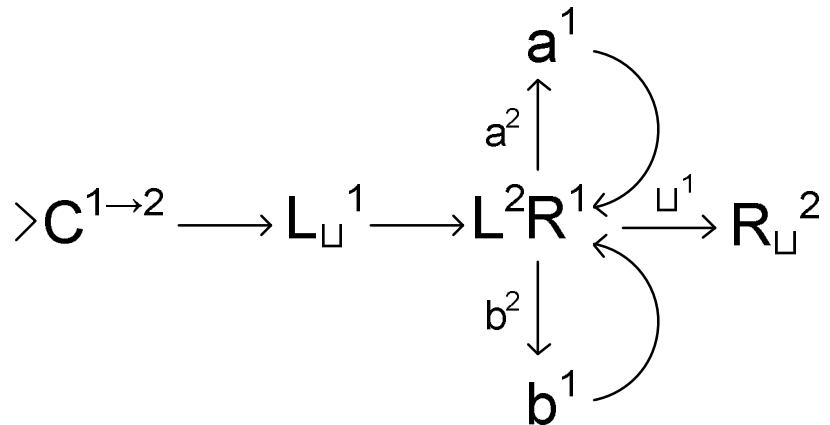
Example 18: Reverse string

- Create a machine schema, which reverses a string!
 - $\Sigma = \{\triangleright, \sqcup, a, b\}$
 - you can use two tapes
 - e.g.: $\sqcup abab\sqcup, \sqcup \rightarrow \sqcup baba\sqcup, \sqcup abab\sqcup$



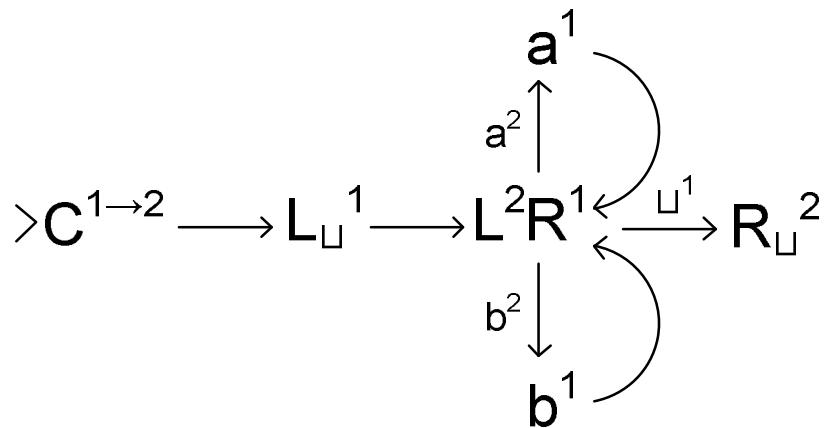
Example 18: Reverse string

- Computation for $\sqcup abab \sqcup$, \sqcup :
 - $\sqcup abab \sqcup, \sqcup$
 - $\sqcup abab \sqcup, \sqcup abab \sqcup$
 - $\sqcup abab, \sqcup abab \sqcup$
 - $\sqcup \underline{abab}, \sqcup abab \underline{}$
 - $\sqcup \underline{bab}, \sqcup abab \underline{}$
 - $\sqcup \underline{bab}, \sqcup \underline{abab}$
 - $\sqcup baab, \sqcup abab \underline{}$



Example 18: Reverse string

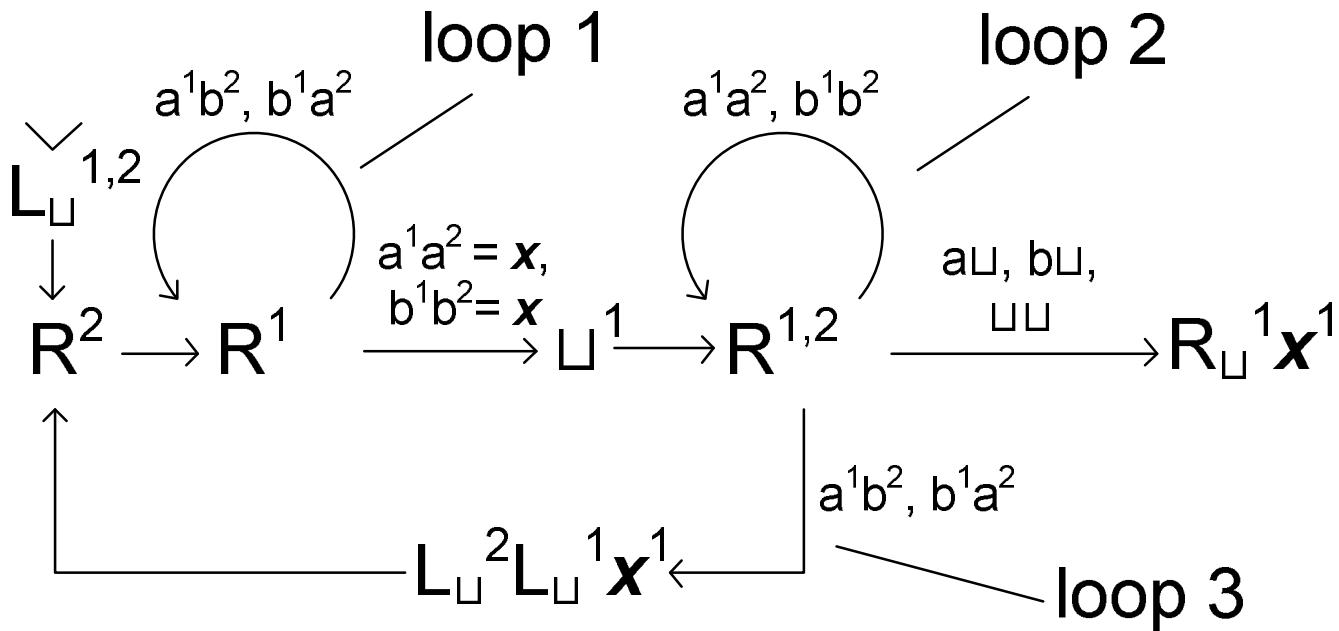
- $\sqcup ba\underline{ab}$, $\sqcup a\underline{bab}$
- $\sqcup babb\underline{b}$, $\sqcup a\underline{bab}$
- $\sqcup abab\underline{b}$, $\sqcup \underline{abab}$
- $\sqcup bab\underline{a}$, $\sqcup \underline{abab}$
- $\sqcup baba\underline{\sqcup}$, $\sqcup \underline{abab}$
- $\sqcup baba\underline{\sqcup}$, $\sqcup abab\underline{\sqcup}$



Example 19: Search substring

- Create a machine schema, which searches a substring in a string!
 - $\Sigma = \{\triangleright, \sqcup, a, b\}$
 - tape 1 contains the string, tape 2 contains the substring
 - the string and the substring cannot be empty
 - position the head on tape 1 to the beginning of the located substring
 - e.g.: $\sqcup a a b a a b \sqcup, \sqcup a b a \sqcup \rightarrow \sqcup a \underline{a} b a a b \sqcup, \sqcup a b a \sqcup$

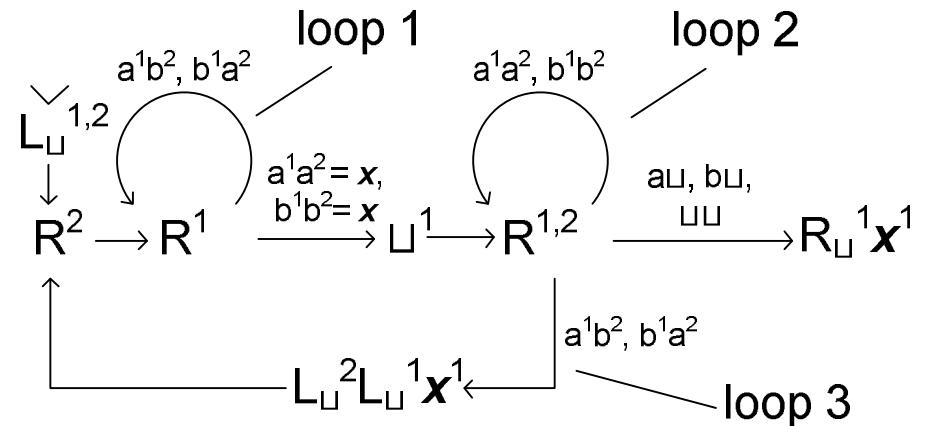
Example 19: Search substring



- Remark: x is a single character

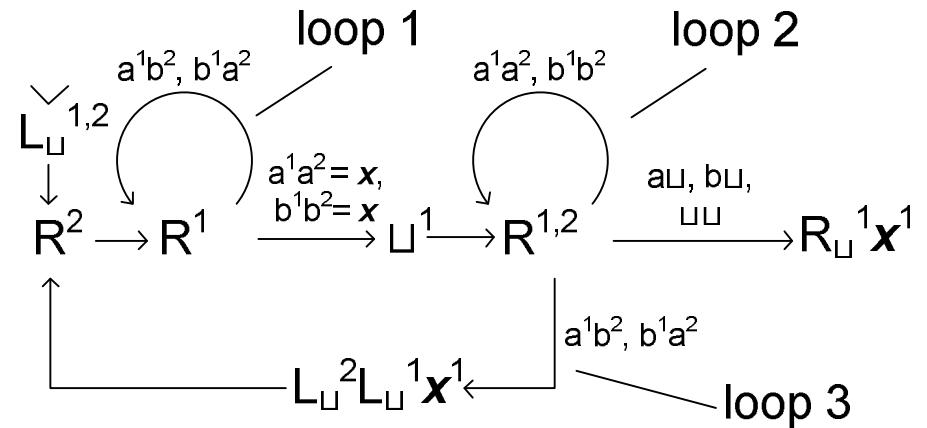
- Logic of the operation:

- loop 1 finds the first character of the substring (tape 2) in the string (tape 1)
- this position is marked by space
- loop 2 cycles while the string and the substring matches
- if the end of the substring is reached then the marked position on tape 1 is restored and the machine halts
- if there is a mismatch then loop 3 restores the marked position and the search is started after this position

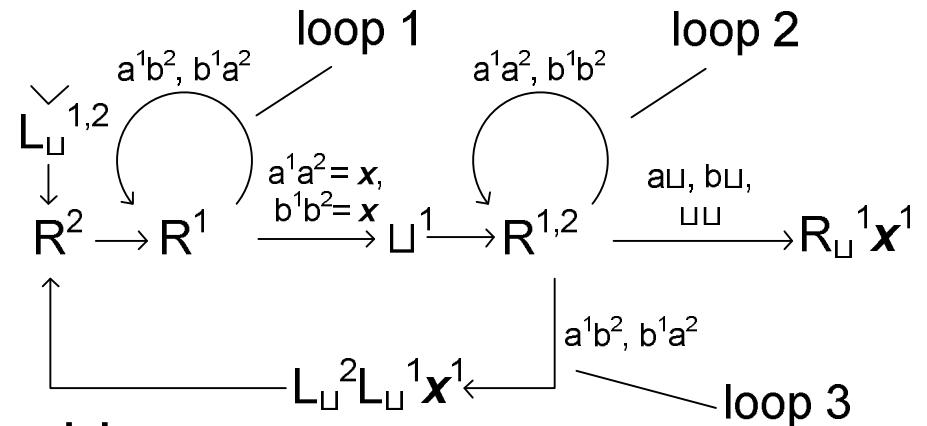


- Computation:

- aaaaabbb, bbb
- aaaabbb, bbb
- aaaabbb, bbb
- aaaabb, bbb
- aaaabb, bbb
- aaaabb, bbb



- aabaabb, abab
- aabaabb, abab
- aaabaabb, abab // $\rightarrow aa, bb$
- aaaabaabb, aabab // $\downarrow ab, ba$
- aaabaabb, abab
- aaabaabb, abab
- aaabaabb, abab // $\rightarrow aa, bb$
- aaabaabb, aaabab
- aaabbaabb, aaaabab
- aaaabbaabb, aaaaabab // $\rightarrow a\underline{a}, b\underline{a}, \underline{a}\underline{a}$
- aaabaabb, abab



Example 19: Search substring

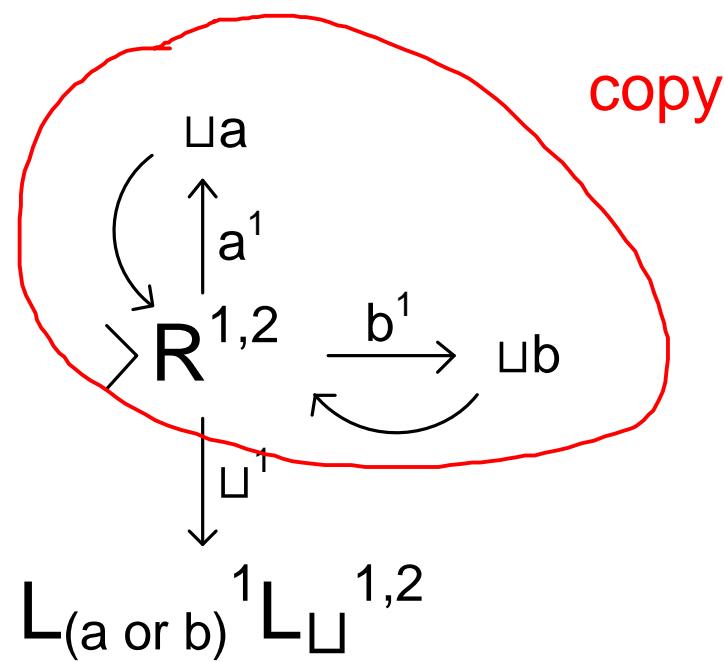
- Homework:
 - give the computation for:
 - bbaababa, bab
 - bbaababa, abba
 - how does the machine stop if the substring is not in the string?
 - give a similar machine with $\Sigma = \{\triangleright, \sqcup, a, b, c\}$

Example 20: Split string

- Create a machine schema, which can split a string into 2 tapes based on the head position!
 - $\Sigma = \{\triangleright, \sqcup, a, b\}$
 - the input must be at least 2 character long
 - after the machine finished
 - the first tape should contain the input from the first character until the head (including the head position)
 - the second tape should contain the rest
 - e.g.: aabab, → aa, bab

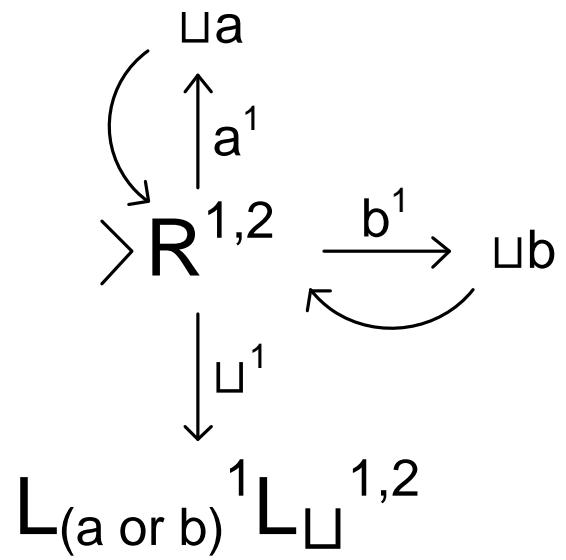
Example 20: Split string

- Create a machine schema, which can cut a string to 2 tapes!



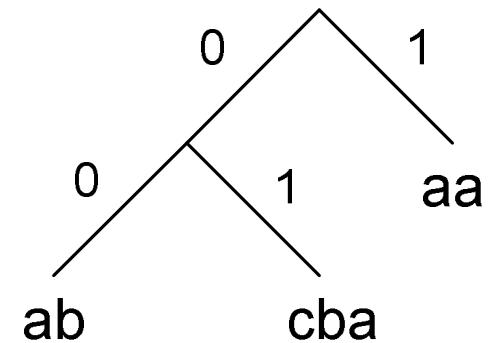
Example 20: Split string

- Computation:
 - aabab,
 - aabab, // $\rightarrow b^1$
 - aaab, b
 - aaab, b // $\uparrow a^1$
 - aaab, ba
 - aaab, ba // $\rightarrow b^1$
 - aaa, ba
 - aa, ba
 - aa, ba



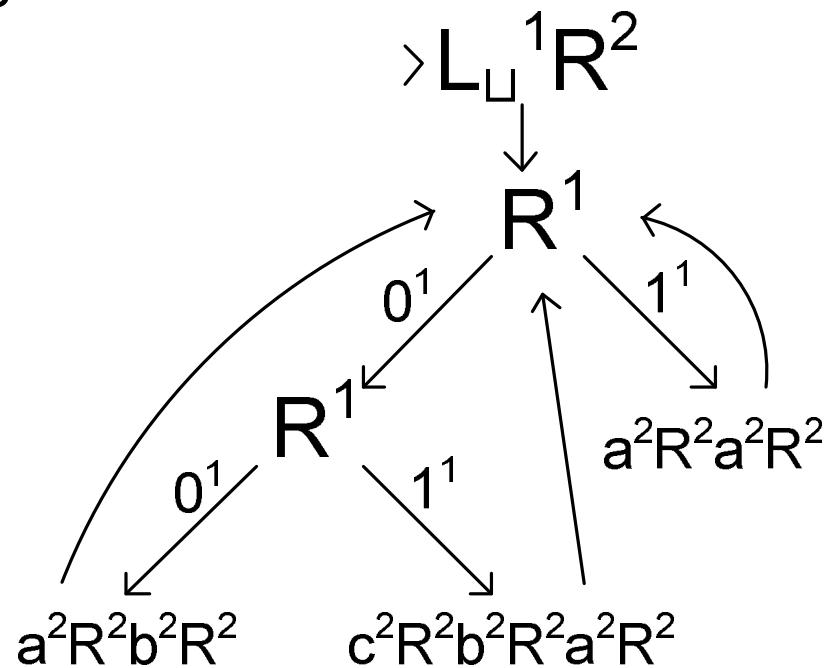
Example 21: Huffman decoding

- Create a machine schema, which can decode a Huffman encoded string!
 - tape 1 contains the coded string, tape 2 will contain the decoded string
 - decoding: change the code to string
 - 1 → aa
 - 00 → ab
 - 01 → cba
 - the creation of the tree is not detailed here
 - e.g.: 101001, → 101001, aacbaabaa



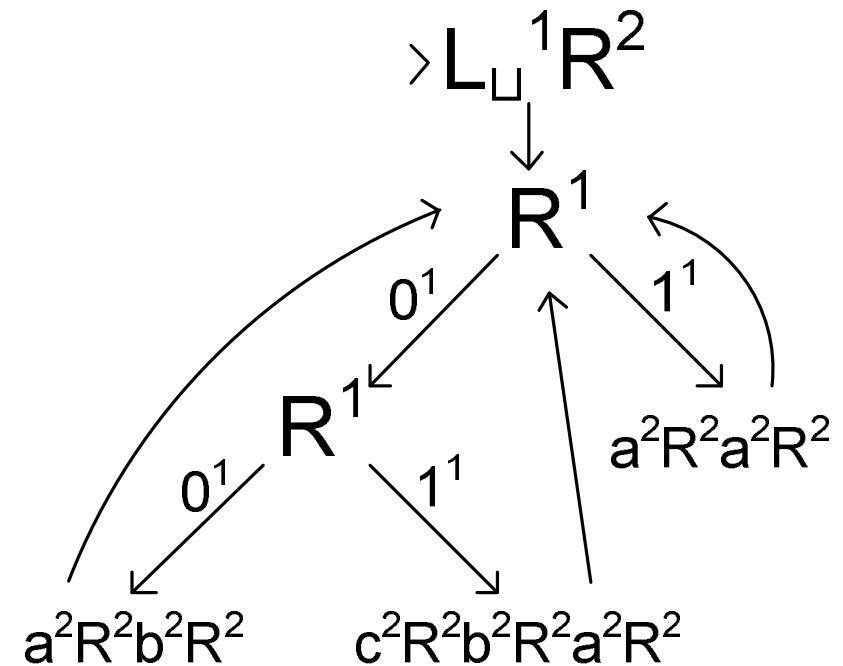
Example 21: Huffman decoding

- Create a machine schema, which can decode a Huffman encoded string!



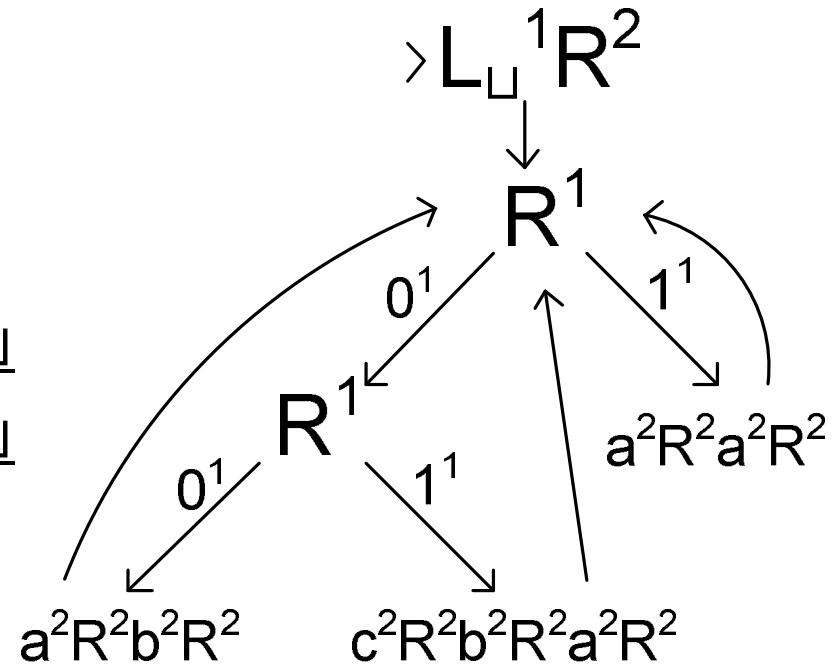
Example 21: Huffman decoding

- Computation:
 - $\sqcup 101001 \sqcup, \sqcup \sqcup$
 - $\sqcup 101001 \sqcup, \sqcup \sqcup$
 - $\sqcup 101001 \sqcup, \sqcup aa \sqcup$
 - $\sqcup 101001 \sqcup, \sqcup aa \sqcup$
 - $\sqcup 101001 \sqcup, \sqcup aa \sqcup$
 - $\sqcup 101001 \sqcup, \sqcup aacba \sqcup$



Example 21: Huffman decoding

- $\llbracket 101\underline{001} \rrbracket$, $\llbracket aacba \rrbracket$
- $\llbracket 1010\underline{01} \rrbracket$, $\llbracket aacba \rrbracket$
- $\llbracket 10100\underline{1} \rrbracket$, $\llbracket aacbaab \rrbracket$
- $\llbracket 10100\underline{1} \rrbracket$, $\llbracket aacbaab \rrbracket$
- $\llbracket 10100\underline{1} \rrbracket$, $\llbracket aacbaabaa \rrbracket$
- $\llbracket 10100\underline{1} \rrbracket$, $\llbracket aacbaabaa \rrbracket$

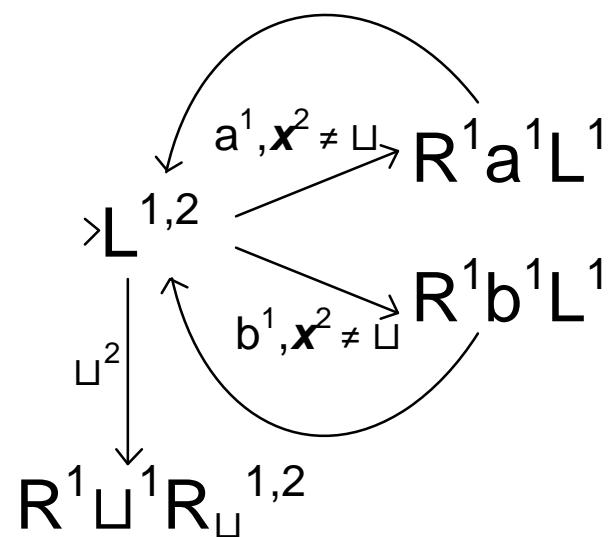


Example 22: Shifting machine ($Sh_{\rightarrow}^{1,2}$)

- Construct a machine schema, which can shift to the right the last n character of a string in tape 1!
 - $\Sigma_1 \in \{a, b\}$
 - n is determined by the length of an other string in tape 2
 - the newly inserted character is the blank
 - e.g.: $\sqcup a a a b a \sqcup, \sqcup^{**} \sqcup \rightarrow \sqcup a a a \sqcup b a \sqcup, \sqcup^{**} \sqcup$
 - $n = 2$

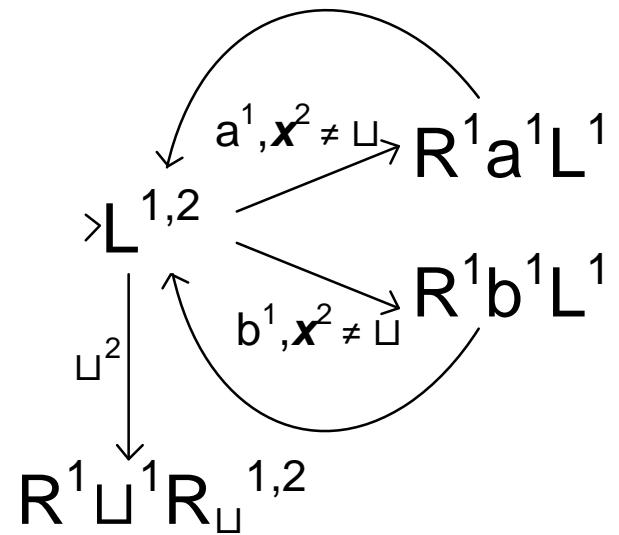
Example 22: Shifting machine ($\text{Sh}_{\rightarrow}^{1,2}$)

- Construct a machine schema, which can shift to the right the last n character of a string in tape 1!



Example 22: Shifting machine ($\text{Sh}_{\rightarrow}^{1,2}$)

- Computation:
 - $\sqcup \text{aba} \sqcup, \sqcup^{**} \sqcup$
 - $\sqcup \text{aba} \sqcup, \sqcup^{**} \sqcup // \rightarrow a^1$
 - $\sqcup \text{abaa} \sqcup, \sqcup^{**} \sqcup$
 - $\sqcup \text{abaa} \sqcup, \sqcup^{**} \sqcup // \rightarrow b^1$
 - $\sqcup \text{abba} \sqcup, \sqcup^{**} \sqcup$
 - $\sqcup \text{abba} \sqcup, \sqcup^{**} \sqcup // \downarrow \sqcup^2$
 - $\sqcup a \sqcup \text{ba} \sqcup, \sqcup^{**} \sqcup$

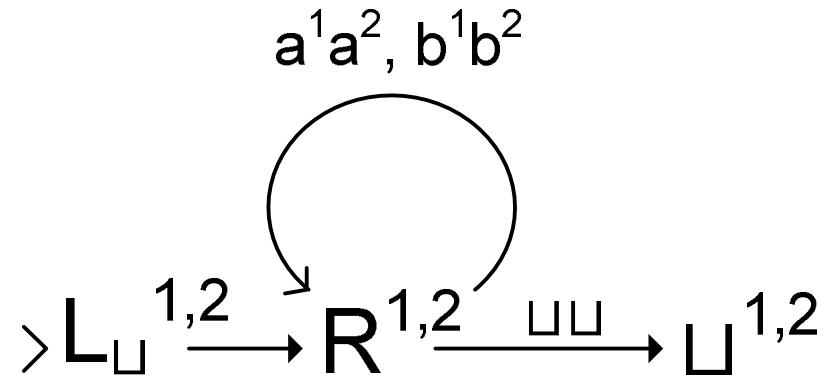


Example 23: Compare machine (T_{CMP})

- Create a machine schema, which can compare 2 strings!
 - $\Sigma_1 \in \{a, b\}$
 - if the two strings match then position the heads after the strings on the spaces, otherwise the head should be somewhere on the strings
 - a machine schema can check if the heads are on spaces ($\rightarrow \sqcup \sqcup$)
 - e.g.: $\sqcup aba \sqcup \sqcup$, $\sqcup aba \sqcup \sqcup \rightarrow \sqcup aba \sqcup \sqcup$, $\sqcup aba \sqcup \sqcup$

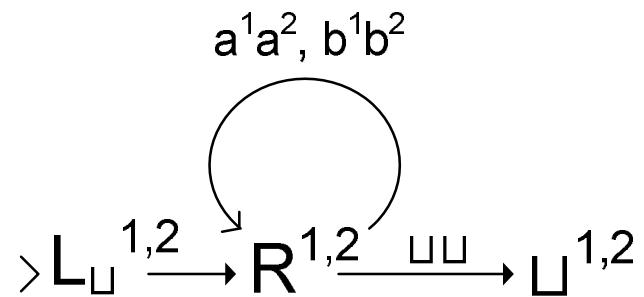
Example 23: Compare machine (T_{CMP})

- Create a machine schema, which can compare 2 strings!
 - the arc " $\rightarrow \sqcup \sqcup$ " is there to sign that, if we go through this arc then the two string equal



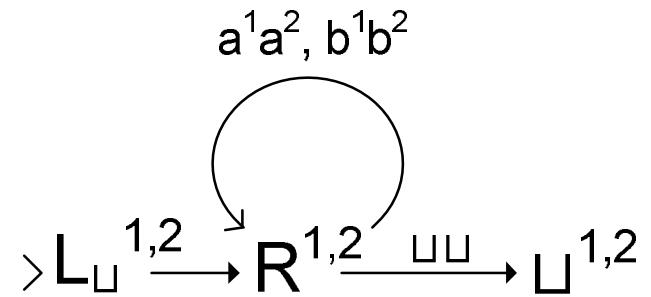
Example 23: Compare machine (T_{CMP})

- Computation:
 - $\sqcup \underline{aba} \sqcup, \sqcup \underline{aba} \sqcup$
 - $\sqcup \underline{aba} \sqcup, \sqcup \underline{aba} \sqcup // \rightarrow \sqcup \sqcup$
 - $\sqcup \underline{aba} \sqcup, \sqcup \underline{aba} \sqcup$



Example 23: Compare machine (T_{CMP})

- Computation:
 - $\sqcup ab\sqcup, \sqcup aa\sqcup$
 - $\sqcup ab\sqcup, \sqcup aau\sqcup$
 - $\sqcup ab\sqcup, \sqcup aau\sqcup$
 - $\sqcup ab\sqcup, \sqcup aau\sqcup // \text{string rejected}$



Example 24: Accept string zzz

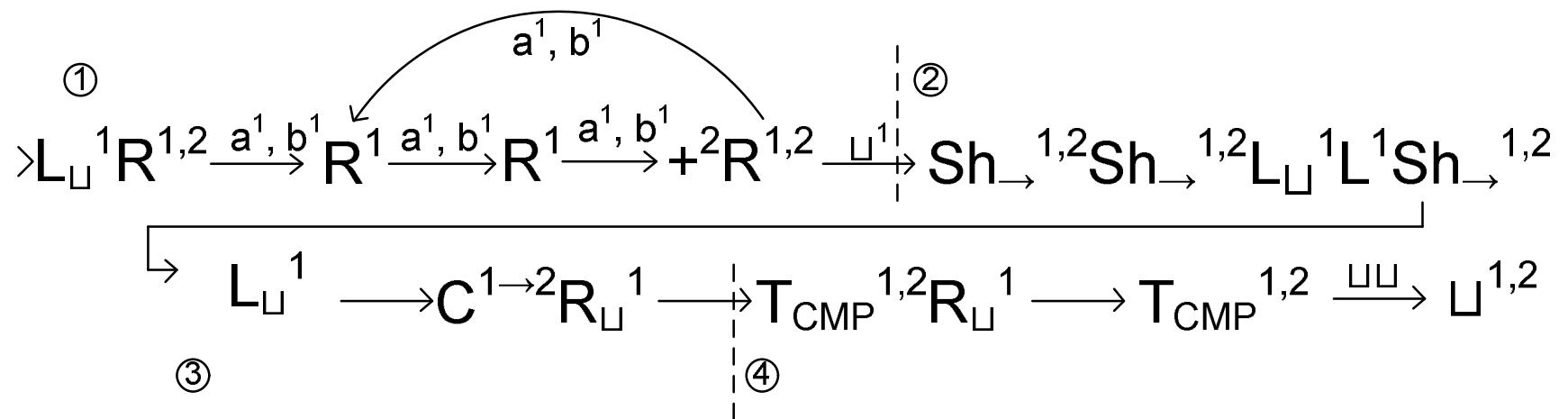
- Create a machine schema, which can accept
 $L(M) = \{w \in \{a, b\}^* \mid w = zzz\}$!
 - if the input is in the zzz form then position the head after the input on the space
 - a machine schema can check if the head is on space ($\rightarrow \sqcup$)
 - you can modify the input and you can use a second tape
 - e.g.: abababb, b → abababb, b++abb

Example 24: Accept string zzz

- beside the usual machines, the following machines can be also used
 - T_{CMP} : compare two strings on two tapes
 - aba, aba → aba, aba
 - $C^{1 \rightarrow 2}$: copy a string from tape 1 to tape 2
 - bab, → bab, bab
 - $Sh_{\rightarrow}^{1,2}$: shifts a string on tape 1 based on the length of another string on tape 2
 - aaaab, ** → aaaba, **

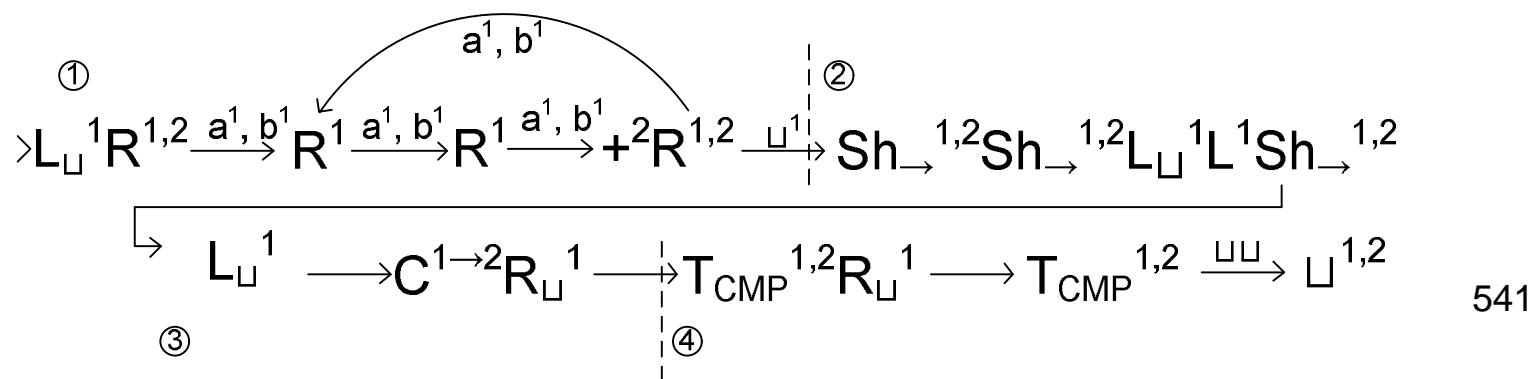
Example 24: Accept string zzz

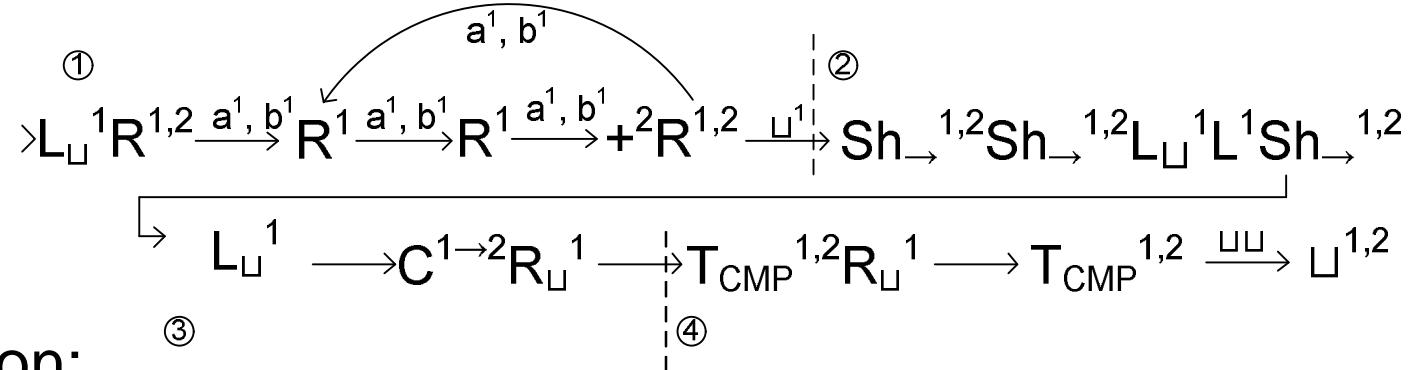
- Create a machine schema, which can accept
 $L(M) = \{w \in \{a, b\}^* \mid w = zzz\}$!



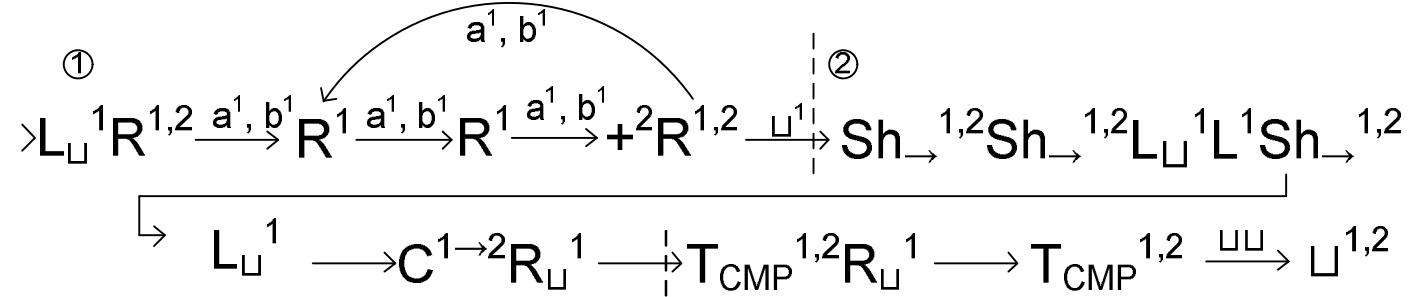
Example 24: Accept string zzz

- Logic of the operation:
 - block 1 write a '+' on tape 2 after 3 right steps on tape 1
 - computation halts if the input's length cannot be divided by 3
 - block 2 splits the string to $\sqcup z \sqcup z \sqcup z \sqcup$ using $Sh_{\rightarrow}^{1,2}$
 - block 3 copies the first substring to the second tape
 - block 4 compares the second and the third substring with the first





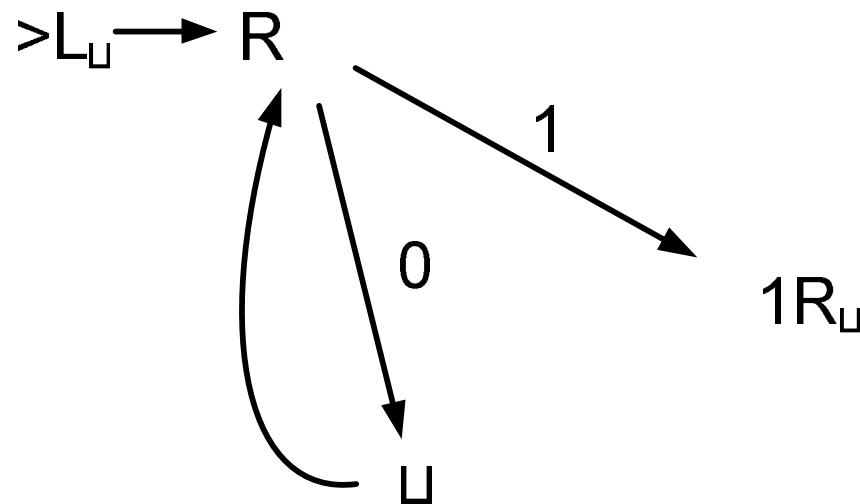
- Computation:
 - ababab\sqcup, \sqcup
 - ababab, \sqcup\sqcup
 - abbabab, \sqcup\sqcup
 - ababab, \sqcup\sqcup
 - ababab, \sqcup+ \sqcup
 - ababab, \sqcup+ \sqcup
 - ababab, \sqcup+ \sqcup
 - ababab\sqcup, \sqcup+ + \sqcup // block 1 ends



- ababab\sqcup, \sqcup++\sqcup
- ababab\sqcup, \sqcup++\sqcup
- abab\sqcupab\sqcup, \sqcup++\sqcup
- abab\sqcupab\sqcup, \sqcup++\sqcup
- ab\sqcupab\sqcupab\sqcup, \sqcup++\sqcup // block 2 ends
- ab\sqcupab\sqcupab\sqcup, \sqcup++\sqcup
- ab\sqcupab\sqcupab\sqcup, \sqcup++\sqcupab\sqcup
- ab\sqcupab\sqcupab\sqcup, \sqcup++\sqcupab\sqcup // block 3 ends
- ab\sqcupab\sqcupab\sqcup, \sqcup++\sqcupab\sqcup
- ab\sqcupab\sqcupab\sqcup, \sqcup++\sqcupab\sqcup
- ab\sqcupab\sqcupab\sqcup, \sqcup++\sqcupab\sqcup // block 4 ends

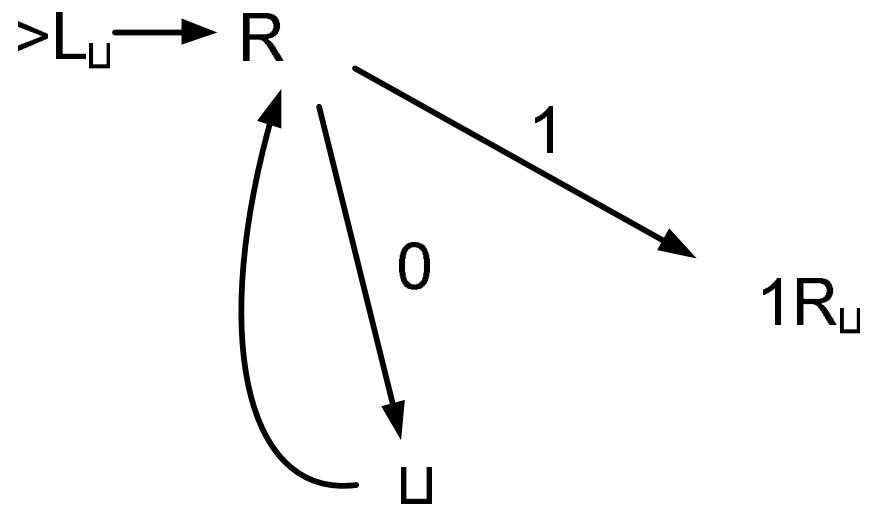
Example 25

- Create a machine schema, which erases all "0"-s from the left side of a binary number!
 - $\Sigma_1 \in \{0, 1\}$
 - e.g.: $\sqcup 000111010 \sqcup \rightarrow \sqcup \sqcup \sqcup \sqcup 111010 \sqcup$



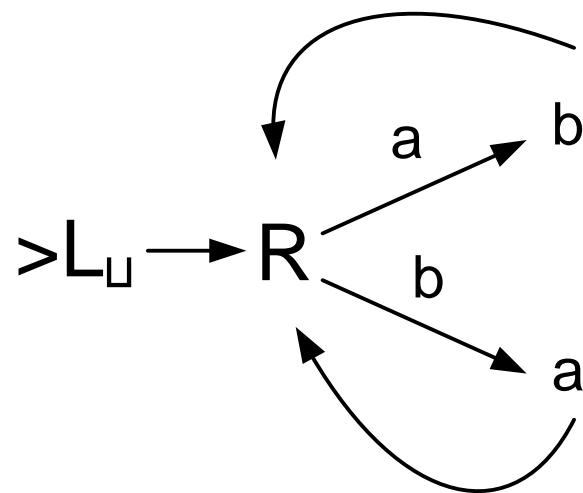
Example 25

- Computation:
 - 000101110
 - 000101110
 - 000101110
 - 00101110
 - 00101110
 - 000101110
 - 000101110
 - 000101110
 - 000101110



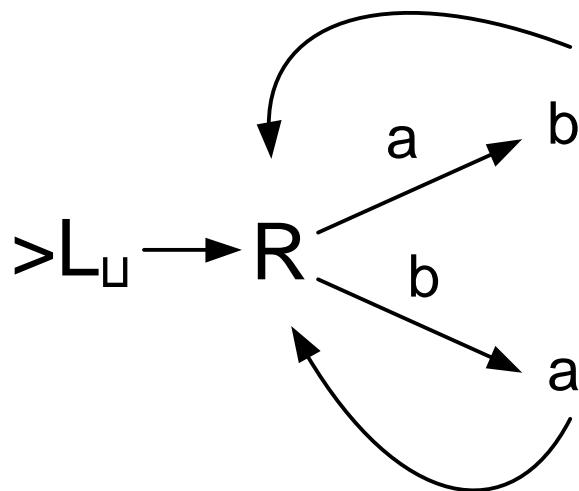
Example 26

- Create a machine schema, which switches all "a" with "b" and vice versa in a string!
 - $\Sigma_1 \in \{a, b\}$
 - e.g.: ababb \rightarrow babaa



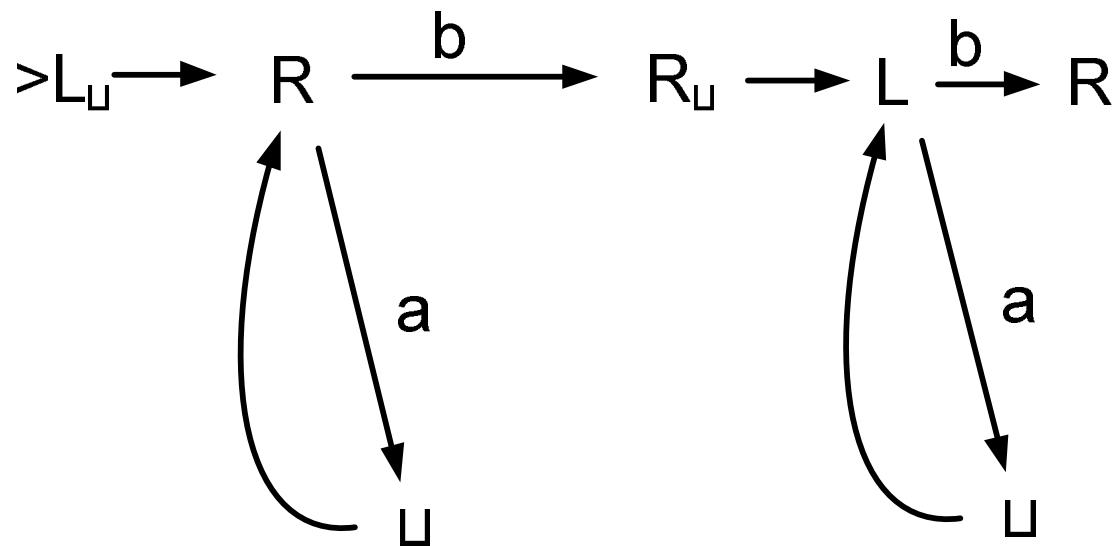
Example 26

- Computation:
 - abbb
 - abbb
 - abbb
 - bbbb
 - bbbb
 - babb
 - babbb
 - baaab
 - baab
 - baaaa
 - baaa



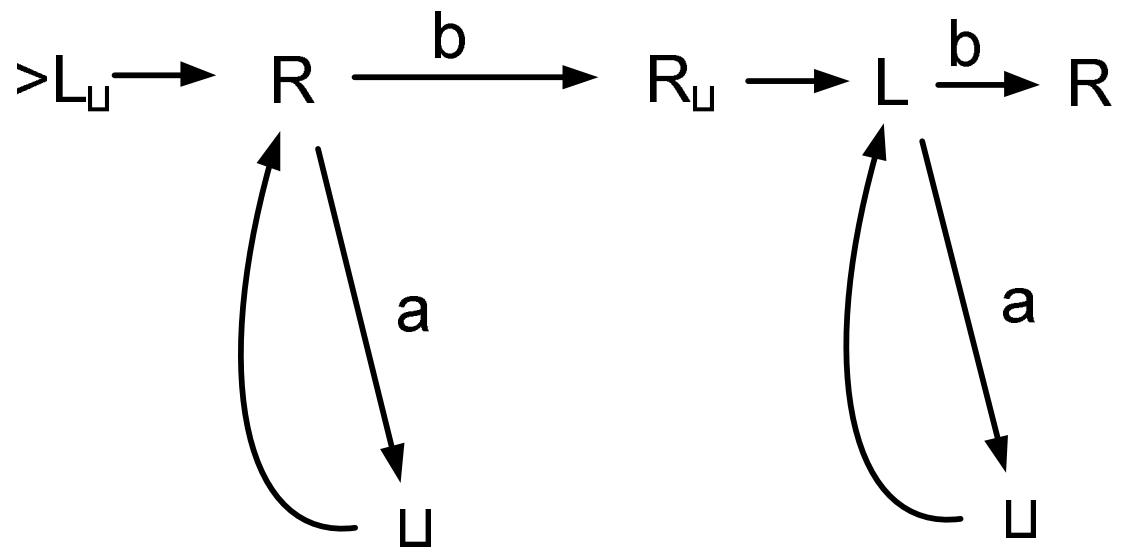
Example 27

- Create a machine schema, which erases all 'a' from the beginning and the end of the input string!
 - $\Sigma_1 \in \{a, b\}$
 - e.g.: abba \rightarrow bb



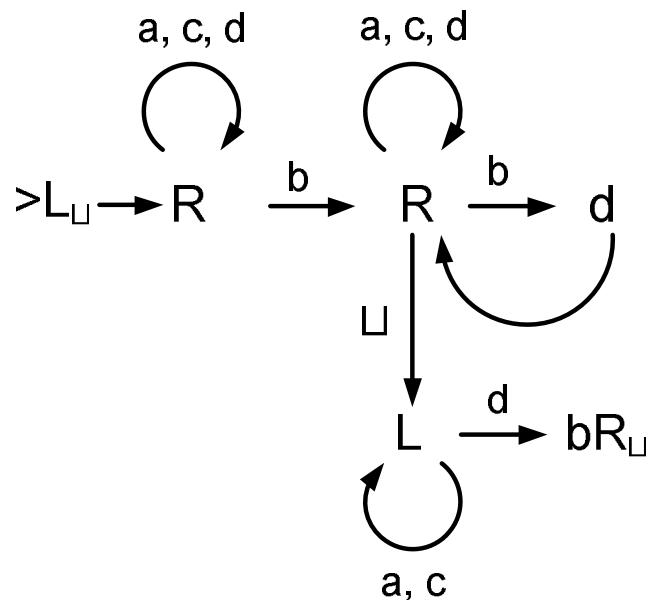
Example 27

- Computation:
 - abba
 - abba
 - aabba
 - abba
 - abba
 - abba
 - abbaa
 - abb
 - abbb
 - abbb



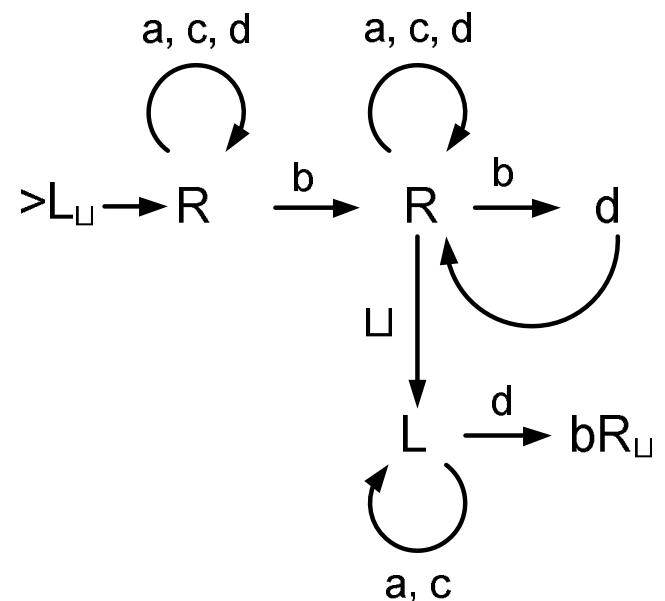
Example 28

- Create a machine schema, which changes all 'b' to 'd' in the input string, except the first and the last ones!
 - $\Sigma_1 \in \{a, b, c, d\}$
 - e.g.: $\sqcup abcbbb\sqcup \rightarrow \sqcup abcd\!d\!bc\sqcup$



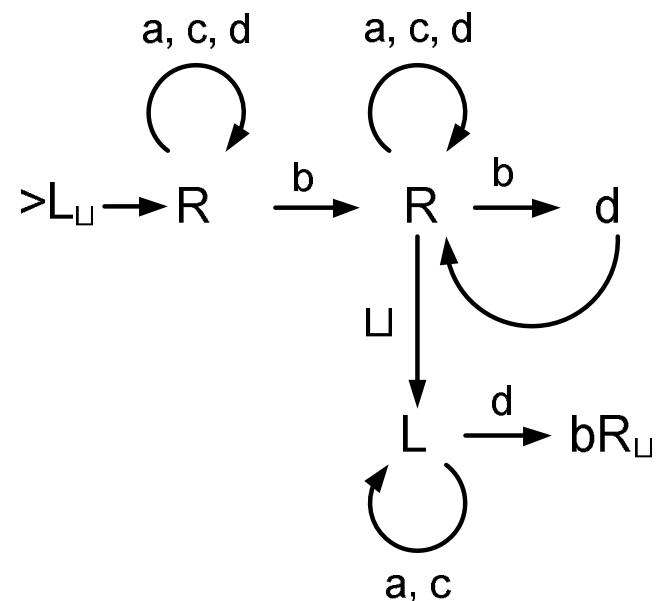
Example 28

- Computation:
 - $\sqcup \underline{abc}bbb\mathbf{c}\sqcup$
 - $\sqcup abcbbb\mathbf{c}$
 - $\sqcup \underline{abc}bbb\mathbf{c}$
 - $\sqcup a\underline{bc}bbb\mathbf{c}$
 - $\sqcup ab\underline{c}bbb\mathbf{c}$
 - $\sqcup abc\underline{\mathbf{b}}bbb\mathbf{c}$
 - $\sqcup abcdbbb\mathbf{c}$
 - $\sqcup abcdb\underline{\mathbf{b}}bc$
 - $\sqcup abcdd\underline{\mathbf{b}}bc$



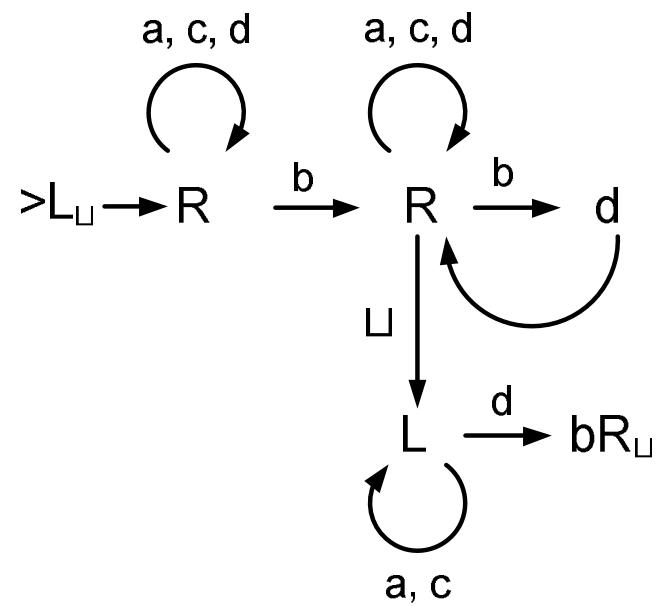
Example 28

- abcddbc
- abcdddc
- abcdddc
- abcdddcL
- abcdddcc
- abcdddc
- abcddbc
- abcddbcL



Example 28

- Is this machine correct always?
 - not if the input contains d
 - e.g.: adda → adba

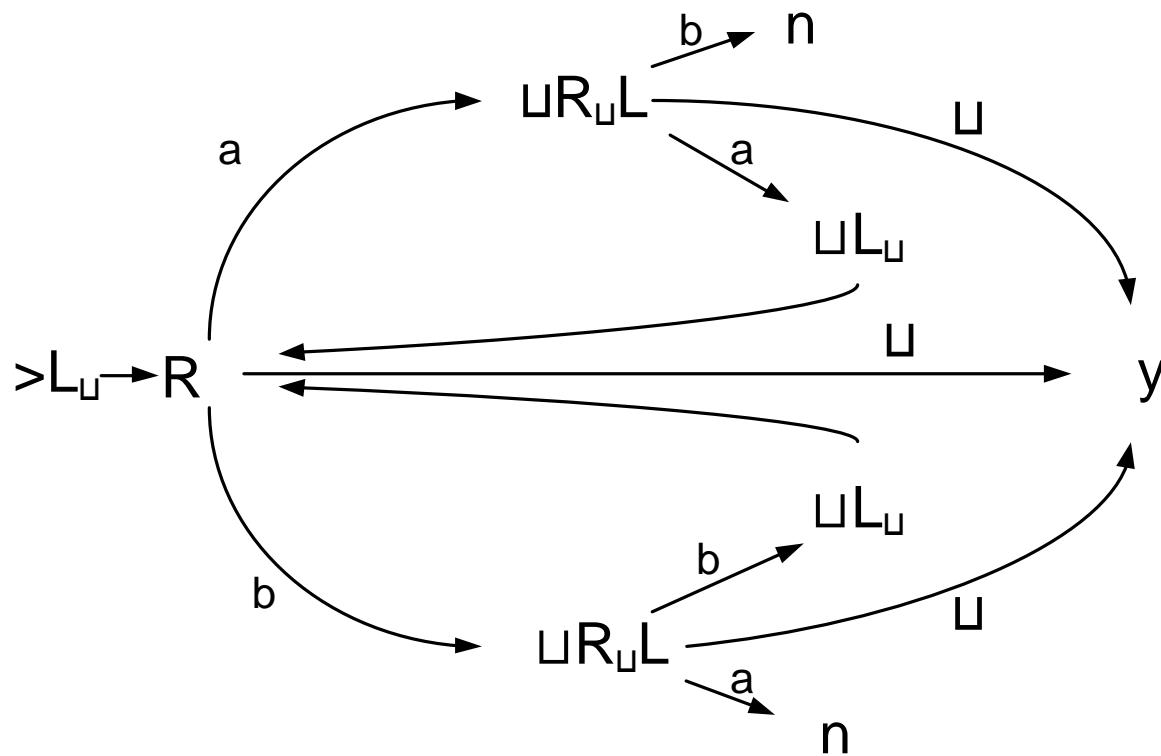


Example 29

- Create a machine schema, which decides
 $L = \{w \in \Sigma^* \mid w = w^R\}$
 - $\Sigma_1 \in \{a, b\}$
 - if the decision is yes then perform the y machine
 - if the decision is no then perform the n machine
 - the input can be modified
 - e.g.: aabbba \rightarrow y
 - y is the halting machine

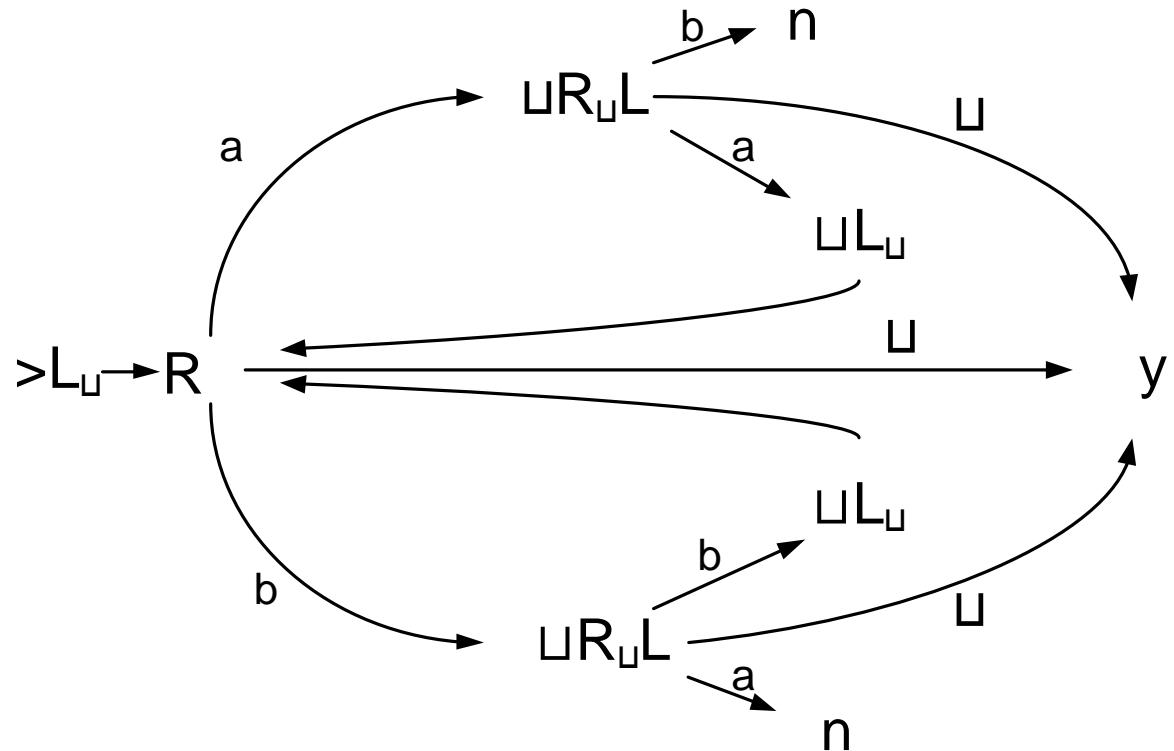
Example 29

- Create a machine schema, which decides
 $L = \{w \in \Sigma^* \mid w = w^R\}!$



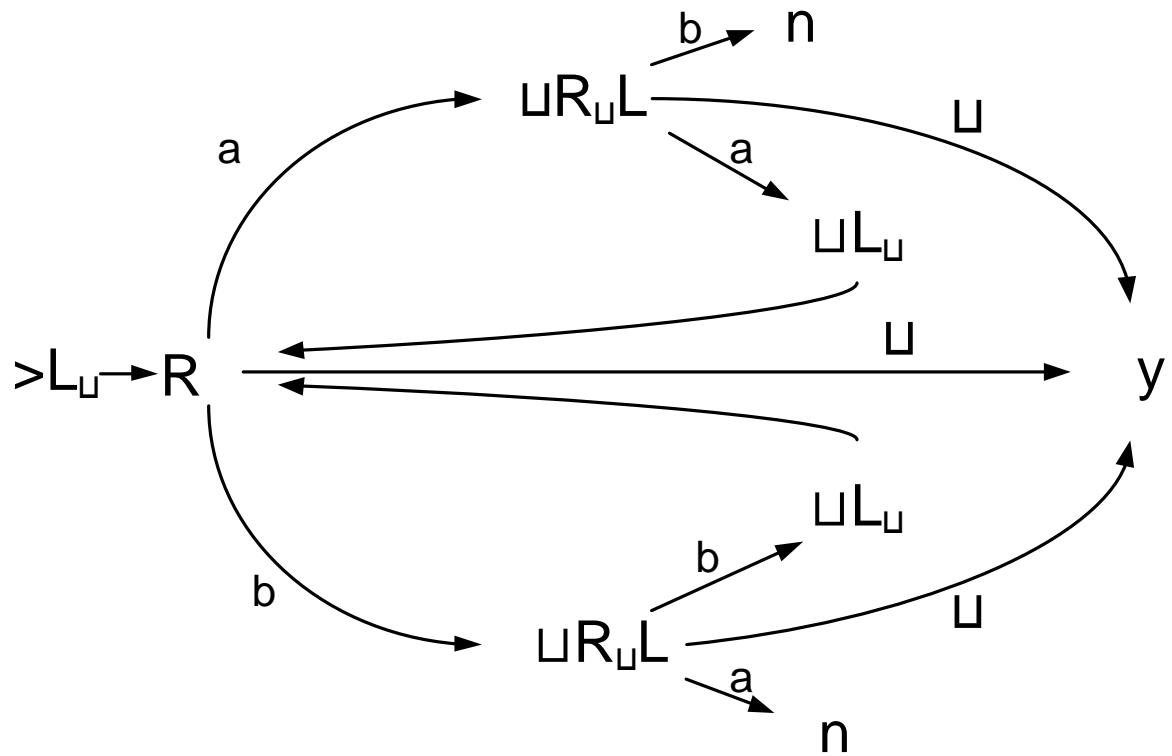
Example 29

- Computation:
 - abba
 - abba
 - abba
 - bba
 - bba
 - bbaa
 - bb
 - bb
 - bb



Example 29

- uub
- uubu
- uub
- uuu
- uu
- uuu // → u
- uuu

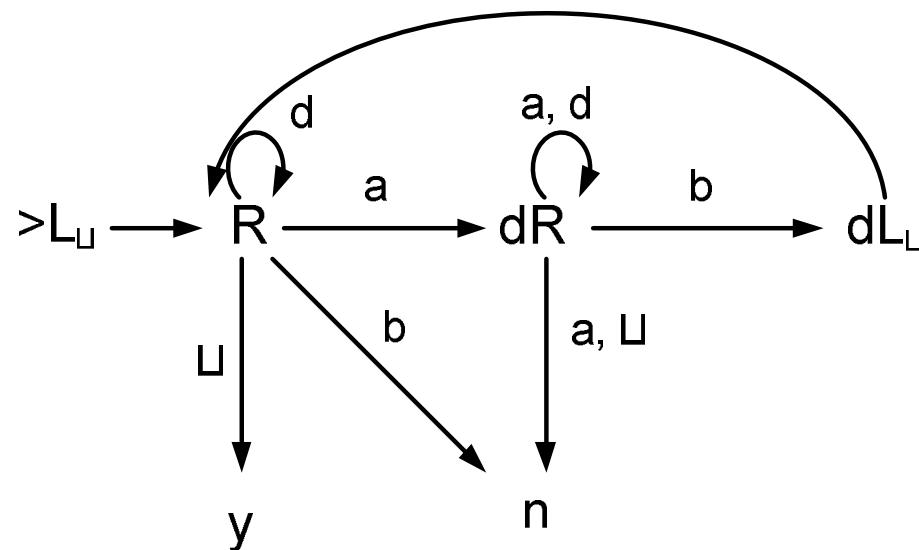


Example 30

- Create a machine schema, which decides
 $L = \{w \in \{a, b\}^* \mid w = a^n b^n\}$
 - if the decision is yes then perform the y machine
 - if the decision is no then perform the n machine
 - the input can be modified
 - e.g.: aabb → y
 - y is the halting machine

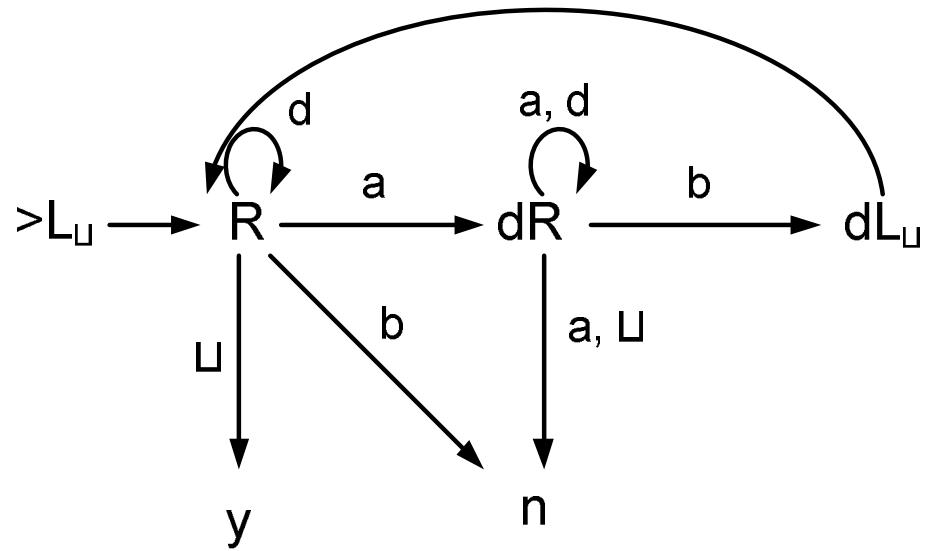
Example 30

- Create a machine schema, which decides
 $L = \{w \in \{a, b\}^* \mid w = a^n b^n\}$!



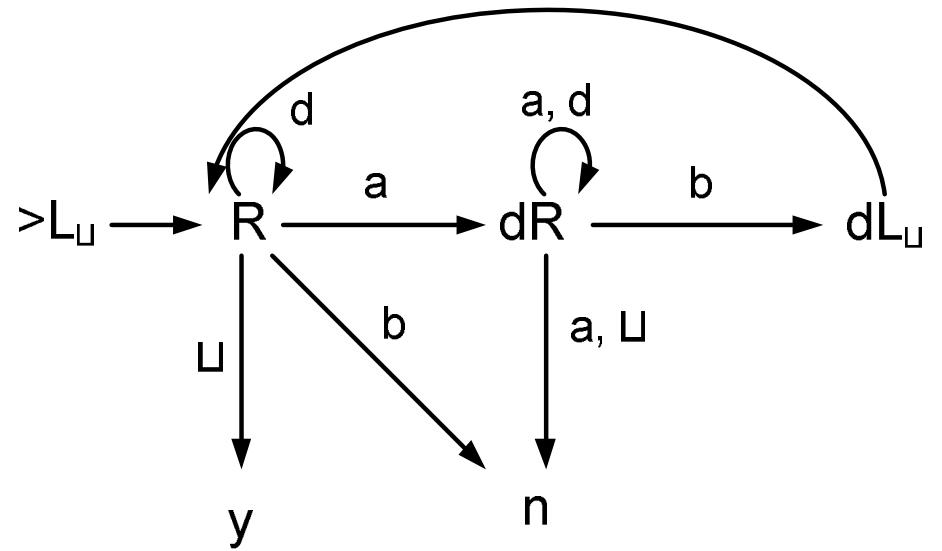
Example 30

- Computation:
 - aabb
 - aabb
 - aabb
 - dabb
 - dabb
 - dabb
 - dadb
 - dad**b**



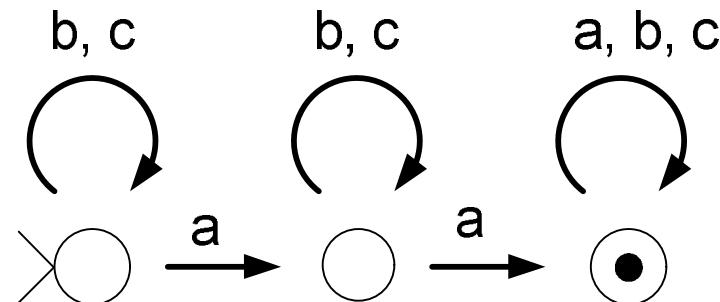
Example 30

- $\sqcup \underline{dadb}$
- $\sqcup d\underline{adb}$
- $\sqcup \underline{dddb}$
- $\sqcup \underline{dddb} // \downarrow \sqcup$
- $\sqcup \underline{dddb} \sqcup$

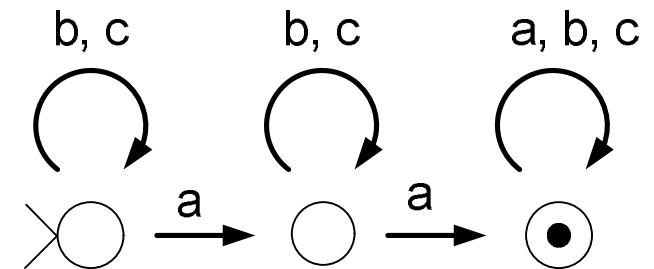


Example 31

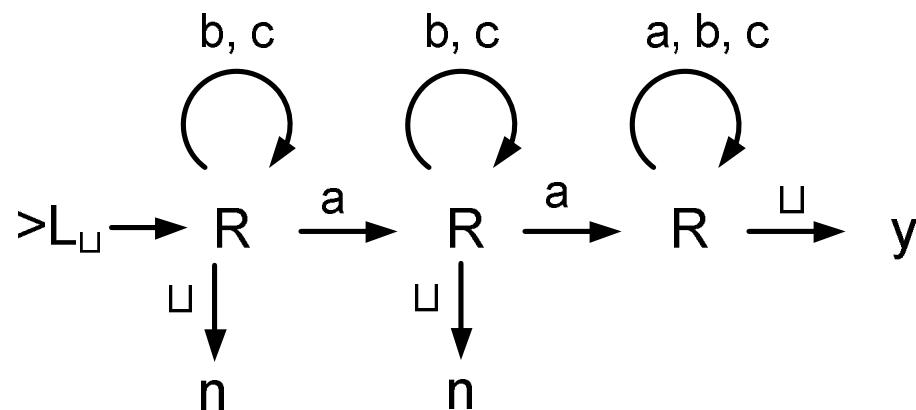
- Create a machine schema, which decides the language of the DFA below!
 - if the decision is yes then perform the y machine
 - if the decision is no then perform the n machine
 - the schema should start from the left side of the input
 - $L = \{w \in \{a, b, c\}^* \mid \#a \geq 2\}$



Example 31



- Create a machine schema, which decides the language of the DFA below!
 - $L = \{w \in \{a, b, c\}^* \mid \#a \geq 2\}$



Example 31

- Computation:
 - abaca
 - abaca
 - abaca // →a
 - abaca
 - abaca // →a
 - abaca
 - abaca
 - abaca // →
 - abaca

